ON INDEX ASSIGNMENT AND THE DESIGN OF MULTIPLE DESCRIPTION QUANTIZERS

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ABSTRACT

The practical design of multiple description quantizers for diversity-based communication is investigated. A simulated annealing based method is proposed for obtaining the optimal index assignment for a multiple-description vector quantizer. This method can be used to design quantizers having an arbitrary number of descriptions with equal or unequal transmission rates. According to the simulation results, the proposed method yields multiple-description quantizers with performance comparable with or better than previously reported results.

1. INTRODUCTION

The main idea of multiple description (MD) source coding [1] is to encode each block of source samples into two or more different descriptions so that diversity-based communication is possible over channels prone to break down. MD coding, including MD vector quantization (MDVQ), has gained much attention recently due to potential applications in communication over packet networks and fading channels. The design of MD quantizers was first studied in [2], where an algorithm for designing MD scalar quantizers (MDSQ) together with two heuristic procedures for choosing a good index assignment (IA) based on asymptotic (high rate) results were presented. The design algorithm, which is a generalization of the well known Lloyd algorithm for quantizer design, is guaranteed to converge to a locally optimal solution. Even though, the extension of this algorithm to MDVQ was considered in [3], the IA selection procedures suggested in [2] did not extend to VQ and no method was suggested for choosing good IA for MDVQ. In general, the IA problem is a combinatorial optimization problem in which the number of possible solutions precludes an exhaustive search over the solution space. The problem is similar to the IA problem arising in noisy channel vector quantizers wherein search algorithms such as simulated annealing (SA) [4] and binary switching algorithm (BSA) have shown to produce good IA. Previously BSA algorithm has also shown to produce good IA for MDVQ [5]. A different approach to MDVQ design, based on deterministic annealing (DA) was presented in [6]. Simulation results for both MDSQ for iid Gaussian source and MDVQ for Gauss-Markov source were presented in [6].

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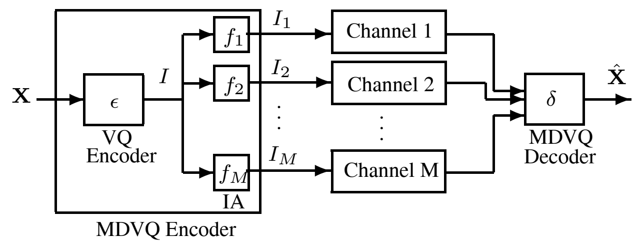

Fig. 1. *A communication system based on MDVQ.*

The main contribution of this paper is a study of MDVQ design based on optimal IA obtained by SA . This method may be used to optimize IA of a fixed MDVQ or to obtain initial IA for iterative design of MD quantizers as proposed in [2] and [3]. It is our observation that this approach provides an effective and a relatively simple way to design unstructured MDVQ with an arbitrary number of descriptions. We compare our results with other known work, including MDSQ designs in [2], MDVQ in [6], and optimal IA in [5].

2. PROBLEM FORMULATION

A block diagram of an MDVQ system is shown in Fig. 1. Let $\mathbf{X} \in \mathbb{R}^k$ be a stationary random ergodic process with pdf $p(\mathbf{x})$. This process is to be quantized and communicated over a diversity system with M independent channels having rates R_m, $m = 1, \ldots, M$. Each channel in this system has two random states: working perfectly (no errors) or broken down (no output). The encoder generates M descriptions for each source vector $\mathbf{X}$ which are transmitted over the M channels. For MD coding to be useful, the M descriptions representing a source vector must not be identical. It is assumed that the encoder has no knowledge as to which of the M channels are functioning at a given time. On the other hand, the decoder can identify the faulty channels and it forms an estimate for the source vector using only the descriptions that it receives over the functioning channels. The MDVQ encoder can be described by two mappings as shown in Fig. 1. The first is a partition of the space of input vector into N non-overlapping cells $\epsilon : \mathbb{R}^k \rightarrow \{1, 2, \ldots, N\}$. The second mapping is the IA in which each cell in the encoder partition is assigned M indices (codewords). That is, given $I = \epsilon(\mathbf{X})$, M indices are generated, $I_m = f_m(I)$, $m = 1, \ldots, M$, where $I_m \in \{1, \ldots, N_m\}$ and $N_m = 2^{kR_m}$.

In this paper, we will focus on the two channel case $M = 2$, for simplicity (the extensions required for $M > 2$ are straightforward). As usual, the mean square error (MSE) will be used to measure the quantizer distortion. Let the encoder ϵ be defined by N cells Ω_i, $i = 1, \ldots, N$, where $\bigcup_i^N \Omega_i = \mathbb{R}^k$ and $\bigcap_i^N \Omega_i = \emptyset$. Let $\mathbf{c}(i)$, $i = 1, \ldots, N$ represent the decoder codebook to be used when both channels are working and $\mathbf{c}_m(j)$, $j = 1, \ldots, N_m$ represent the decoder codebook to be used when only the channel m is working, $m = 1, 2$. Also, let μ be the probability that both descriptions are received, and μ_m be the probability that only m^{th} description is received. These probabilities are easily obtained if the failure (or loss) probability of each channel is given. We will assume that when both channels fail, re-transmissions are performed until at least one description is received. Then, the MSE of the MDVQ system is given by

$$D = \mu D_0 + \mu_1 D_1 + \mu_2 D_2 \tag{1}$$

where $D_0 = E\|\mathbf{X} - \mathbf{c}(i)\|^2$, $D_1 = E\|\mathbf{X} - \mathbf{c}_1(i_1)\|^2$, and $D_2 = E\|\mathbf{X} - \mathbf{c}_2(i_2)\|^2$, with $i_1 = f_1(i)$ and $i_2 = f_2(i)$ being the output of MDVQ encoder for the input $\mathbf{x}$ such that $\epsilon(\mathbf{x}) = i$. The term D_0 represents the *central distortion* whereas the term $\mu_1 D_1 + \mu_2 D_2$ represents the *side-distortion*. Given the rate constraints R_1 and R_2, an optimal MDVQ is defined as the one which minimizes D in (1). As any other VQ optimization problem, the closed form solution of the MDVQ design problem is difficult. On the other hand, iterative procedures similar to the Lloyd algorithm has been used to design locally optimal MD quantizers [2], [3]. The necessary conditions for the optimality of the encoder and the decoder are given below.

Optimal Decoder- Given an encoder ϵ, and IA functions f_1 and f_2, the optimal code vectors are the minimum MSE (MMSE) source estimates given by

$$\mathbf{c}^*(i) = E\{\mathbf{X}|i\}, \tag{2}$$

$$\mathbf{c}_m^*(i_m) = E\{\mathbf{x}|i_m\},\ m = 1, 2 \tag{3}$$

where $i = 1, \ldots, N$ and $i_m = 1, \ldots, N_m$. The *central codebook* $\mathbf{c}$ simply consists of centroids of the encoder cells $\Omega_i, i = 1, \ldots, N$. Note that the central codebook is independent of IA. The *side codebooks* can be given in terms of the central codebook as

$$\mathbf{c}_m^*(j) = \sum_{i=1}^{N} \mathbf{c}^*(i) P(I = i | f_m(i) = j), \tag{4}$$

$j = 1, \ldots, N$

Optimal Encoder- Given a set of decoder codebooks $\mathbf{c}$, $\mathbf{c}_1$, and $\mathbf{c}_2$, and IA functions f_1 and f_2, the MSE in assigning a source vector $\mathbf{x}$ to the i^{th} cell of the encoder ϵ is

$$D_i(\mathbf{x}) = \mu\|\mathbf{x} - \mathbf{c}(i)\|^2 + \mu_1\|\mathbf{x} - \mathbf{c}_1(i_1)\|^2 + \mu_2\|\mathbf{x} - \mathbf{c}_2(i_2)\|^2, \tag{5}$$

where $i_1 = f_1(i)$ and $i_2 = f_2(i)$. Then, the optimal encoder is given by (superscript denotes transpose)

$$\epsilon^*(\mathbf{x}) = i \iff \beta_i - 2\mathbf{x}^T\alpha_i \leq \beta_j - 2\mathbf{x}^T\alpha_j\ \forall j \neq i, \tag{6}$$

where $\alpha_i = \mu\mathbf{c}(i) + \mu_1\mathbf{c}_1(i) + \mu_2\mathbf{c}_2(i)$ and $\beta_i = \mu\|\mathbf{c}(i)\|^2 + \mu_1\|\mathbf{c}_1(i)\|^2 + \mu_2\|\mathbf{c}_2(i)\|^2$, $i = 1, \ldots, N$.

The problem of MDVQ design may also be formulated as a constrained optimization problem in which the central distortion is minimized subject to constraints on side distortion [2], [3]. The solution to this problem is obtained by setting $\mu = 1$ and interpreting $\mu_1 = \lambda_1$ and $\mu_2 = \lambda_2$ as Lagrangian multipliers in (1).

3. INDEX ASSIGNMENT IN MDVQ

Note that, in MDVQ, the mapping $F : \{I_1, I_2, \ldots, I_M\} \rightarrow I$ is unique. Hence, the central distortion D_0 in (1) is not a function of IA. However, the mapping from any other sub-set of indices $\{I_j, I_l, \ldots, I_m\}$ to I need not be unique, meaning that if one or more descriptions are lost, the corresponding encoder cell cannot be exactly determined. In this case, the decoder has to produce the best estimate of the source vector based on the received set of indices. The resulting side-distortion can be minimized by the optimal selection of IA. That is, given the encoder ϵ and decoder codebooks $\mathbf{c}$, $\mathbf{c}_1$, and $\mathbf{c}_2$, the optimal IA is obtained by minimizing $\mu_1 D_1 + \mu_2 D_2$ in (1) with respect to f_1 and f_2. This is equivalent to minimizing the cost function

$$J(f_1, f_2) = \sum_{i=1}^{N} \Big\{ [\mu_1\|\mathbf{c}_1(f_1(i))\|^2 - 2\mathbf{g}_i^T\mathbf{c}_1(f_1(i))] + \mu_2[\|\mathbf{c}_2(f_2(i))\|^2 - 2\mathbf{g}_i^T\mathbf{c}_2(f_2(i))] \Big\} P_i, \tag{7}$$

where $\mathbf{g}_i$ is the centroid of Ω_i and $P_i = P(\mathbf{X} \in \Omega_i)$. When the codebooks satisfy the optimality conditions in (2) and (4), (7) simplifies to

$$J(f_1, f_2) = \sum_{i=1}^{N} \big[\mu_1\|\mathbf{c}(i) - \mathbf{c}_1(f_1(i))\|^2 + \mu_2\|\mathbf{c}(i) - \mathbf{c}_2(f_2(i))\|^2\big] P_i. \tag{8}$$

This describes MSE due to using code vectors from the side codebooks instead of the central codebook at the decoder.

Let A be an $N_1 \times N_2$ two-dimensional matrix and call it the *IA matrix* (in M channel problem, this will be an M-dimensional matrix). The mappings $f_1(i)$ and $f_2(i)$ are an assignment of the encoder cell Ω_i to the $(f_1(i), f_2(i))^{th}$ element of the matrix A. In particular, the N non-zero elements of the IA matrix are given by

$$A(f_1(i), f_2(i)) = i, \; i = 1, \ldots, N. \tag{9}$$

If only the diagonal elements are used, we have the ordinary (single description) VQ. On the other hand MDVQ results when non-diagonal elements are also used. When $N < N_1 N_2$, not all elements of A are used and these represent unused IA combinations. Given the two rates R_1 and R_2, and an encoder partition ϵ, there are $(N_1 N_2)!/(N_1 N_2 - N)!$ possibilities for filling the IA matrix. Hence in most cases, it is impractical to find the optimal IA matrix using an exhaustive search. In the following, we propose the use of SA for minimizing the cost function in (7) with respect to instances of the IA matrix. SA has been used in various combinatorial optimization problems including code design, see [4] and the references therein. Apart from the cost function used, the IA optimization algorithm used in this paper is similar to the noisy channel IA optimization algorithm described in [4]. The algorithm, as applied to MDVQ, can be summarized as follows:

1. Set initial temperature $T = T_0$ (a high value); Select an initial IA matrix $A = A_0$.

2. Randomly *perturb* A to A' and evaluate the change ΔJ in the cost function in (7).

3. If $\Delta J < 0$, replace A by A'; If $\Delta J \geq 0$, replace A by A' with probability $exp(-\Delta J/T)$.

4. If the number of iterations which resulted in a cost decrease exceeds a prescribed limit k_l or if too many iterations (k_m) have been performed without any cost decrease goto Step 5); Otherwise goto Step 2).

5. Lower the temperature as $\alpha T \rightarrow T$; If T is below some prescribed freezing temperature T_f or if the current solution appears stable, terminate the algorithm; Otherwise goto Step 2)

The space of candidate solutions in our problem consists of all possible instances of the IA matrix A. Thus, the perturbation of a given solution A to A' can be achieved by interchanging two randomly chosen elements of A. In our implementation, we used $T_0 = 10$, $T_f = 10^{-10}$, $\alpha = 0.98$, $k_l = 5$, and $k_m = 500$.

4. EXPERIMENTAL RESULTS

In this section, the effect of IA optimization on the performance of MD quantizers is investigated by designing two-dimensional ($d = 2$) MDVQ with $M = 2$ descriptions for Gaussian sources. Both an iid source and a first-order Gauss-Markov (GM) source with a correlation coefficient of 0.9 have been considered. The results presented in this section have been obtained using a training set of 400,000 source samples.

Fig. 2 shows the performance of MDVQ designed for iid source as a function of description ("packet") loss probability. In this case, the mean of the source has been used as the decoder output when both descriptions are lost, so that these results can be compared with those presented in [5] (it is worth noting that curve (b) in Fig. 2 appears identical to the performance curve of optimal system in Fig. 4 of [5]). Fig. 3 shows a similar set of results for the GM source. In each case, the initial encoder is a source optimized VQ encoder obtained by the Lloyd algorithm while the initial IA has been picked randomly. The curve (a) in both Figs. 2 and 3 indicate the performance of the initial system. The curve (b) is the performance of the system in which IA has been optimized using the SA algorithm (encoder has not been changed). The curve (c) has been obtained by iteratively optimizing the initial encoder and the codebooks using the optimality conditions given in Sec. 2 (*i.e.* similar to the Lloyd algorithm), without explicit optimization of initial IA. Finally, the system of curve (d) has been obtained by first optimizing the initial IA using the SA algorithm and then iteratively optimizing the encoder partition and the codebooks as in the case of curve (c).

It can be seen that IA optimization alone can yield a good MDVQ. On the other hand, even if the initial IA is randomly picked, improving the encoder partition and decoder codebooks by Lloyd iterations leads to an MD quantizer with somewhat better performance. However, the quantizers obtained by these two approaches are quite different. In the former case, the IA is optimized keeping the encoder partition fixed. Hence, the encoder resolution is not changed and the central distortion remains unchanged. It is the reduced side-distortion that improves the overall performance. In the latter case, the central distortion is traded-off for side distortion by modifying the encoder partition, so as to minimize the overall distortion. In this case, the encoder resolution N^* in the final design can be less than that in the initial system (some cells get merged during the Lloyd iterations), which introduces more redundancy into the transmitted indices. From both figures it can be seen that best performance is obtained when Lloyd iterations are carried-out on a system with optimized IA. However, note that as the loss probability is increased (beyond about 0.4 here), IA optimization alone appears to be sufficient and optimizing the encoder in addition provides no significant advantage.

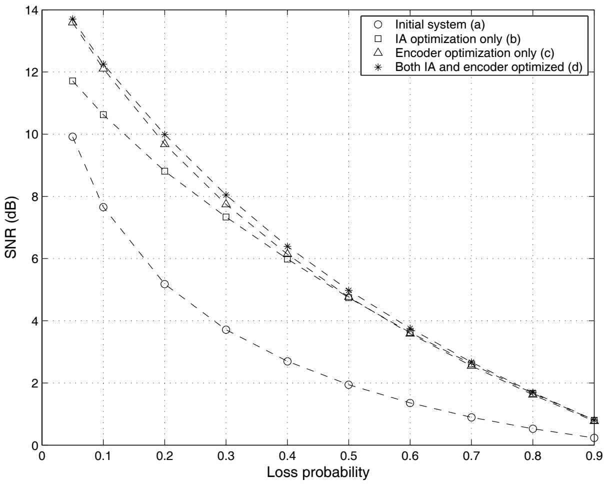

Fig. 2. *Performance of MDVQ for iid Gaussian source ($M = 2$, $k = 2$, $R_1 = R_2 = 2$ bits/sample).*

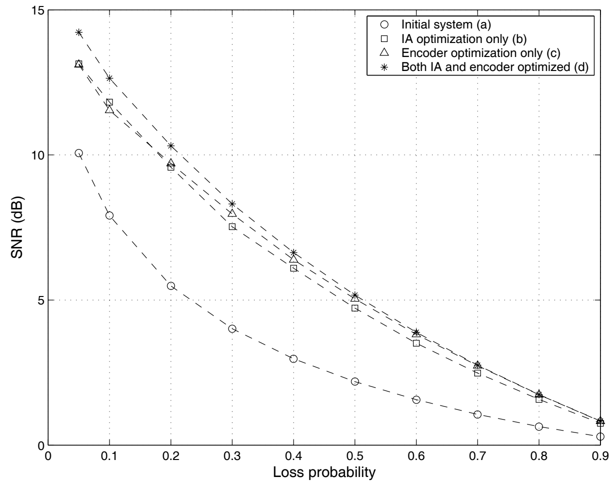

Fig. 3. *Performance of MDVQ for Gauss-Markov source ($M = 2$, $k = 2$, $R_1 = R_2 = 2$ bits/sample).*

At higher loss probabilities, the side distortion (which is a function of IA) dominates over central distortion.

Finally, in Tables 1 and 2, MDSQ and MDVQ with SA based IA (SA-MDSQ and SA-MDVQ respectively) are compared with those obtained by deterministic annealing (DA) as reported in [6]. According to these results, MD quantizers with optimized IA appear to perform somewhat better than DA-based designs. We have also compared the performance of SA-MDSQ and that reported in Table III of [2] and found SA-MDSQ to be slightly better.

5. CONCLUDING REMARKS

SA based IA optimization method can be effectively used to design MDVQ with unbalanced descriptions. The performance of MDVQ obtained by this method was found to be comparable or better than previously reported results. In most cases, the IA optimization alone appears to yield a good MDVQ. At low packet loss probabilities however, optimizing the encoder does yield an additional improvement that may be significant.

Table 1. Comparison of SA-MDSQ and MDSQ from Fig.1 of [6] for iid Gaussian source with $R_1 = R_2 = 3$ bits/sample, $\lambda_1 = 0.006$, $\lambda_2 = 0.012$.

	MDSQ from [6]	SA-MDSQ
D_0 (dB)	-26.5	-27.17
D_1 (dB)	-4.60	-5.15
D_2 (dB)	-12.51	-14.09
D (dB)	-23.02	-23.74

Table 2. Comparison of SA-MDVQ and MDVQ from Fig.3 of [6] for Gauss-Markov source with $k = 2$, $R_1 = R_2 = 1.5$ bits/sample, $\lambda_1 = \lambda_2 = 0.01$.

	MDVQ from [6]	SA-MDVQ
D_0 (dB)	-15.13	-15.11
D_1 (dB)	-1.76	-1.93
D_2 (dB)	-0.19	-2.06
D (dB)	-13.20	-13.62

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