A GAME THEORETIC APPROACH FOR COOPERATIVE MIMO SCHEMES WITH CELLULAR REUSE OF THE RELAY SLOT

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ABSTRACT

An iterative game theory based algorithm is proposed to allocate the resources in cooperative schemes for the downlink. Both Amplify and Forward (AF) and Decode and Forward (DF) cooperative schemes are considered with cellular reuse of the relay slot. Multiple antennas can be used at the involved stations. Using the algorithm proposed, the simultaneous relay powers are decided in a decentralized way using mean channel level measures and mean values of noise and interference power. The cell capacity gain for the cooperative schemes using the decentralized algorithm is evaluated by means of simulation both for the AF and DF approaches.

1. INTRODUCTION

In the downlink (DL) of cellular systems, cooperation among users at the physical layer level is a promising approach for capacity and/or range increase [1], [3], [7], [8]. In these schemes, the signals received from the base station (BS) and the relay station (RS) are combined at the user equipment (UE). Therefore, cooperative schemes can be seen as a generalization of the typical multihop approach where a relaying terminal retransmits the symbols from the base station or central controller (thus providing range extension). The main advantage of the cooperative schemes, with respect to classical relaying strategies, is that cooperation creates a “virtual” MIMO system that may offer significant capacity gains in fading channels with respect to existing systems. A key issue for effective cellular capacity gain in cooperative or typical multihop networks is the reuse of the relay link. Up to now this issue has not been paid very much attention in the literature, with the exception of [7]. The reuse of the relay link is strongly influenced by the proper relay power allocations along with the selection of the ‘best’ neighbors to cooperate.

The main contribution of this paper is a decentralized approach to perform the assignment of resources in the relay link. The capacity gain thus obtained is evaluated considering the whole cell with reuse of the relay link, both for the AF and the DF cases.

2. CAPACITY IN COOPERATIVE SCHEMES WITH REUSE OF THE RELAY SLOT

There are two different approaches for cooperative transmission, according to the role played by the relaying terminal: the amplify and forward (AF) scheme and the decode and forward scheme (DF) [3], [8]. In the AF approach the relay amplifies and retransmits the signal received from the BS. In the DF scheme the relay station decodes the received signal and retransmits the decoded and regenerated symbols.

For both AF and DF scheme, two orthogonal channels (2 slots in a TDMA system) are allocated per user connection. In order to obtain some capacity improvement with respect to the non-cooperative scheme, where only 1 channel is required, it is mandatory to have some reuse of the relay slot.

Assuming a TDMA strategy for the DL, with the BS serving K users, the frame structure is shown in figure 1. First, the K DL transmissions are allocated. At the end of the frame, K' simultaneous retransmissions from the corresponding relays are allocated in a single slot, with $K' \leq K$. Therefore, the effective capacity of a single cooperative connection has to be multiplied by a factor $K/(K+1)$ instead of a factor 1/2 corresponding to the relay slot non-reuse case. When allocating simultaneous relay transmissions in the relay slot, the relay channels are not obviously mutually orthogonal, so proper assignment of powers and relays will strongly influence the cell capacity.

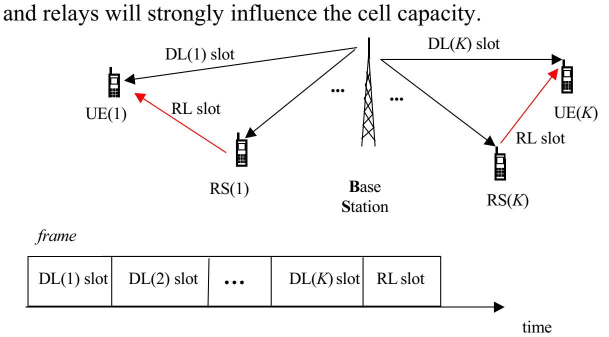

Figure 1 *Channel time allocation for K users with reuse of the RL slot*

Let’s denote M, N and R the number of antennas at the source (BS), destination (UE) and relay station (RS) respectively. In the following $\mathbf{H}_{0,k}$, $\mathbf{H}_{1,r}$ and $\mathbf{H}_{2,r\text{-}k}$ will denote the channel matrices containing respectively the channel coefficients in the direct link (BS to k-th UE), the 1st hop (BS to r-th RS) and the 2nd hop also denoted as the relay link (r-th RS to k-th UE). The channel coefficients will include a path loss component and a zero-mean complex Gaussian component accounting for the Rayleigh fading.

For the AF approach, the signal received at the k-th UE, cooperating with RS r, during the downlink (DL) slot and the relay link (RL) slot, can be modeled as:

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$$\begin{bmatrix} \mathbf{y}_k^{(DL)} \\ \mathbf{y}_k^{(RL)} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{0,k} \\ \mathbf{H}_{2,r-k}\mathbf{G}_r\mathbf{H}_{1,r} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \mathbf{I}_N & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{2,r-k}\mathbf{G}_r & \mathbf{I}_N \end{bmatrix} \begin{bmatrix} \mathbf{n}_k^{(DL)} \\ \mathbf{n}_r^{(DL)} \\ \mathbf{n}_k^{(RL)} \end{bmatrix} + \mathbf{i}_k \tag{1}$$

where $\mathbf{x}_k$ is the signal transmitted to the k-th user, $\mathbf{G}_r$ is a linear combining matrix at the r-th relay, $\mathbf{I}_N$ denotes the NxN identity matrix. Finally $\mathbf{n}_k^{(DL)}$ and $\mathbf{n}_r^{(DL)}$ are the noise vector received at the k-th UE and r-th RS during the DL slot, while $\mathbf{n}_k^{(RL)}$ is the noise vector at the UE during the RL slot. Eq. (1) can be written in a more compact way as follows:

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{n}_k + \mathbf{i}_k \tag{2}$$

The interference vector $\mathbf{i}_k$ contains interference caused by relays r' collaborating with users k':

$$\mathbf{i}_k = \sum_{\substack{r'=1 \\ r'\neq r}}^{K'} \begin{bmatrix} \mathbf{0} \\ \mathbf{H}_{2,r'-k}\mathbf{G}_{r'}\mathbf{H}_{1,r'} \end{bmatrix} \mathbf{x}_{k'} + \begin{bmatrix} \mathbf{0} \\ \mathbf{H}_{2,r'-k}\mathbf{G}_{r'} \end{bmatrix} \mathbf{n}_{r'}^{(DL)} \tag{3}$$

Assuming the channel unknown at the transmitter side, it is known that an isotropic transmission is a robust solution under channel uncertainty [5], then the instantaneous capacity for a single cooperative connection under the AF approach is:

$$C_{AF} = \frac{K}{K+1}\log_2\left(\mathbf{I}_{2N} + \frac{P_{BS}}{M}\mathbf{H}_k\mathbf{H}_k^H\mathbf{R}_{in,k}^{-1}\right) \tag{4}$$

with P_{BS} the power transmitted by the BS and $\mathbf{R}_{in,k}$ the received noise and interference correlation matrix. Note that, for uniform power allocation at the BS, the power transmitted by the r-th relay is given by:

$$p_r = trace\left\{\mathbf{G}_r^H\mathbf{G}_r\left(\frac{P_{BS}}{M}\mathbf{H}_{1,r}\mathbf{H}_{1,r}^H + \sigma_r^2\mathbf{I}_R\right)\right\} \tag{5}$$

with σ_r^2 the noise variance at the r-th relay.

For the DF approach, the signal received at the k-th UE, cooperating with RS r, during the downlink (DL) slot and the relay link (RL) slot, can be modeled as:

$$\begin{bmatrix} \mathbf{y}_k^{(DL)} \\ \mathbf{y}_k^{(RL)} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_{0,k} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{2,r-k}\sqrt{\frac{p_r}{P_{BS}}} \end{bmatrix}}_{\mathbf{H}_k} \begin{bmatrix} \mathbf{x}_k^{(DL)} \\ \mathbf{x}_k^{(RL)} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_k^{(DL)} \\ \mathbf{n}_k^{(RL)} \end{bmatrix} + \mathbf{i}_k \tag{6}$$

where the signal transmitted by the RS $\mathbf{x}_k^{(RL)}$ in the RL does not need to be linearly related to the signal transmitted by the BS $\mathbf{x}_k^{(DL)}$ in the DL. The vector $\mathbf{i}_k$ contains interference caused by relays r' collaborating with users k':

$$\mathbf{i}_k = \sum_{\substack{r'=1 \\ r'\neq r}}^{K'} \begin{bmatrix} \mathbf{0} \\ \mathbf{H}_{2,r'-k}\sqrt{\frac{p_{r'}}{P_{BS}}}\mathbf{x}_{k'}^{(RL)} \end{bmatrix} \tag{7}$$

Under the DF approach, the instantaneous capacity for a single user cooperative connection can be shown to be [6]:

$$C_{DF} = \frac{K}{K+1}\min\left\{\log_2\left(\mathbf{I}_R + \frac{P_{BS}}{M\sigma_r^2}\mathbf{H}_{1,r}\mathbf{H}_{1,r}^H\right),\right.$$
$$\left.\log_2\left(\mathbf{I}_{2N} + \frac{P_{BS}}{M}\mathbf{H}_k\mathbf{H}_k^H\mathbf{R}_{in,k}^{-1}\right)\right\} \tag{8}$$

For the assignment of transmitted power, we consider that the relays can inform the UE about the state of its link with the BS. Also, we consider that the decision is driven by the mean channel levels and mean noise and interference measures. Let γ_0, γ_1 and γ_2 be the mean signal to noise plus interference ratio at the direct link, first hop and RL respectively:

$$\gamma_0 = \frac{P_{BS}}{L_{0,k}\sigma_k^2} \qquad \gamma_1 = \frac{P_{BS}}{L_{1,r}\sigma_r^2} \qquad \gamma_2 = \frac{\dfrac{p_r}{L_{2,r-k}}}{\sigma_k^2 + \sum_{\substack{r'=1 \\ r'\neq r}}^{K'}\dfrac{p_{r'}}{L_{2,r'-k}}} \tag{9}$$

with $L_{0,k}$ the channel loss in the direct link for UE k, $L_{1,r}$ is the channel loss in the 1st hop for the r-th relay and $L_{2,r-k}$ is the channel in the RL (2nd hop) between relay r and user k. $L_{2,r'-k}$ denotes the channel loss between user k and the relay r' assigned to user k'. σ_k^2 and σ_r^2 are the noise variance at the k-th UE and the r-th RS respectively. Finally, p_r and $p_{r'}$ are the power transmitted by the relay r and r' respectively.

When the number of antennas at the BS, RS and UE verifies that $M \geq R \geq N$, the capacity achieved for the UE k cooperating with relay r can be approximated in the AF by:

$$\tilde{C}_{AF} = \frac{K}{K+1}\left(N\log_2(1+\gamma_0) + N\log_2\left(1+\frac{\gamma_1\gamma_2}{1+\gamma_1+\gamma_2}\right)\right) \tag{10}$$

assuming uncorrelated channel components, large number of antennas and $M \geq R \geq N$:

$$\frac{1}{M}\mathbf{H}_{0,k}\mathbf{H}_{0,k}^H \cong \frac{\mathbf{I}_N}{L_{0,k}}; \quad \frac{1}{M}\mathbf{H}_{1,r}\mathbf{H}_{1,r}^H \cong \frac{\mathbf{I}_R}{L_{1,r}}; \frac{1}{R}\mathbf{H}_{2,r-k}\mathbf{H}_{2,r-k}^H \cong \frac{\mathbf{I}_R}{L_{2,r-k}} \tag{11}$$

$$\mathbf{H}_{2,r-k}\mathbf{G}_r\mathbf{G}_r^H\mathbf{H}_{2,r-k}^H \cong \frac{trace\left(\mathbf{G}_r\mathbf{G}_r^H\right)}{L_{2,r-k}}\mathbf{I}_N \tag{12}$$

Using (11) the DF approach capacity can be approximated by:

$$\tilde{C}_{DF} = \frac{K}{K+1}\min\left\{R\log_2(1+\gamma_1),\right.$$
$$\left.N\log_2(1+\gamma_0) + N\log_2(1+\gamma_2)\right\} \tag{13}$$

3. GAME THEORY FOR DISTRIBUTED RESOURCES ASSIGNMENT

Game theory is a tool which allows to analyze the interaction of decision-makers with conflicting objectives. It has been long used by economists and in the late years it has been also proposed to solve some problems in communications systems, as it is power control in CDMA wireless systems [4]. In RL resources assignment, there can be observed the three components defined in a non-cooperative *game* [4] in which all the users follow the same rules: 1) a set of players: the UEs={1,2,…,K} of the cell with potentially conflicting objectives, 2) a set of possible actions for each player, which can enter in conflict with the selections of other nodes and it is also dependent on other's decisions: the relay power and/or the relay to cooperative with, 3) and the most important aspect, a set of utility functions to map action profiles into the real numbers.

Let us assume the users in the cell have chosen a RS around based on proximity. In this case the *K*-user non-cooperative game Γ consists of the selection of power for the previously assigned relay. Let the power vector $\mathbf{p} = [p_{r(1)}, \ldots, p_{r(K)}]^T \in P$ denote the outcome of the game where P is the set of all power vectors. User *k* has the strategy set $P_{r(k)} = [0,..,P_{\max}]$ and a utility function $u_k(p_{r(k)}, \mathbf{p}_{r(-k)})$, with $p_{r(k)}$, the power of the relay cooperating with the *k*-th user and $\mathbf{p}_{r(-k)}$ the vector containing the power of the relays cooperating with other users.

3.1 The proposed utility function

The utility function proposed in this contribution is:

$$u_k(\mathbf{p}) = f(p_{r(k)}, \mathbf{p}_{r(-k)})g(p_{r(k)}) = \frac{\tilde{C}(p_{r(k)}, \mathbf{p}_{r(-k)})}{p_{r(k)} + P_{BS}} \quad (14)$$

The function $f(p_{r(k)}, \mathbf{p}_{-r(k)})$ is the approximated capacity of the cooperative scheme (eq. (10) or eq. (13)). While capacity increases with $p_{r(k)}$ and tends to a saturation value, the 2nd function is decreasing in the transmitted power. It accounts for the total transmitted power, including the BS and the relay: $g(p_{r(k)}) = 1/\left(p_{r(k)} + P_{BS}\right)$. The proposed utility function gives the theoretical maximum number of bits/Hz without error we can transmit per unit of employed energy. The consideration of the total required power, including the BS power, allows fair comparison of capacity with respect to the non-cooperative scheme in terms of equal transmitted power. Hence, it is possible to identify when the use of cooperative transmission does not improve capacity. In the sequel, it is assumed that the BS transmitted power is fixed.

The solution that is most widely used for game theoretic problem is the Nash equilibrium. The Nash equilibrium is an action profile at which no user may gain by unilaterally deviating (no user has any incentive to change selected power):

$$u_k(p_{r(k)}, \mathbf{p}_{r(-k)}) \geq u_k(p_{r(k)}', \mathbf{p}_{r(-k)}) \text{ for any } p_{r(k)}' \in P_k \quad (15)$$

Proposition The game Γ=[*K*, {$P_{r(k)}$}, {u_k}] has a least one equilibrium point.

Proof: A Nash equilibrium exists in a game [2] if for all *k*=1,…,*K*: 1) $P_{r(k)}$ is a non-empty, convex, and compact subset of some Euclidean space $\Re^K$ and 2) $u_k(\mathbf{p})$ is continuous in **p** and quasi-concave in $p_{r(k)}$.

By definition the action sets $P_{r(k)}$ are non-empty and convex. Each $P_{r(k)}$ is closed, since it includes the boundary points 0 and P_{max}, and it is also bounded, since all the power values are between the boundary points. Therefore, the action sets $P_{r(k)}$ are compact. Thus, the 1st condition is satisfied.

It remains to show that the utility function $u_k(\mathbf{p})$ is quasi-concave in $p_{r(k)}$. A function $y: X \to \mathbb{R}$ is quasi-concave if and only if either y is monotonic or there is $x^* \in X$ such that *y* is nondecreasing on $X \cap (-\infty, x^*]$ and nonincreasing on $X \cap [x^*, \infty)$.

For those values of *p* nulling the 1st derivative of the utility function (possible maximum or minimum) the 2nd derivative is:

$$u''(p) = \frac{f''(p)}{f(p)} - 2\left(\frac{g'(p)}{g(p)}\right)^2 + \frac{g''(p)}{g(p)}; if \quad u'(p) = 0 \quad (16)$$

Substituting $g(p) = 1/\left(p + P_{BS}\right)$, eq. (16) simplifies to $f''(p)/f(p)$. For the AF approach, the capacity increases logaritmically with *p*, so $f''(p)/f(p) < 0$. Therefore, the utility function has no minimum, or equivalently it is monotonic or it has a single maximum. Hence, the conditions for quasi-concavity are satisfied. For the DF approach, from 0 to some $p_{r(k)}^*$ the capacity is limited by the 2nd hop link (RL) and increases logaritmically with the relay power. As a consequence of that $f''(p)/f(p) < 0$, so in this region the utility function is monotonic or it has a single maximum. For powers greater than $p_{r(k)}^*$, the capacity is limited by the 1st hop link and so constant with respect to the relay power, and then the utility function is decreasing. Therefore, also for the DF approach, the utility function is quasi concave in $p_{r(k)}$.

3.2 Powers computation

Suppose the relays power are updated at time instant $\{\tau_1, \tau_2, \ldots\}$. The sequence of powers is generated as follows:

1) Set the initial power vector **p**(0)=**p** with **p** is any vector in the strategy space *P* and then set *n*=1.
2) For every τ_n compute

$$\hat{p}_{r(k)}(\tau_n) = \arg \max_{p_{r(k)} \in P_k} u_k\left(p_{r(k)}, \mathbf{p}_{r(-k)}(\tau_{n-1})\right) \quad k = 1 \ldots K \quad (17)$$

Note that the utility function for every user depends on the relay powers in previous iteration. Nevertheless, the computation of the utility function is performed based on mean noise and interference measures and the mean channel level between the user and corresponding relay, using (10) or (13). The solution of eq. (17) is a NE of powers for a given choice of relay stations. The iterative procedure continues until all players find that the change in their power levels is less than a pre-defined bound, or an upper limit on the number of iterations is reached.

4. SIMULATIONS RESULTS AND CONCLUSIONS

For the simulations, the following scenario has been considered: a square area of 900x900 m^2, one BS in the center of the cell using omnidirectional antennas, UEs uniformly distributed in the cell and Rayleigh flat fading channels. The distance loss model is usual inverse law with propagation exponent to be 4. The BS transmission power is 30 dBm and the UE and RS have maximum transmission power of 23 dBm. The thermal noise at the RS and UE stations is –115.2 dBm. We consider that each user has chosen a relay station around.

Figure 2 shows a specific realization of the described scenario. For this specific realization of the scenario, figure 3 shows the evolution in the transmitter power for the relays assigned to users 2, 3 and 6 for three different initializations of the iterative algorithm, considering the AF scheme, and the users associated to a close relay in the neighbourhood. Note that the

RS cooperating with user 2 converges to zero, that it is user 2 decides not to use cooperation with the relay chosen. It can be observed that, with independence of the initial power value, the algorithm converges, after a few iterations, to the same stationary point for the relays power. The same behaviour, not plotted in the figure for clarity, is observed for the rest of users.

Note that, as the resources will be assigned based on average channel states, the instantaneous cell capacity will be random with a cumulative function that can be obtained through simulations. Figure 4 and figure 5 show the cumulative function of the cell capacity for M=1 and M=2 antennas at the BS with and without cooperation. Figure 4 corresponds to the AF approach with R=1 antenna at the RS while figure 5 corresponds to the DF approach with R=2 antennas at the RS.

Although the RS amplifies and retransmits the received noise in the AF approach, this scheme outperforms the DF approach if the RS uses only 1 antenna. The reason is that in such a situation the capacity of the DF approach is limited by the BS-RS link. When the number of relay antennas is R=2N, the BS-RS channel and the equivalent cooperative channel seen by the UE have both M transmitting antennas and 2N receiving antennas. Thus, it will not be the BS-RS link the one that limits capacity (considering the chosen RS has better channel conditions in terms of path loss than the UE).

The AF capacity for M=2 BS antennas is clearly superior to the capacity obtained using only the 2x1 direct link and approaches to the performance of a 2x2 MIMO system . The difference with respect to the MIMO approach is due to: 1) the fact of having a reuse greater than 1 but not infinite in the RL, 2) to the interference at the RL, and 3) the possibility of having a RS which is not closer to the BS than the corresponding UE. This last aspect is equivalent to have one of the antennas at a MIMO system in permanent fading. Using R=2 antennas at the RS there is a gain when decoding at the relay (see figure 5). This slight gain, which can be higher for a greater relay noise level, is achieved at the cost of a more complex relay terminal.

5. REFERENCES

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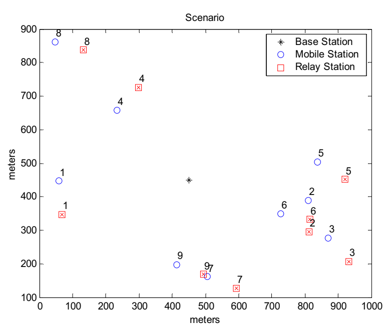

Figure 2 *Realization of the scenario.*

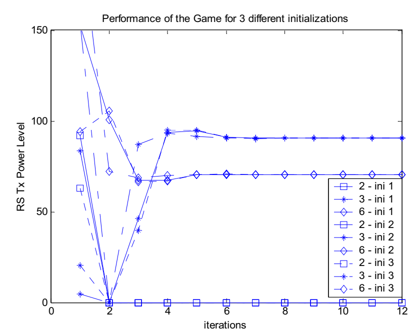

Figure 3 *Relay power evolution for the users 2,3 and 6 with 3 different initializations of the algorithm.*

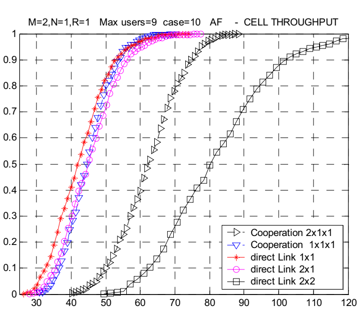

Figure 4 *Cumulative function of the cell capacity with (MxRxN) and without cooperation (MxN). AF approach with R=1 antenna at the RS.*

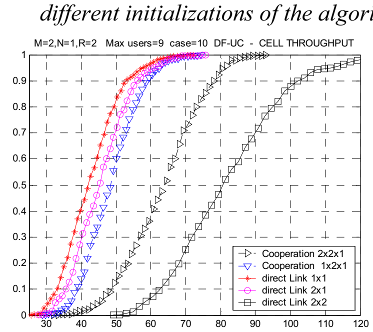

Figure 5 *Cumulative function of the cell capacity with (MxRxN) and without cooperation (MxN). DF approach with R=2 antennas at the RS.*