ON THE MAXIMUM ACHIEVABLE RATES IN WIRELESS MESHED NETWORKS: CENTRALIZED VERSUS DECENTRALIZED SOLUTIONS

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ABSTRACT

In this work we provide the optimal coding strategy for meshed wireless networks, where more links are active simultaneously, assuming as optimality criterion the rates of all the links. We formulate the rate maximization problem as a Multi-objective Optimization Problem (MOP). Assuming a multi-carrier modulation for each user, we show how to allocate the power of each user optimally according to a centralized power distribution algorithm. We also propose a decentralized (suboptimal) but simpler algorithm, based on the idea of Nash equilibrium (NE). Finally, we compare the two strategies showing that the loss, in terms of information rate, of the decentralized strategy based on the Iterative Water-filling Algorithm can be very small with respect to the optimal centralized solution, as the distance between the interfering links is just a few times the distance of each link, thus making the decentralized approach a viable solution.

1. INTRODUCTION

In this paper, we consider a wireless network composed of Q sources (S) and as many destinations (D), operating simultaneously, and we wish to find out the coding strategy for each user that, for a given user power budget, maximizes the rate of all links. This is a multi-objective optimization problem (MOP) and its solution is not known in the general case. To simplify the solution, we assume that the channels are time-invariant within each transmitted block and that all users adopt a multi-carrier modulation (MCM) strategy. Using MCM, the problem is converted into the search for the optimal power spectral allocation for each user. The problem has applications in a series of currently interesting scenarios, like ad-hoc (i.e. infrastructureless) networks, sensor networks and multihop networks, in the preliminary phase where more mobile terminals send their data to relay terminals simultaneously. The problem has been tackled very recently, for example in [6], [2], [3], [4], [9], [7]. In [2] it was proposed a centralized algorithm that finds the power loading maximizing the weighted rate-sum in the case of two active links numerically, under the assumption that the objective function is convex. However, each user's rate, in the presence of interference is not a convex function of the powers for any channel realizations . In [6], a sum-rate maximization problem with a weighted power sum constraint was solved. However, the maximization of the sum-rate does not assure the maximization of all rates in the sum and it does not prevent a user to get a null rate. Therefore, there is no guarantee of optimality in the solutions proposed in [6] and [2]. In [3], the difficult problem of finding the capacity region of a Gaussian interference channel was studied in detail, but under the assumption of strong interference. Unfortunately, this assumption is too restrictive and it does not fit the setup of our problem. In this paper, we formulate the problem as a true MOP and we propose: i) a centralized (optimal) algorithm that reaches the Pareto optimal solutions of MOP for any channel realizations, number Q of links S-D and power budget; ii) a decentralized (suboptimal) algorithm, based on the idea of Nash equilibrium (NE), that approaches the optimal solution with negligible losses in situations of practical interest. We derive the sufficient conditions that insure the NE to be unique and, finally, we show how to modify the cost functions in order to make the NE's coincide with the Pareto optimal solutions.

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2. SYSTEM MODEL AND PROBLEM FORMULATION

Throughout the paper, we use the following setup. Each user transmits blocks of N symbols using linear precoding. We denote with $\boldsymbol{s}(n)$ the n-th block of information symbols and with $\boldsymbol{x}(n) = \boldsymbol{F}\boldsymbol{s}(n)$ the corresponding transmitted block, where $\boldsymbol{F}$ is an $N \times N$ full-rank matrix. All channels are FIR, time-invariant, with maximum order L. We denote with $h_{kl}(n)$ the impulse response between the k-th S and the l-th D. Two nodes S and D are considered *paired*, when $k = l$. We append a cyclic prefix (CP) of order L to each block to facilitate elimination of inter-block interference (IBI) and channel equalization. We assume, without any loss of generality, that the information symbols are uncorrelated with variance σ_s^2, and that the receiver noise vector $\boldsymbol{\eta}(n)$ is white Gaussian, with covariance matrix $\boldsymbol{C}_\eta = \sigma_n^2 \boldsymbol{I}$. All nodes S transmit simultaneously. Assuming perfect synchronization and no coordination[1] among the S-D pairs, the N-size vector $\boldsymbol{y}_k(n)$, received by the k-th node D, after discarding the guard interval, is

$$\boldsymbol{y}_k(n) = \boldsymbol{H}_{kk}\boldsymbol{F}_k\boldsymbol{s}_k(n) + \sum_{j=1,\, j\neq k}^{Q} \boldsymbol{H}_{jk}\boldsymbol{F}_j\boldsymbol{s}_j(n) + \boldsymbol{\eta}_k(n), \quad (1)$$

with $k = 1, \ldots, Q$. Thanks to the insertion of the CP, $\boldsymbol{H}_{kk}$ is an $N \times N$ circulant Toeplitz matrix with (i, j) entry $h_{kk}((i-j) \mod N)/r_{kk}^{\alpha}$, where $h_{kk}(n)$ is the the channel impulse response; we single out the distance r_{kk} between the k-th S-D pair, because it will be useful in our ensuing derivations to show the sufficient conditions for the uniqueness of the Nash equilibrium. We assume that the transmitted power propagates with an attenuation factor, as a function of the travelled distance r, $1/r^{2\alpha}$, with $\alpha \geq 1$. Since no cooperation[2] among pairs is allowed, the second term on the

[1] In this context, coordination means that encoding and/or decoding are performed jointly among all the S's and D's.

[2] In practice, this assumption is reasonable, because the transmitter and the receivers are physically separated.

right-hand side of (1) represents the multi-user interference (MUI) received by the k-th D node caused by the other active S-D links. Therefore, the Q input-output relationships (1) can be modelled as a Gaussian Interference Channel (GIC) with Q transmitters and Q receivers. Our goal is to find out the coding matrices $\{\boldsymbol{F}_k\}_{k=1}^{Q}$ that maximize the information rates of all cooperating pairs S-D *jointly*, subject to the constraint that each transmitter has a maximum power P_T. The full characterization of the problem would require the derivation of the capacity region (CR) of the GIC, but this is still an open problem. Only partial results have been achieved so far under the condition of strong interference (see e.g. [3] and the references therein). However, in [3] it was necessary to assume that the interference level had to be stronger than the useful signal and this is not applicable to our case. Furthermore, reaching the boundary of the CR would require some cooperation among different pairs in their respective coding strategies. This operation would be highly difficult to implement in a wireless infrastructureless network and it would require such a waste of resources for signalling among all the nodes to make the resulting rate maximization meaningless.

Thus, we approach the problem making the following assumption: a1) no interference cancellation is performed at the receiver, so that MUI is treated as additive colored Gaussian noise; a2) no cooperation among different pairs is allowed. Thus, a sub-optimal[3] distributed coding scheme is adopted, where the encoding/decoding is performed from each pair *independently* and no multi-access scheme is embedded in the codebook of each pair; a3) all channels $\{\boldsymbol{H}_{kj}\}_{k,j=1}^{Q}$ are perfectly known to all transmitters and receivers. Only for the sake of simplicity, we also assume that a4) multi-carrier modulation is performed from each pair, but no constraint on the bandwidth that each pair may use, is imposed. The interfering nature of the system makes inadequate the use of a single figure of merit (e.g. the sum rate) to maximize the rates of all links simultaneously, because increasing the rate of some link would increase the interference on the other links and then decrease their rates. Dealing with a multi-objective function, we have to define what we mean by optimal vector. In this work, we adopt the Pareto optimality criterion [1]. This means that indicating with R_k, the information rate of the k-th link, with $\boldsymbol{p}_k$ the set of powers allocated by the k-th user over the available N sub-carriers, and with $\mathcal{D}$ the domain of admissible powers, i.e. the set of values satisfying the power constraints, the solution $\boldsymbol{p}^* := (\boldsymbol{p}_1^*, \ldots, \boldsymbol{p}_Q^*)$ is Pareto-optimal iff it is Pareto-dominant, i.e. there does not exist any other vector $\boldsymbol{p} := (\boldsymbol{p}_1, \ldots, \boldsymbol{p}_Q) \in \mathcal{D}$, with $\boldsymbol{p} \neq \boldsymbol{p}^*$, such that $R_k(\boldsymbol{p}) \geq R_k(\boldsymbol{p}^*)$ for $k = 1, 2, \ldots, Q$, with at least one of the above inequalities satisfied in strict-sense.

Having established the optimality criterion, under a1)-a4) the optimal tradeoff among the rates of all links can be found only by solving the following multi-objective maximization problem

$$\begin{aligned} &\{\boldsymbol{p}_1^*, \cdots, \boldsymbol{p}_Q^*\} = argmax\{R_1, R_2, \ldots, R_Q\}, \text{ with} \\ &R_k = \frac{1}{N}\sum_{i=0}^{N-1} \log\left(1 + \frac{1}{\Gamma}\frac{|H_{kk}(i)|^2 p_k(i)/r_{kk}^{\alpha}}{\sigma_n^2/\sigma_s^2 + \sum_{j\neq k}^{Q}\frac{1}{r_{jk}^{\alpha}}|H_{jk}(i)|^2 p_j(i)}\right) \\ &\text{subject to} \quad \sigma_s^2 \sum_{i=0}^{N-1} p_k(i) \leq P_T, \ \ k = 1, \ldots, Q, \end{aligned} \tag{2}$$

where R_k is computed as the maximum mutual information between the transmitted block $\boldsymbol{x}_k(n)$ and the received block $\boldsymbol{y}_k(n)$, *assuming the other received signals as additive noise*; $H_{jk}(i)$ are the samples of the channel transfer function, i.e. $H_{jk}(i) = \sum_{q=0}^{L} h_{jk}(q)\, e^{-j2\pi iq/N}$; $p_k(i)$ is the power allocated over the i-th sub-carrier from the k-th S; $\boldsymbol{p}_k := \{p_k(0), \ldots, p_k(N-1)\}\forall k$ and Γ denotes the SNR-gap that depends on the target error-probability, the coding and the modulation scheme. The stars indicate the optimal solutions in the Pareto sense. The trade-off surface (with respect to the componentwise inequality) in the objective space of MOP[4] represents the largest rate region (RR) achievable under the given power constraint and the assumptions a1)-a4). Unfortunately, the MOP maximization (2) is not convex, because the rates are neither convex nor concave with respect to the power vectors. Hence, the classical optimization techniques involving the maximization of weighted sum rate provides, in general, only a sub-optimal solution.

[3]Generally, the distributed encoding/decoding we have assumed is sub-optimal, because in achieving the capacity of frequency-selective GIC, even if the encoding/decoding process is performed independently, the codebook of each user has to be generated jointly [3].

3. CENTRALIZED SOLUTION

The largest RR achievable, under a1)-a4), can be found numerically using the Normal-Boundary Intersection (NBI) algorithm, proposed in [5]. The NBI provides the upper right boundary of the objective feasible set of MOP (2) and the corresponding optimal power allocation. Note that if such a boundary is convex, then all the points on the boundary are Pareto-optimal, otherwise some of them could be dominated [5]. However, to be implemented, the NBI algorithm needs a centralized control with full knowledge of all system parameters. After solving the MOP, the central control unit has to transfer the optimal power allocation vectors to all transmitters. Alternatively, a simpler iterative algorithm based on a gradient descent method can be found reformulating the MOP as a competitive non cooperative game, having the following structure $\mathscr{G} = (\Omega,\ \{\mathscr{P}_k\}_{k\in\Omega},\ \{\Phi_k\}_{k\in\Omega},\ \boldsymbol{H}, \sigma_n^2,\ \sigma_s^2)$, where $\Omega := \{1, 2, \ldots, Q\}$ is the set of pairs indices; $\mathscr{P}_k$ is the set of the admissible strategies (power distribution) for the k-th player; Φ_k are the payoff functions (rates); the power distribution $\boldsymbol{p}_k := \{p_k(i)\}_{i=1}^{N} \in \mathscr{P}_k$ over the N available sub-carriers, subject to the power constraint $P_T = \sigma_s^2 \sum_{i=1}^{N} p_k(i)$, represents the game strategy for the k-th player. The game structure, i.e. the channels $\boldsymbol{H} := \{\boldsymbol{H}_{ij}\}_{i,j=1}^{Q}$ and the variances σ_n^2 and σ_s^2 are assumed to be known to all players. Moreover, only pure strategies are allowed. In such a game, each player competes with the others in order to maximize its own payoff function Φ_k, regardless of all other players. In this competition, if there exists a Nash Equilibrium (NE), it means that there is an optimum strategy profile $\overline{\boldsymbol{p}} := (\overline{\boldsymbol{p}}_1, \overline{\boldsymbol{p}}_2, \ldots, \overline{\boldsymbol{p}}_Q) \in \mathscr{P} := \mathscr{P}_1 \times \ldots \times \mathscr{P}_Q$ such that, for each pair, setting $\boldsymbol{p}_{-k} := [\boldsymbol{p}_1, .., \boldsymbol{p}_{k-1}, \boldsymbol{p}_{k+1}, .., \boldsymbol{p}_Q]$, it has to be, for all $k \in [1,\, Q]$,

$$\Phi_k(\overline{\boldsymbol{p}}_k, \overline{\boldsymbol{p}}_{-k}) \geq \Phi_k(\boldsymbol{p}_k, \overline{\boldsymbol{p}}_{-k}),\ \forall \boldsymbol{p}_k \neq \overline{\boldsymbol{p}}_k \in \mathscr{P}_k. \tag{3}$$

The relationship between the solutions of MOP and the NE's of the game $\mathscr{G}$ is given by the following

Theorem 1. The Pareto Optimal (PO) solutions of MOP (2) are the NE's of the game $\mathscr{G}$ having the following pay-off functions

$$\Phi_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k}) = R_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k}) + \frac{1}{\lambda_k}\sum_{j\neq k=1}^{Q} \lambda_j\, R_j(\boldsymbol{p}_k, \boldsymbol{p}_{-k}), \tag{4}$$

[4]Note that, if the feasible set of the objective function in (2) is not convex, not every point lying on the trade-off surface is globally Pareto-optimal.

[5]The convex combination of the objective functions fails to obtain the points lying on the non convex part of the trade-off curve.

where $\boldsymbol{\lambda} \in \mathscr{R}^Q$ with $\boldsymbol{\lambda} \succ \mathbf{0}$, $R_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k})$ is given by (2) and $k \in [1,\ Q]$.

Proof. It is straightforward to check that the global optimal solution of the following maximization problem

$$\{\bar{\boldsymbol{p}}\} = \operatorname{argmax}_{\boldsymbol{p} \in \mathscr{P}} \sum_{j=1}^{Q} \lambda_j\, R_j(\boldsymbol{p}), \tag{5}$$

for any given $\boldsymbol{\lambda} \succ \mathbf{0}$ is a point on the Pareto boundary of MOP. If $\bar{\boldsymbol{p}}$ is an optimal solution of (5), it has to be

$$\sum_{j=1}^{Q} \lambda_j\, R_j(\bar{\boldsymbol{p}}) \geq \sum_{j=1}^{Q} \lambda_j\, R_j(\boldsymbol{p}),\ \forall \boldsymbol{p} \neq \bar{\boldsymbol{p}}. \tag{6}$$

Setting $\boldsymbol{p} = (\boldsymbol{p}_k, \bar{\boldsymbol{p}}_{-k})$, (6) becomes

$$\begin{aligned} & R_k(\bar{\boldsymbol{p}}_k, \bar{\boldsymbol{p}}_{-k}) + 1/\lambda_k \textstyle\sum_{j\neq k=1}^{Q} \lambda_j\, R_j(\bar{\boldsymbol{p}}_k, \bar{\boldsymbol{p}}_{-k}) \geq \\ & \qquad R_k(\boldsymbol{p}_k, \bar{\boldsymbol{p}}_{-k}) + 1/\lambda_k \textstyle\sum_{j\neq k=1}^{Q} \lambda_j\, R_j(\boldsymbol{p}_k, \bar{\boldsymbol{p}}_{-k}) \\ \iff\ & \Phi_k(\bar{\boldsymbol{p}}_k, \bar{\boldsymbol{p}}_{-k}) \geq \Phi_k(\boldsymbol{p}_k, \bar{\boldsymbol{p}}_{-k}), \quad \forall k \in [1,\ Q], \end{aligned} \tag{7}$$

where $\Phi_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k}) := R_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k}) + 1/\lambda_k \sum_{j\neq k=1}^{Q} \lambda_j R_j(\boldsymbol{p}_k, \boldsymbol{p}_{-k})$. The last inequality in (7) defines the NE (see (3)) of the game $\mathscr{G}$ [6]. Thus, the Pareto boundary of MOP can be found by computing the NE of the game $\mathscr{G}$, provided that such a NE is *unique*. For each choice of the weight vector $\boldsymbol{\lambda} \succ \mathbf{0}$, we get a (usually different) PO point of MOP. In [9] we derived the sufficient conditions (SC) for the existence and uniqueness of the NE in the game $\mathscr{G}$, applying the conditions of [8] to the game $\mathscr{G}$. Even though we omit the derivations here for lack of space, we wish to remark that the SC do not require the convexity of the objective functions in (2). Building on the general iterative algorithm proposed in [8], we propose an iterative algorithm based on gradient descent method that, under the SC, is proved to converge to the unique NE. Differently from NBI, that requires the computation of the optimal solution to be performed on a central unit having all the information, this algorithm could be implemented on each node, but each node should have, at each iteration, knowledge of the (partial) results achieved by all other nodes. Therefore, also in this case, it is necessary to have a high degree of signalling among all the nodes.

Before concluding this section , it is worth pointing out that the SC derived in [9] are only sufficient, i.e. a unique NE for $\mathscr{G}$ might exist even if they are not met. Experimentally, we have found that the above algorithm has always converged to the same optimal solutions, regardless of the SC, the channels and the power budgets for each pair. However, even if the proposed algorithm can be performed iteratively among the pairs, at each step, each link has to know the channels, the rates and the power allocations of all the other links.

4. DECENTRALIZED STRATEGIES

The globally optimal solutions of the MOP seen before can be reached only if a centralized system having full knowledge of the system is available or if the nodes exchange their partial results. Either way, these solutions are not attractive for an ad-hoc network. Hence, a more practical and interesting problem to solve is the simultaneous rate maximization of the active links requiring minimum centralized control and minimum (possibly null) exchange among the sources. To this end, here we approach (2) introducing the following additional constraint: a5) no source is allowed to know rate, channel transfer functions and power allocation of the other links. Each node S is assumed to know only its own channel and the covariance matrix $\boldsymbol{C}_{w_k}$ of the interference arriving at its own D node. Under such a constraint, the optimal solution is the stable operation point where each link has maximized its own rate, with all others viewed as additive colored noise. In formulas, the optimal powers are the solutions of the following maximization problems

$$\{\bar{\boldsymbol{p}}_k\} = \operatorname{argmax}_{\boldsymbol{p}_k \in \mathscr{P}_k} \left\{R_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k})\right\}, \quad \forall k \in [1,\ Q]. \tag{8}$$

From (3) and (8), it follows that the solutions of (8) are the NE's of a strategic non-cooperative game $\tilde{\mathscr{G}} = \left(\Omega,\ \{\mathscr{P}_k\}_{k\in\Omega},\ \{R_k\}_{k\in\Omega},\ \{\boldsymbol{C}_{w_k}\}_{k=1}^{Q}\right)$, where each player (link) competes against the others to maximize only its own information rate R_k (pay-off function), under a power constraint. In [9] we proved the following

Theorem 2. Given the game $\tilde{\mathscr{G}}$, there exists at least one stable NE. If the following conditions hold true

$$r_{ij}^{\alpha} > r_{ii}^{\alpha}(Q-1) \left[\max_k \left\{|H_{ij}(k)|^2/|H_{ii}(k)|^2\right\}\right], \ \forall i,j \in [1,\ Q], \tag{9}$$

then the NE of $\mathscr{G}$ is unique.

Interestingly, expression (9) has a physical interpretation: In order to assure the uniqueness of the competitive equilibrium, a minimum distance between the cooperating pairs has to be guaranteed. Such a distance corresponds to the maximum level of interference that may be tolerated by each pair and, as we expected, it depends on i) the number Q of pairs; ii) the distance r_{ii} between the S and D in each pair; and iii) the worst ratio $\max_k \left\{|H_{ij}(k)|^2/|H_{ii}(k)|^2\right\}$ between the channel transfer function of all interference links and the channel transfer function of direct link.

Note that, directly from the definition of NE (3), it follows that $R_k(\overline{\boldsymbol{p}}_k, \overline{\boldsymbol{p}}_{-k}) = \max_{\boldsymbol{p}_k} R_k(\boldsymbol{p}_k, \overline{\boldsymbol{p}}_{-k}) \geq \max_{\boldsymbol{p}_k} \min_{\boldsymbol{p}_{-k}} R_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k})$, which means that, if a5) is introduced, each link, differently from (4), is able to maximize, at least, its worst rate. Generally, these rates are Pareto dominated by the solutions of MOP (i.e the NE's of $\mathscr{G}$). However, if (9) holds true, the unique NE of $\tilde{\mathscr{G}}$ corresponds to the best rates achievable under a5) and for the given power budget.

A simpler *distributed* algorithm that reaches the NE can be obtained as follows. From the definition of NE, we deduce that, for each NE, the optimal power allocation strategy for every player of the game $\tilde{\mathscr{G}}$, must be the water-filling power distribution over the available sub-carriers subject to the power constraint P_T and regarding the interference due to the other players as additive (colored) noise. Hence, the power allocation reaching one NE must be solution of the following system of implicit equations

$$\begin{aligned} \overline{p}_k(i) &= \left(\frac{1}{\mu_k} - \frac{\sigma_n^2+\sigma_s^2\sum_{j\neq k}^{Q}|\widetilde{H}_{jk}(i)|^2\overline{p}_j(i)}{\sigma_s^2|\widetilde{H}_{kk}(i)|^2}\right)^{+}, \\ \frac{1}{\mu_k} &= \frac{P_T+\sum_{i\in I_k}\frac{\sigma_n^2+\sigma_s^2\sum_{j\neq k}^{Q}|\widetilde{H}_{jk}(i)|^2\overline{p}_j(i)}{\sigma_s^2|\widetilde{H}_{kk}(i)|^2}}{N_k},\ i \in I_k,\ k \in \Omega, \end{aligned} \tag{10}$$

where I_k is the set of sub-carriers allocated for the k-th pair and N_k the cardinality of I_k. Since the game $\tilde{\mathscr{G}}$ admits at least one stable NE, the existence of a simultaneous water-filling solution (10) is guaranteed. It follows that an iterative procedure among the players (S-D links), where at every step, each player performs the single-user water-filling power distribution (10), regarding the interference from the other players as noise, if it converges, it has

[6]Note that, thanks to our approach based on a game theory framework, we are able to find the global solutions of (5), without requiring the convexity of the objective function.

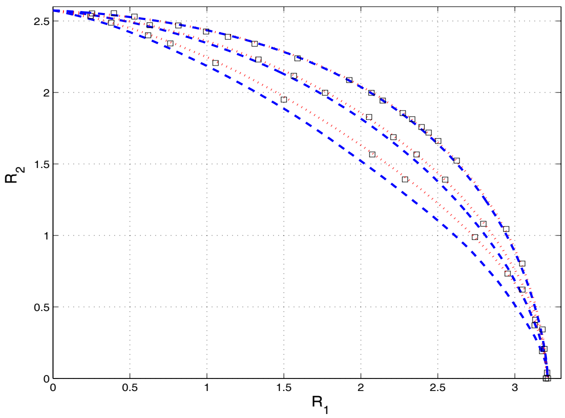

Fig. 1. Rate region achieved for $r_{ij}/r_{ii} = 2, 3, 5$ by i) NBI algorithm (red dashed curve); ii) IWFA (blue dotted curve); iii) GTA (square points).

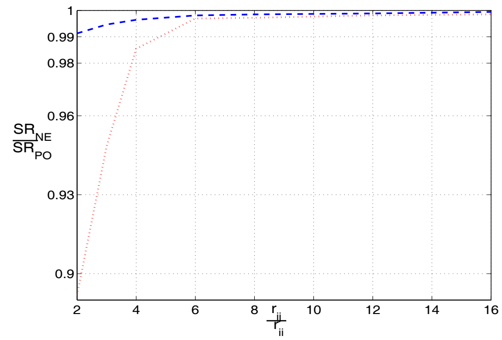

Fig. 2. Sum-rate ratio SR_{NE}/SR_{PO} vs. r_{ij}/r_{ii}, for i) SNR$=$ 5dB (blue dashed curve); ii) SNR$=$ 15dB (red dotted curve).

to converge to one of the stable NE's, from any starting point. If conditions (9) hold true, the iterative water-filling algorithm always converges to the *unique* NE [4], [10].
Comparing the payoff functions of the games $\mathscr{G}$ and $\tilde{\mathscr{G}}$, an interesting interpretation arises: the "socially" optimum of rate maximization can be achieved only if all players cooperate to find out the power allocation strategies. In fact, all the players in the game $\mathscr{G}$ maximize individually the same pay-off function (4). Pursuing an individual optimum provides a worse rate for *all* the players, instead. In the next section we quantify this rate loss.

5. PERFORMANCE AND CONCLUSIONS

We compare now the different centralized and decentralized solutions. We have simulated our algorithms using the following setup. The number of active links is $Q = 2$. The size of each transmitted block is $N = 32$; the channels are simulated as FIR filters of order $L = 6$, whose taps are iid complex Gaussian random variables with zero mean and unit variance; the additive noise $\boldsymbol{\eta}_r(n)$, for $r = 1, 2$ is assumed to be drawn from a complex white Gaussian random process with zero mean and variance σ_n^2, for each component. For the sake of simplicity, we have assumed also $r_{ii} = r_{jj}$ and $r_{ij} = r_{ji}$ for all $i, j = 1, 2$. In Fig.1 we report the achievable RR associated with the optimal power control algorithm based on NBI (red dashed curve) and IWFA (blue dotted curve), obtained under a total power constraint P_T, with $P_T/\sigma_n^2 = 20$ dB and for the ratios $r_{ij}/r_{ii} = 2, 3, 5$. On the boundary of the optimal RR, we have drawn some points obtained by using the algorithm based on the game theory approach (call it GTA). We report also the points obtained by GTA when the SC's are not satisfied. From Fig. 1, we infer that: i) the rates reached by IWFA are very close to the optimum, also in the case of high level of interference; ii) GTA converges to the optimal points also if the SC's are not met. We have pointed out that, if IWFA converges, the final NE is not, in general, a global optimum of (2). In order to quantify this rate loss, we introduce the normalized sum-rate ratio, defined as SR_{NE}/SR_{PO}, where SR_{NE} denotes the sum-rate reached by IWFA for a given channels set, whereas SR_{PO} is the maximum available sum-rate achievable by using the algorithms NBI and GTA. In Fig.2, we report the ratio SR_{NE}/SR_{PO}, averaged on 100 independent channel realizations in case of two cooperating S/D pairs using the same transmit power, as a function of r_{ij}/r_{ii}, for $SNR := \sigma_s^2/\sigma_n^2 = 5$ dB (blue dashed curve) and $SNR = 15$ dB (red dotted curve). Experimentally, we have found that the algorithms have always converged to the same value, regardless of SC's, and the channels and the power budgets for each pair. Interestingly, the rate loss is negligible as soon as $r_{ij}/r_{ii} > 2$. In summary, in this paper we have proposed and compared centralized and decentralized power control algorithms for meshed wireless networks. The centralized algorithm is optimal in the sense that enlarges the RR reached by the existing algorithms and provides the largest RR achievable, under the assumptions a1)-a4). From the comparison between the centralized and the decentralized IWFA algorithm, we have shown that IWFA achieves solutions very close to the optimum (with a rate loss less than 1%). Since IWFA can be implemented with a minimal centralized control and with a controlled convergence speed, IWFA appears to be a choice to be preferred to any other centralized algorithm.

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