# Resource Allocation Strategies for Wireless Ad-hoc Networks

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***Abstract*— In this paper, we propose resource allocation strategies for a class of wireless networks with a clustering protocol. The nodes are assumed stationary and establish connections with the master node according to a priority scheme that relates to their distances from the master node. The paper considers bandwidth allocation with soft and hard constraints, as well as power allocation strategies. Simulation results illustrate the performance of the proposed strategies.**

## I. Introduction

Utility based pricing and resource allocation strategies have received considerable attention in recent years (see, e.g., [1]–[5]). In these and other related works, several measures of fairness have been proposed. In this paper, we propose adaptive bandwidth allocation strategies that minimize proportional blocking probabilities (as defined in the sequel) for a class of ad-hoc wireless networks. We also propose a proportionally fair power allocation method for the nodes. Both these resource allocation issues are addressed for a wireless network that adopts a clustering protocol. The protocol is summarized as follows. The space is divided into $M+1$ virtual geographical cells, each containing $N$ nodes with one additional node acting as a master node. A frequency slot is allocated to each node that wishes to communicate with the master node in a cell. We allow for frequency reuse across cells in a manner similar to that in mobile cellular systems. The master nodes perform important tasks like routing and congestion control for high data rate communications, as well as handling some data processing and network information for nodes connected to them. Being a master node is power consuming and hence the nodes are made to take turns as master nodes with equal probability, but with the constraint that there can be only one master node in a cell at any time. Priority to be a master node is given to the node that has lowest interference from other cells. The nodes communicating in the same frequency slot in other cells cause interference with this cell and this interference is measured in terms of the signal-to-interference ratio (SIR) defined as follows. The SIR for node $i$ at time $k$ on an uplink channel is defined by

$$\gamma_i(k) = \frac{G_{ii} p_i(k)}{\sum\limits_{j \in \mathcal{A}} G_{ij} p_j(k) \; + \; \sigma^2} \tag{1}$$

where, for each time instant $k$, $G_{ij}$ denotes the channel gain from the $j-$th node to the intended master node of the $i-$th

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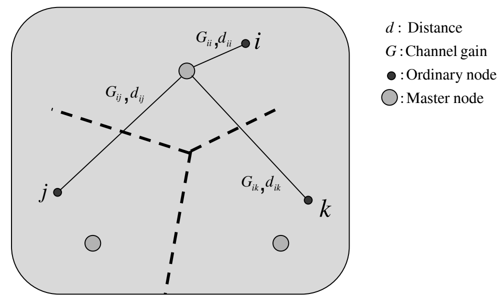


Fig. 1. *A schematic representation with three cells, three master nodes, and active and interfering nodes. The active node is node $i$ and the interfering nodes are nodes $j$ and $k$ from other cells.*

node, $p_i$ is the transmission power from the $i-$th node, and $\sigma^2$ is white Gaussian noise power at the receiver of the master node that node $i$ is connected to. Moreover, $\mathcal{A}$ denotes the set of all nodes that are interfering with node $i$ from all cells - see Fig. 1. We assume that the transmission power of each node at every instant satisfies $P_{\min} \le p_i(k) \le P_{\max}$.

A packet is transmitted from a source to its final destination through intermediary master nodes. We assume that each node has a buffer of sufficient size to store routed packets. The operation of the nodes in any cell follows a periodic cycle. Each cycle starts with a set-up phase when a node is chosen as a master node. The set-up phase is followed by a transmission phase during which all nodes in a cell that can communicate with the master node send their packets through available frequency slots. In the set-up phase, every node in a cell expresses its desire to be the master node with a probability that is equal to all nodes. When there is contention, the node with lowest interference power from other cells is chosen as the master node. Once a particular node is chosen as a master node, it lets all other nodes know through a broadcast in that cell that it is the master node for the current cycle. At the end of this set-up phase, it is decided based on the number of available frequency slots, say $Q_l$, for cell $l$, which nodes connect with the master node during the transmission phase. Nodes express a desire for connection with probabilities that are inversely proportional to their distances from the master node. If the number of nodes that express a desire to connect with the master node is more than $Q_l$, then the master chooses $Q_l$ nodes that are closest to it. Note that the number of slots $Q_l$ in a cell $l$ denotes the bandwidth available to that cell. We assume that each of the frequency slots has the same

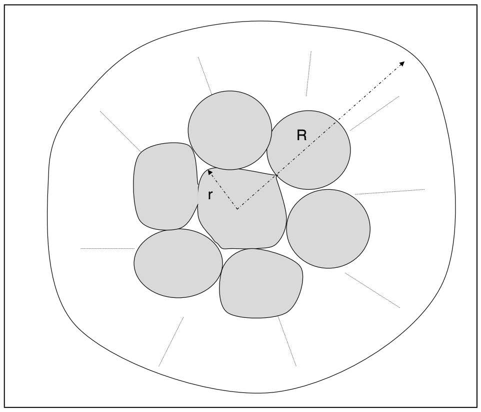


Fig. 2. *Nodes beyond a radius R do not cause interference with the nodes inside a cell of radius r.*

bandwidth.

In the transmission phase, routing decisions occur. We assume that the nodes in a cell are uniformly distributed inside a circle of radius $r$. Only nodes that use the same frequency slot as a particular node $i$ in other cells cause interference with node $i$. We assume that these interfering nodes are located within radius $R$ from the master node that $i$ is connected to. We also assume that each geographical cell is surrounded by $M$ other cells within an area $A_c$ and that these cells can cause interference as shown in Figure 2. We shall subsequently rely on the following definition.

**Definition:** The blocking probability in a cell $l$ is defined as $\mathrm{Prob}(\bar{Z} > Q_l)$, where $\bar{Z}$ is the average number of nodes that express a desire to connect with the master node and $Q_l$ is the number of frequency slots available in the cell. $\diamondsuit$

The rest of the paper is organized as follows. In section II, we propose a strategy to allocate the available frequency slots to different cells in the network without placing a hard constraint on the total number of frequency slots. We do this by first deriving a utility function in terms of a proportional blocking probability across cells and optimizing this function with respect to the number of frequency slots in each cell. In section III we use a similar utility function as in section II, but include hard constraints involving the number of frequency slots. In section IV, we describe a power allocation strategy for the nodes in the network.

## II. Bandwidth Allocation with Soft Constraints

Consider a cell $l$ in the network and let us order the nodes in the cell from $i = 1$ to $i = N$. Let $B_i$ denote the event that node $i$ expresses a desire to connect with the master node. Without loss of generality, we assume that the events $\{B_i\}$ are independent. Let $\mathcal{B}_N$ denote the sigma algebra formed by the events $\{B_1, .....B_N\}$. Let

$$Z_N = \sum_{i=1}^{N} I(B_i)$$

where $I(B_i)$ is the indicator function; it is equal to 1 if event $B_i$ occurs and 0 otherwise. The variable $Z_N$ denotes the total number of nodes that express a desire to connect with the master node; its value is a function of $N$. Let $Z_k = \sum_{i=1}^{k} I(B_i)$ denote the number of nodes that express a desire to connect with the master node if there were instead $k$ nodes in the cell. We first recall the following lemma.

**Lemma 1** (**Azuma's Inequality** [6]) Suppose $\{Y_0 = 0, Y_1, Y_2, Y_3, ....\}$ is a martingale sequence such that for each $k$, $|Y_k - Y_{k-1}| \leq c_k$, where $c_k$ may depend on $k$. Then, for all $k \geq 1$ and for any $\mu > 0$,

$$P(Y_k \geq \mu) \leq \exp\left\{-\mu^2 / \left(2\sum_{j=1}^{k} c_j^2\right)\right\}$$

where $P(\cdot)$ denotes probability of the event. $\diamondsuit$

It can be shown that $Y_k$ defined by

$$Y_k = Z_k - \sum_{i=1}^{k} P(B_i) \tag{2}$$

with $Y_0 = 0$, is a martingale. Here $P(B_i)$ denotes the probability of event $B_i$. Moreover, it can be seen that $|Y_k - Y_{k-1}| \leq 1$. It can be assumed under mild assumptions that for the protocol described in the previous section, there exists a positive scalar $\alpha < 1$ such that

$$\sum_{i=1}^{k} P(B_i) < \frac{1}{1-\alpha} \tag{3}$$

Now applying Azuma's inequality with $\mu = Q_l - 1/(1-\alpha)$, we get

$$P\left\{Y_k \geq Q_l - \frac{1}{1-\alpha}\right\} \leq \exp\left\{-\frac{(Q_l - \frac{1}{1-\alpha})^2}{2k}\right\} \tag{4}$$

which implies that

$$P\left\{\sum_{i=1}^{k} I(B_i) \geq Q_l\right\} \leq \exp\left\{-\frac{(Q_l - \frac{1}{1-\alpha})^2}{2k}\right\} \tag{5}$$

Considering that cell $l$ has $N$ nodes, we get

$$P\left\{\sum_{i=1}^{N} I(B_i) \geq Q_l\right\} \leq \exp\left\{-\frac{(Q_l - \frac{1}{1-\alpha})^2}{2N}\right\} \tag{6}$$

In other words, the probability that a node in cell $l$ is blocked is upper bounded by the right hand side of (6). We now allow each cell $l$ to maximize the following utility function:

$$U(Q_l) = \mu_l \sum_{l=1}^{M+1} \left\{1 - \exp\left(-\frac{\left(Q_l - \frac{1}{1-\alpha}\right)^2}{2N}\right)\right\} - \nu_l \left\{Q - \sum_{l=1}^{M+1} Q_l\right\}^2 \tag{7}$$

where $\mu_l$ and $\nu_l$ are proportionality constants. The first term in the above utility function proportionally minimizes the probability that a node is blocked in every cell and the second term imposes a soft constraint on the total number of frequency slots to be $Q$. Since the utility function is continuous and concave in each of the variables $Q_l$, for $Q_l > \frac{1}{1-\alpha}$, it admits an equilibrium in the region $Q_l > \frac{1}{1-\alpha}$, $l = 1, 2, ...., M+1$. The following gradient ascent algorithm seeks the equilibrium [9]:

$$Q_l(k+1) = Q_l(k) + 2\epsilon\nu_l \left\{ Q - \sum_{l=1}^{M+1} Q_l \right\}$$
$$-\frac{1}{N}\epsilon\mu_l \left\{ Q_l - \frac{1}{1-\alpha} \right\} \times \exp\left\{ -\frac{\left(Q_l - \frac{1}{1-\alpha}\right)^2}{2N} \right\} \quad (8)$$

with initial conditions $Q_l \gg 1/(1-\alpha)$, $l = 1, 2, ..., M+1$, and where $\epsilon$ is a positive step size. The actual number of slots in each cell will be chosen as the closest integer to each of the $Q_l$ in equilibrium.

## III. Bandwidth Allocation with hard constraints

We now consider an alternate optimization problem that optimizes a utility function with hard constraints on the bandwidth. The constraints also incorporate the scenario of finite buffer capacity in a master node. Let $T$ be the time span of the transmission cycle between a node in a cell and its master node. Let $f$ denote the rate of transmission of each node in bits per second. Then the amount of data received by the master node in this interval is

$$Q_l \times f \times T$$

Let the available buffer during a transmission cycle in a cell $l$ be $b_l$. We then pose the following optimization problem assuming a fixed rate of transmission $f$ by all nodes. We seek the optimal $Q_l$ as follows:

$$\max_{\{Q_l\}} \sum_{l=1}^{M+1} \mu_l \left\{ 1 - \exp\left( -\frac{\left(Q_l - \frac{1}{1-\alpha}\right)^2}{2N} \right) \right\}$$

subject to

$$Q_l f T \leq b_l, \quad l = 1, 2, ..., M+1$$
$$\sum_{l=1}^{M+1} Q_l \leq Q$$

where $Q$ is the total number of frequency slots available in the network. Let $\mathcal{P}$ denote the set

$$\mathcal{P} = \{Q_l : Q_l f T \leq b_l, l = 1, 2, ..., M+1\}$$

We propose the following modified Poljak (steepest ascent) algorithm [7]–[9] to seek the equilibrium values of $Q_l$. Consider positive sequences $\{\alpha_k\}$ and $\{\beta_k\}$ with the following properties:

$$\begin{aligned} \lim_{k\to\infty} \alpha_k &= 0, \quad \lim_{k\to\infty} \beta_k = 0 \\ \sum_{k=1}^{\infty} \alpha_k &= \infty, \quad \sum_{k=1}^{\infty} \beta_k = \infty, \quad \lim_{k\to\infty} \frac{\alpha_k}{\beta_k} = 0 \end{aligned}$$

An example of such a pair of sequences is $\alpha_k = 1/k$, $\beta_k = 1/\sqrt{k}$. We then update the bandwidth in the following manner. At each iteration $k$,

$$\begin{aligned} &\text{if } \sum_{l=1}^{M+1} Q_l(k) \leq Q \\ &\qquad Q_l(k+1) = [Q_l(k) + \alpha_k U'(Q_l(k))]_{\mathcal{P}} \\ &\text{else} \\ &\qquad Q_l(k+1) = Q_l(k) - \beta_k \epsilon \end{aligned} \quad (9)$$

with initial conditions $Q_l \gg 1/(1-\alpha)$, $l = 1, 2, ..., M+1$, where $[.]_{\mathcal{P}}$ denotes projection onto the set $\mathcal{P}$ and $\epsilon$ is a small positive constant. The projection operation is such that if $Q_l f T > b_l$, then $Q_l$ is chosen to satisfy $Q_l f T = b_l$. Moreover, $U'(.)$ denotes the derivative of the following utility function with respect to $Q_l$:

$$U(Q_l) = \sum_{l=1}^{M+1} \mu_l \left\{ 1 - \exp\left( -\frac{\left(Q_l - \frac{1}{1-\alpha}\right)^2}{2N} \right) \right\}$$

It can be shown in a manner similar to [7] that the above algorithm converges to the equilibrium that maximizes the given utility function.

## IV. Joint Power and Rate Allocation

Consider a particular frequency slot. Let $i = 1, 2, ..., M+1$ index the nodes that are transmitting at this frequency slot in all cells. Let $p_i$ denote the power at which node $i$ transmits to the respective master node in its cell. Let $f_i$ denote the flow rate at node $i$. Now, in view of Shannon's capacity formula [10], the flow rate $f_i$ demands an SIR level $\gamma_i$ that is given by

$$f_i = \frac{1}{2}\log_2[1 + \gamma_i] \quad (10)$$

Let $\bar{x}_i$ denote the dB value of a variable $x_i$, namely $\bar{x}_i = 10 \log x_i$. Now, usually, during normal network operation, $\gamma_i \gg 1$ and, hence, $f_i$ in (10) is proportional to $\bar{\gamma}_i$. We can then pose the following utility maximization problem for $f_i$ and $p_i$:

$$\max_{\{P_{\min} \leq p_i \leq P_{\max}\}} \left( \sum_{i=1}^{M+1} \kappa_i(\bar{\gamma}_i - \delta_i p_i) \right)$$

where $\delta_i$ is a positive constant and the $\{\kappa_i\}$ are proportionality factors. The above utility function maximizes the rate by restricting the power consumption in a soft manner. Note that the above problem has an equilibrium since the cost function is continuous and concave in each of its variables $p_i$; each $p_i$ belongs to a convex set. We again call upon the modified Poljak algorithm as given in the previous section:

$$p_i(k+1) = [p_i(k) + \alpha_k U'(p_i(k))]_{\mathcal{S}} \quad (11)$$

to update the power levels of the individual nodes where now

$$U(p_i) = \sum_{i=1}^{M+1} \kappa_i(\bar{\gamma}_i - \delta_i p_i)$$

and $\gamma_i$ depends on $p_i$ as in (1). Moreover, $\mathcal{S}$ denotes the set

$$\mathcal{S} = \{p_i : P_{\min} \leq p_i \leq P_{\max}, i = 1, 2, ....M + 1\}$$

with $[.]_{\mathcal{S}}$ being a projection onto $\mathcal{S}$. The projection onto $\mathcal{S}$ sets $p_i$ to $p_i = P_{\max}$ if $p_i > P_{\max}$ and $p_i = P_{\min}$ if $p_i < P_{\min}$.

## V. Simulations

To illustrate the convergence of the algorithm in Section II, we distribute the frequency slots to three cells with a soft constraint on the total number of slots to be $Q = 400$. Figure 3 shows the convergence of the frequency slots to equilibrium values by using (8). The simulation parameters chosen for the utility function are:

$$\mu_1 = 0.1,\ \mu_2 = 1,\ \mu_3 = 0.1,\ \nu_1 = 0.1,\ \nu_2 = 0.2,\ \nu_3 = 0.15$$

and $\epsilon = 0.1$. Figure 4 illustrates the performance of the algorithm in section III. The simulation parameters for Figure 4 are

$$fT = 1,\quad Q = 700,\quad \alpha_k = 1/k,\quad \beta_k = 1/\sqrt{k},\quad b_l = 300$$

for $l = 1, 2, ..., M + 1$. Figure 5 shows the convergence of the power levels for three mutually interfering nodes in three different cells that use the same frequency slot. Here, $P_{\max} = 10$ and $P_{\min} = 0.1$.

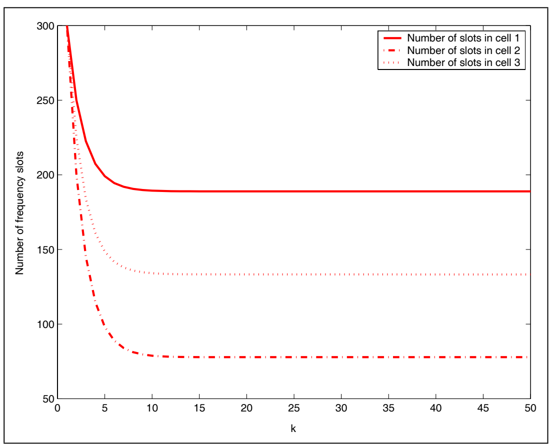


Fig. 3. *Convergence of the number of frequency slots within a three cell network using algorithm (8).*

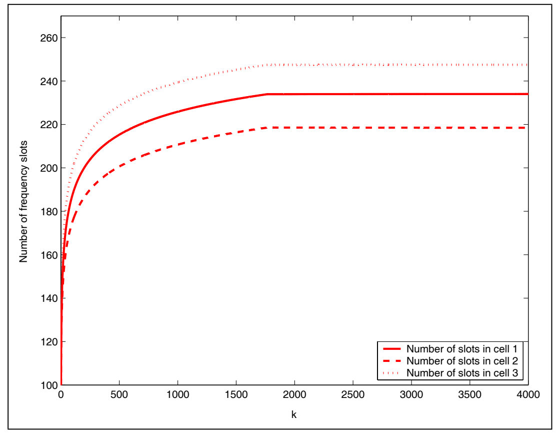


Fig. 4. *Convergence of the number of frequency slots within a three cell network using algorithm (9).*

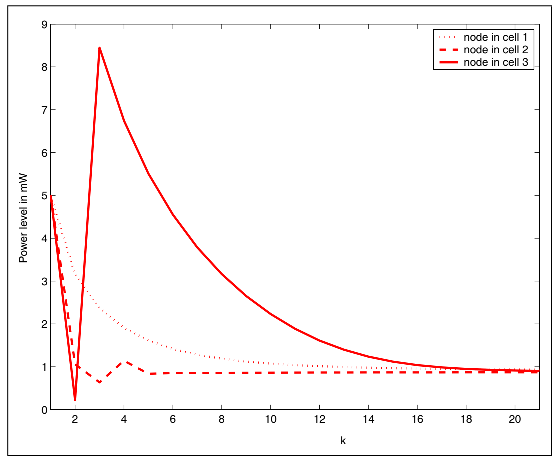


Fig. 5. *Convergence of the power levels to the equilibrium using algorithm (11).*

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