DIVERSITY ANALYSIS OF ORTHOGONAL SPACE-TIME MODULATION FOR DISTRIBUTED WIRELESS RELAYS

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ABSTRACT

For ad hoc mobile networks, parallel wireless relays with space-time modulation have recently been discovered, and their potential to increase network capacity has been found to be significant. In this paper, we provide an analytical study of the diversity factor achievable by two relays with orthogonal space-time modulation. Unlike two transmitters or two receivers in the conventional MIMO setting for which the averaged bit error rate (BER) is proportional to $1/SNR^2$, the averaged BER of two relays is shown to be proportional to $\ln(SNR)/SNR^2$. This insight explains a difference between the diversity of multiple relays and the diversity of multiple receivers/transmitters. A switching scheme applied to two relays is also analyzed.

1. INTRODUCTION

For outdoor none-line-of-sight mobile communications, the wavelength must not be too small and the conventional antenna arrays may not be applicable for mobile users. For example, with carrier frequency below 900 MHz, only a single antenna is generally possible for most mobile users. The conventional architecture designed for ad hoc mobile networks only allows communications on a node-to-node basis where there is no antenna diversity to conquer small scale fading. When two nodes communicate with each other, all other nodes in a neighborhood of the two nodes are muted (i.e., stopped from transmission) by the current ad hoc protocols to avoid interference. This can be a waste of resources because researchers have recently discovered that mobile nodes in a neighborhood of the source and the destination can jointly participate in the data transmission. The cooperation of mobile nodes may result in an improved capacity [7], [2], [3], [9], [11], [6], [5]. A study on wireless relays with space-time modulations further discovered that more than 10 dB power saving is achievable by using only eight relays, and a (conservative) empirical value of the diversity factor (in terms of bit error rate) achievable by r relays is about $r/2$ [6], [5].

In this paper, we report an analytical insight into wireless relays with space-time modulation. In particular, we show a diversity analysis of two relays with orthogonal space-time modulation. Our analysis concludes that the diversity factor of two relays is governed by

$$d = 2 - \ln[\ln(SNR)]/\ln(SNR)$$

instead of $d = 2$. This diversity factor is different from that for two receivers/transmitters, which reveals a distinction between multiple relays and multiple receivers/transmitters. In other words, the theory developed for MIMO systems with space-time modulation is not directly transportable to relays. Mobile relays with space-time modulation commands a new branch of theory on space-time processing.

Also included in our analysis is a simple switching scheme applied at the relays. We show that such a simple scheme can lead to a further reduction of bit-error-rate(BER).

The paper is organized as follows. In section 2, we provide a brief review of two relays with orthogonal space-time modulation, and express the conditional BER at the destination served by the two relays. In section 3, we sketch a derivation of the closed-form expression of the BER averaged over the small scale fading. In section 4, we provide the diversity analysis. In section 5, the performance of the switching scheme is illustrated. Finally, section 6 concludes the paper.

2. CONDITIONAL BER AT THE DESTINATION ASSISTED BY RELAYS

We assume two relays assisting a source and a destination. Each of the two relays estimates two consecutive symbols, $s(1)$ and $s(2)$, transmitted from the source, applies orthogonal space-time modulation to the two symbols, and then forwards the space-time modulated symbols to the destination. The relays concurrently execute the above operations, and no symbols are exchanged between them. More specifically, the output symbols of the two relays are defined as:

$$\begin{pmatrix} x_1(1) & x_2(1) \\ x_1(2) & x_2(2) \end{pmatrix} = \begin{pmatrix} s_1(1) & -s_2(2)^* \\ s_1(2) & s_2(1)^* \end{pmatrix} \quad (1)$$

where $x_i(j)$ is the j^{th} symbol output from the i^{th} relay, and $s_i(j)$ is the j^{th} symbol received by i^{th} relay. The symbols received by the relays are modelled as $s_i(j) = a_i s(j) + n_i(j)$ where a_i is the small scale fading factor between the source and the i^{th} relay and $n_i(j)$ is the complex white Gaussian noise of variance σ_n^2. The structure in (1) resembles the Alamouti code [1].

The received symbols at the destination can be described as:

$$y(j) = \sum_{i=1}^{2} b_i x_i(j) + v(j) \quad (2)$$

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or equivalently,

$$\begin{pmatrix} y(1) \\ y(2)^* \end{pmatrix} = \begin{pmatrix} b_1a_1 & -b_2a_2^* \\ b_2^*a_2 & b_1^*a_1^* \end{pmatrix} \begin{pmatrix} s(1) \\ s(2)^* \end{pmatrix} + \begin{pmatrix} b_1n_1(1) - b_2n_2(2)^* \\ b_1^*n_1(2)^* + b_2^*n_2(1) \end{pmatrix} + \begin{pmatrix} v(1) \\ v(2)^* \end{pmatrix} \tag{3}$$

where b_i is the small scale fading factor between the i^{th} relay and the destination. $v(i)$ is the additional complex white Gaussian noise of variance σ_v^2 at the destination. In (3) the coefficient matrix of the original symbol vector is complex orthogonal and the last two noise terms are white and of the variance $\sigma_n^2 \sum_1^2 |b_i|^2 + \sigma_v^2$.

An optimal detection of each symbol at the destination can be based on each individual component of the following sufficient statistics:

$$\begin{pmatrix} r(1) \\ r(2)^* \end{pmatrix} = \begin{pmatrix} b_1a_1 & -b_2a_2^* \\ b_2^*a_2 & b_1^*a_1^* \end{pmatrix}^H \begin{pmatrix} y(1) \\ y(2)^* \end{pmatrix} \tag{4}$$

The error rate is governed by the SNR of this statistics. If the relays are close to the destination, then the following approximation may hold:

$$\sigma_n^2 \sum_{i=1}^{2} |b_i|^2 + \sigma_v^2 \approx \sigma_n^2 \sum_{i=1}^{2} |b_i|^2 \tag{5}$$

Therefore, the SNR of $r(j)$ is given by the following expression:

$$SNR_I = SNR_0 \frac{\sum_{i=1}^2 |a_i|^2 |b_i|^2}{\sum_{i=1}^2 |b_i|^2} \tag{6}$$

where SNR_I is the (instantaneous) SNR conditional upon the small scale fadings, SNR_0 is a nominal SNR. All of a_i and b_i are assumed to be i.i.d circular complex Gaussian random variables with unit variance.

We now add a generalization based on switching, i.e, at the i^{th} relay, $|a_i|^2$ is compared with a predetermined threshold T. If the threshold is exceeded, the relay is left open. Otherwise, the relay is turned off. In other words, a_i is multiplied by g_i defined as:

$$g_i = \begin{cases} 1, & |a_i|^2 \geq T; \\ 0, & |a_i|^2 < T. \end{cases} \tag{7}$$

Then, the conditional SNR at the destination is

$$SNR_I = SNR_0 \frac{\sum_{i=1}^2 |a_i|^2 g_i |b_i|^2}{\sum_{i=1}^2 g_i |b_i|^2} \tag{8}$$

Note that $\underline{g} = [g_1, g_2]^T$ is a function of the random variables a_1 and a_2 and the threshold T.

Assume that each symbol comes from the QPSK constellation and the noise is white Gaussian. Then, the conditional BER of an optimal symbol detector is known to be

$$BER(\underline{a}, \underline{b}) = Q(\sqrt{SNR_I} | \underline{a}, \underline{b})$$

where Q(.) is the Gaussian Q-function [8].

3. CLOSED-FORM EXPRESSION OF THE AVERAGED BER

We now sketch a derivation of the closed-form expression of the averaged BER by following three steps. In the first step, we derive $\overline{BER}(T|\underline{b}, \underline{g})$, which is the averaged BER respect to $\underline{a}$ but conditional upon $\underline{b}$ and $\underline{g}$. In the second step, we derive $\overline{BER}(T|\underline{b})$, which is the average of $\overline{BER}(T|\underline{b}, \underline{g})$ with respect to $\underline{g}$ conditional upon $\underline{b}$. Finally, we consider the average of $\overline{BER}(T|\underline{b})$ with respect to $\underline{b}$.

3.1. Expression of $\overline{BER}(T|\underline{b}, \underline{g})$

We denote β_i as $|a_i|^2$ conditional upon $g_i = 1$. Therefore probability density function(PDF) of β_i is

$$p(\beta_i) = \exp(-\beta_i)\exp(T)$$

with $\beta_i \geq T$.

With the help of (4.2) in [13]:

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp(\frac{-x^2}{2\sin^2\theta})\, d\theta$$

and (3.466.1) in [4]:

$$\int_0^{+\infty} \frac{\exp(-u^2x^2)}{x^2+\beta^2}\, dx = (1 - erf(u\beta))\frac{\pi}{2\beta}\exp(u^2\beta^2)$$
$$\Re(\beta) > 0; |\arg(u)| < \pi/4$$

where $erf(x)$ is the standard Gaussian error function, we can show that:

$$\begin{aligned}
& \overline{BER}(T|\underline{b}, \underline{g} = [1,1]^T) \\
= \; & \mathbf{E}_{\underline{\beta}}[Q(\sqrt{SNR_0 \tfrac{\sum_{i=1}^2 \beta_i |b_i|^2}{\sum_{i=1}^2 |b_i|^2}})] \\
= \; & \mathbf{E}_{\underline{\beta}}[\tfrac{1}{\pi}\textstyle\int_0^{\pi/2} \exp(-SNR_0 \frac{\frac{\sum_{i=1}^2 \beta_i |b_i|^2}{\sum_{i=1}^2 |b_i|^2}}{2\sin^2\theta})\, d\theta] \\
= \; & \tfrac{1}{\pi}\textstyle\int_0^{\pi/2} \frac{1}{SNR_0 \frac{|b_1|^2}{2\sum_{i=1}^2 |b_i|^2 \sin^2\theta} + 1} \\
& \frac{1}{SNR_0 \frac{|b_2|^2}{2\sum_{i=1}^2 |b_i|^2 \sin^2\theta} + 1} \exp(-SNR_0 \tfrac{T}{2\sin^2\theta})\, d\theta \\
= \; & \tfrac{0.5\exp[T(1+r)]}{(r-1)\sqrt{2\alpha r + 2\alpha + 1}}[1 - erf(\sqrt{T(1 + r + 0.5\alpha^{-1})})] \\
& + 0.5[1 - erf(\sqrt{0.5T\alpha^{-1}})] - \tfrac{0.5}{r-1}\tfrac{r^{1.5}\exp[T(1+r^{-1})]}{\sqrt{2\alpha + (2\alpha+1)r}} \\
& \times [1 - erf(\sqrt{T(1 + r^{-1} + 0.5\alpha^{-1})})]
\end{aligned} \tag{9}$$

where r and α are

$$r = |\frac{b_1}{b_2}|^2 \text{ and } \alpha = SNR_0^{-1} \tag{10}$$

Similarly, we can show that:

$$\begin{aligned}
& \overline{BER}(T|\underline{b}, \underline{g} = [1,0]^T) \\
= & \overline{BER}(T|\underline{b}, \underline{g} = [0,1]^T) \\
= & \frac{1}{\pi}\int_0^{\pi/2} \frac{1}{SNR_0 \frac{1}{2\sin^2\theta} + 1} \\
& \times \exp(-SNR_0 \frac{T}{2\sin^2\theta})\, d\theta \\
= & 0.5(1 - erf(\sqrt{0.5T\alpha^{-1}})) - \\
& \frac{0.5\exp(T)}{\sqrt{1+2\alpha}}(1 - erf(\sqrt{T(0.5\alpha^{-1}+1)})) \qquad (11)
\end{aligned}$$

When $\underline{g} = 0$, both relays are turned off and hence $\overline{BER}$ is 0.5 obviously.

3.2. Expression of $\overline{BER}(T|\underline{b})$

$$\begin{aligned}
& \overline{BER}(T|\underline{b}) \\
= & \mathbf{E}_{\underline{g}}[\overline{BER}(T|\underline{b}, \underline{g})] \\
= & \overline{BER}(T|\underline{b}, \underline{g} = [1,1]^T)P(\underline{g} = [1,1]^T) + \\
& 2\overline{BER}(T|\underline{b}, \underline{g} = [0,1]^T)P(\underline{g} = [0,1]^T) \\
& +0.5P(\underline{g} = [0,0]^T) \qquad (12)
\end{aligned}$$

Based on (9), (11), (12) and $P(g_i = 1) = \exp(-T)$, we can obtain a close form of averaged BER only conditional upon $\underline{b}$.

When $T = 0$, the corresponding BER is

$$\begin{aligned}
\overline{BER}(T = 0|\underline{b}) = & 0.5 - \frac{r}{2(r-1)\sqrt{2\alpha r^{-1} + 2\alpha + 1}} \\
& + \frac{1}{2(r-1)\sqrt{2\alpha r + 2\alpha + 1}} \qquad (13)
\end{aligned}$$

3.3. Expression of $\overline{BER}(T)$

Averaging $\overline{BER}(T|\underline{b})$ with respect to $\underline{b}$ is the same as averaging with respect to r since the scaling on $\underline{b}$ does not change SNR or BER, i.e, $\overline{BER}(T|\underline{b}) = \overline{BER}(T|\underline{r})$. According to (10), the PDF of r is a $F(2,2)$ distribution which is $(r+1)^{-2}$ [10]. Therefore, the final averaged BER is

$$\overline{BER}(T) = \int_0^{+\infty} \overline{BER}(T|r)(r+1)^{-2}\, dr \qquad (14)$$

When $T = 0$, (14) becomes

$$\overline{BER}(T = 0) = B_1 + B_2 \qquad (15)$$

where

$$\begin{aligned}
B_1 = & 0.5 + \frac{\ln(\sqrt{2\alpha+1} + \sqrt{4\alpha+1})}{2\sqrt{4\alpha+1}} \\
& +(\alpha - 0.5)\ln(1 + \sqrt{2\alpha+1}) - 0.5\sqrt{2\alpha+1} \\
B_2 = & -\frac{(4\alpha+1)^{-0.5} + 2\alpha - 1}{4}\ln(2\alpha)
\end{aligned}$$

4. DIVERSITY ANALYSIS

When $r \to 0$, the limit of (13) is

$$\overline{BER}(T = 0|r = 0) = 0.5[1 - (2\alpha+1)^{-0.5}] \sim 0.5\alpha$$

where the approximation holds for $\alpha \ll 1$. This equation implies a diversity factor equal to 1, which is expected (since there is only one effective relay when $r \to 0$). We can prove that $\overline{BER}(T = 0|r)$ achieves its minimum when $r = 1$, and the minimum is

$$\begin{aligned}
& \overline{BER}(T = 0|r = 1) \\
= & -\frac{\alpha}{(4\alpha+1)^{3/2}} + 0.5 - \frac{0.5}{\sqrt{4\alpha+1}} \\
\sim & 3\alpha^2 \qquad (16)
\end{aligned}$$

This equation implies a diversity factor 2, which is also expected [12] (since the two relays are equivalent to two receivers when $r = 1$). From the above two extreme cases, we can expect that the average of $\overline{BER}(T = 0|r)$ over r should have a diversity factor less than 2.

To prove the above prediction, we simply need to consider (15). We first observe that

$$\begin{aligned}
B_1 & \sim 0.5\alpha^2 \\
B_2 & \sim 6\alpha^2\ln(0.5\alpha^{-1})
\end{aligned}$$

Therefore, if $\ln(\alpha^{-1}) \gg 1$, then

$$\overline{BER}(T = 0) \sim 6\alpha^2\ln(\alpha^{-1})$$

Then, we can show that the diversity factor (i.e., the slope of the BER curve) is

$$\begin{aligned}
d(\alpha) & \triangleq -\frac{\ln(\overline{BER}(T = 0))}{\ln(\alpha^{-1})} \\
& \sim \frac{-\ln(\ln\alpha^{-1}) - 2\ln\alpha}{\ln\alpha^{-1}} \\
& = 2 - \frac{\ln(\ln SNR_0)}{\ln SNR_0} \qquad (17)
\end{aligned}$$

It is noted that the diversity factor is generally less than 2 and approaches 2 only when the SNR is extremely large.

5. OPTIMAL SWITCHING

The optimal value of the threshold T is hard to find since it must minimize (14). Here we consider the following choices of the threshold:

$$\begin{aligned}
T_0 & = SNR_0^{-1}\exp(-SNR_0^{-1}) \\
T_1 & = SNR_0^{-1} \\
T_2 & = \frac{2^{2\times R} - 1}{SNR_0}
\end{aligned}$$

where T_2 mimics a threshold given in [7] (here $R = 1$), T_1 differs from T_2 by a constant factor, and T_1 is equivalent to T_0 for large SNR.

We compare the following two ratios versus r in Figure 1:

$$\gamma(T_1|r) \triangleq \frac{\overline{BER}(T = T_1|r)}{\overline{BER}(T = 0|r)}$$

and

$$\gamma(T_0|r) \triangleq \frac{\overline{BER}(T = T_0|r)}{\overline{BER}(T = 0|r)}$$

We only need to consider $r = 1 \rightarrow +\infty$ since the other part for $r = 0 \rightarrow 1$ is symmetric. The figure shows that T_0 keeps the ratio less than 1 for all SNR_0 and all r. T_1 works poorly only for very small SNR_0 that are not of practical interest.

In Figure 2, We provide numerical results of (14) with four different choices of the threshold. The four thresholds are $T = 0$, $T = T_0$, $T = T_1$ and $T = T_2$. We can see that T_0 is the best among the four.

6. CONCLUSION

We have presented an analytical study of the diversity factor of two relays with orthogonal space-time modulation. Our result suggests that the diversity notion developed for multiple receivers/transmitters is not directly applicable to multiple relays. For two relays with orthogonal space-time modulation, the diversity factor is

$$2 - \ln[\ln(SNR)]/\ln(SNR)$$

instead of 2. We have also considered a switching scheme embedded in the space-time modulation, which further reduces the averaged BER.

7. REFERENCES

[1] S. M. Alamouti, “A simple transmit diveristy technique for wireless communications,” *IEEE J. Select. Area Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[2] P. A. Anghel, G. Leus, and M. Kaveh, “Multi-user space-time coding in cooperative networks,” in *Proc. of IEEE ICASSP’2003*, vol. IV, Hong Kong, April 2003, pp. 73–77.

[3] S. Barbarossa and G. Scutari, “Cooperative diversity through virtual arrays in multihop networks,” in *Proc. of IEEE ICASSP’2003*, vol. IV, Hong Kong, April 2003, pp. 210–212.

[4] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 5th ed. San Diego, CA: Academic Press, 1994.

[5] Y. Hua, Y. Mei, and Y. Chang, “Parallel wireless mobile relays with space time modulations,” in *IEEE Workshop on Statistical Signal Processing*, St. Louis, Missouri, Sept. 28-Oct. 1, 2003.

[6] ——, “Wireless antennas - making wireless communications perform like wireline communications,” in *IEEE AP-S Topical Conference on Wireless Communication Technology*, Honolulu, Hawaii, Oct.15-17, 2003, pp. 1–27.

[7] J. N. Laneman and G. W. Wornell, “Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks,” in *IEEE Globecom 2002*, Taipei, Taiwan, 2002.

[8] J. G. Proakis, *Digital Communication*, 4th ed. McGraw-Hill, 2001.

[9] A. Scaglione and Y. W. Hong, “Opportunistic large arrays: Cooperative transmission in wireless multihop ad hoc networks to reach far distances,” *IEEE Trans. on Signal Processing*, vol. 51, no. 8, pp. 2082–2092, August 2003.

[10] L. L. Scharf, *Statistical Signal Processing:Detection, Estimation and Time series Anaylsis*. Addison-Welsey Publishing Company, 1991.

[11] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperative diversity - part ii: Implementation aspects and performance analysis,” *IEEE Trans. on Communications*, to appear.

[12] M. K. Simon, “Evaluation of averaged bit error probability for space-time coding based on a simpler exact evaluation of pairwise error probability,” *J. Commun. Networks*, vol. 3, no. 3, pp. 257–267, 2001.

[13] M. Simon and M. Alouini, *Digital Communication Over Generalized Fading Channels: A Unified Approach to Performance Analysis*. New York:Wiley, 2002.

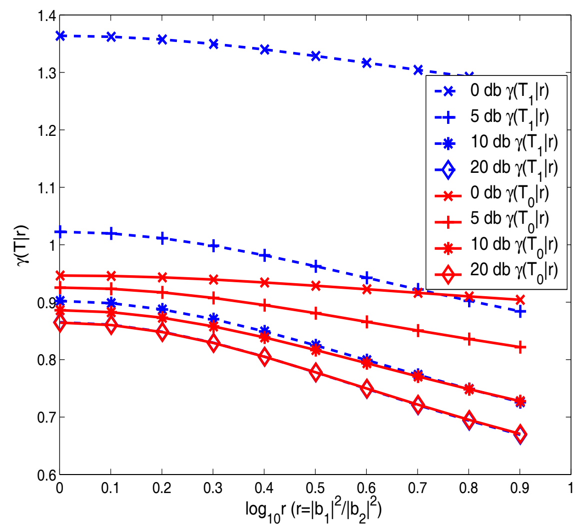

Fig. 1. $\gamma(T|r)$ versus r. $SNR_0 = 0, 5, 10, 20$ db.

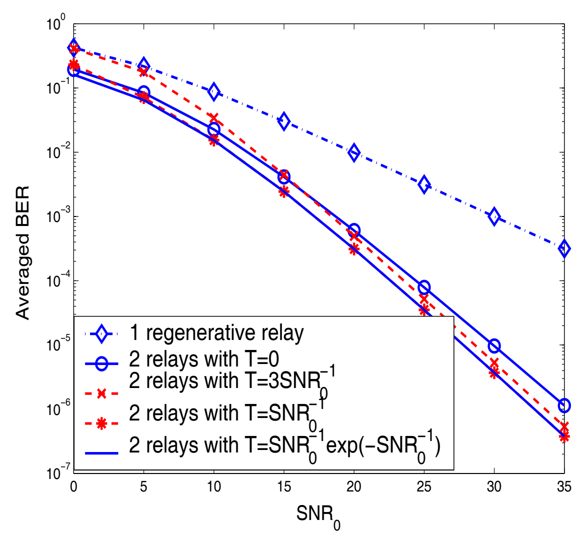

Fig. 2. Averaged BER versus the nominal SNR.