ADAPTIVE MULTIUSER OPPORTUNISTIC FAIR TRANSMISSION SCHEDULING IN POWER-CONTROLLED CDMA SYSTEMS

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ABSTRACT

We consider the problem of multiuser opportunistic fair transmission scheduling (OFS) in power-controlled CDMA systems. OFS is an important technique in wireless networks to achieve fair and efficient resource allocation. Power control is an effective resource management technique in CDMA systems. Given a certain user subset and the channel states, the optimal power control scheme can be calculated. The multiuser OFS problem then refers to the optimal user subset selection at each interval to maximize instantaneous system throughput subject to some fairness constraint. We propose discrete stochastic approximation algorithms to adaptively select user subsets with the maximal system throughput. We also consider the time-varying channel scenarios where the algorithm can track the time-varying optimal user subset. We present simulation results to show the performance of the proposed algorithms in terms of the throughput optimization, the fast convergence, the excellent time-varying tracking capability, and the fairness.

1. INTRODUCTION

Opportunistic fair transmission scheduling (OFS) is an important resource management technique in wireless networks [1, 2, 3]. It aims at balancing two conflicting goals, fairness and utilization of resource, and thus, in general, a tradeoff is needed. Most of the existing works treat the single user scheduler [1, 2]. In [3], an multiuser scheduler is proposed and a general framework is presented based on [1, 2]. Note that, instead of incorporating the physical-layer constraints and implementation details, perfect channel knowledge is assumed and simplified models are employed in the above approaches. Specifically, in [2], the physical-layer condition for each user is captured by a single value of channel-dependent metric; in [3], it is described by a per bit power cost for certain signal-to-interference-plus-noise ratio (SINR).

Power control is an effective resource management technique in CDMA systems. SINR-based power control (SBPC) aims at eliminating the near-far effects by balancing the SINRs of the users. Several approaches have already been developed [4, 5, 6, 7].

Note that the optimal power control scheme can be calculated for each user subset. The multiuser OFS problem then refers to the optimal user subset selection at each scheduling interval to maximize the instantaneous system throughput subject to the fairness constraint. Straightforward implementation of the user subset selection suffers from several problems in practice. One is the high computational complexity, for the number of candidates is normally large. Another problem is that the perfect channel knowledge is not available. In addition, when the channel is time-varying, the algorithm should be able to track the time-varying optimal user subset. In this paper, we propose discrete stochastic approximation algorithms to achieve the optimal user subset selection. The approach is based on the advanced stochastic optimization techniques [8, 9], which has recently been applied to solve some other problems in wireless communications [10]. The algorithm optimizes the performance metric over a set of user subsets, where the performance metric cannot be analytically evaluated but can be estimated using noisy channel observations. Moreover, it adopts a fixed step-size so that the time-varying optimal user subset can be adaptively tracked.

The remainder of this paper is organized as follows. In Section 2, the OFS architecture and the signal model of power-controlled CDMA systems are described, and the user subset selection is formulated as a discrete stochastic optimization problem. Section 3 presents the discrete stochastic approximation algorithm, and then the optimal throughput scheduler for power-controlled CDMA systems is proposed. Moreover, the time-varying user subset tracking algorithm is developed. Simulation results are given in Section 4, and Section 5 contains the conclusions.

2. SYSTEM DESCRIPTIONS

2.1. Multiuser Opportunistic Fair Transmission Scheduling

Figure 1 shows the generalized OFS architecture [3], which consists of a scheduler and a controller. The scheduler chooses a number of users at each interval to maximize the weighted instantaneous system throughput. The controller guarantees the fairness among the users by adjusting the weights of the users.

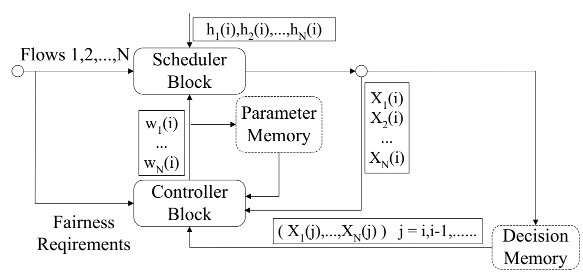

Fig. 1. Generalized architecture of OFS.

Suppose that there are totally N users in the system. At each scheduling interval i, the inputs to the scheduler are the data flows, the channel conditions $\{h_1(i), \cdots, h_N(i)\}$, and the weights

$\boldsymbol{w}(i) = [w_1(i), \cdots, w_N(i)]^T$. Denote θ as a user subset and $|\theta|$ as the number of users in θ. Let Θ be the set of all possible user subsets. Denote $\boldsymbol{H}_\theta(i)$ as the channel set of the users in θ and $\boldsymbol{x}(i) = [X_1(i), \cdots, X_N(i)]^T \geq 0$ as the instantaneous rates of the users. Define the objective function $\Phi(\boldsymbol{H}_\theta(i)) \triangleq \sum_{n\in\theta} w_n(i)X_n(i)$ as the instantaneous total weighted rate for θ. Then the user subset selection is formulated as the discrete optimization problem

$$\theta^*(i) = \arg\max_{\theta\in\Theta} \Phi(\boldsymbol{H}_\theta(i)) = \max_{\theta\in\Theta} \sum_{n\in\theta} w_n(i)X_n(i), \qquad (1)$$

where $\theta^*(i)$ denotes the optimal user subset for the interval i.

For the controller, the inputs at each interval i are the throughput priorities of the users $\psi = [\psi_1, \cdots, \psi_N]^T$ and $\boldsymbol{x}(i)$. The deterministic fairness constraint is given by [3] $\frac{\psi_1}{E\{X_1(i)\}} = \cdots = \frac{\psi_N}{E\{X_N(i)\}}$. Define $\boldsymbol{y}(i, \boldsymbol{w}(i)) \triangleq \frac{\psi}{\sum_{j=1}^N \psi_j} - \frac{\boldsymbol{x}(i,\boldsymbol{w}(i))}{\sum_{j=1}^N X_j(i,\boldsymbol{w}(i))}$, where $\boldsymbol{x}(i, \boldsymbol{w}(i))$ denotes the decision under given $\boldsymbol{w}(i)$. To guarantee the fairness constraint, $\boldsymbol{w}(i)$ is then updated by [2, 3]

$$\boldsymbol{w}(i+1) = \boldsymbol{w}(i) + \nu(i)\boldsymbol{y}(i, \boldsymbol{w}(i)), \qquad (2)$$

where $\nu(i) = \frac{1}{i}$ is the step-size.

2.2. Discrete Stochastic Optimization Formulation

Henceforth, it is assumed that $\boldsymbol{H}_\theta(i)$ and $\boldsymbol{w}(i)$ remain fixed for each i. For notational simplicity, we then drop the index i. Note that in practice, the channels in $\boldsymbol{H}_\theta$ are estimated and thus noisy. Denote $\{\hat{\boldsymbol{H}}_\theta(m),\ m = 1, 2, \cdots\}$ as a sequence of the noisy estimates of $\boldsymbol{H}_\theta$. For each $\hat{\boldsymbol{H}}_\theta(m)$, we can compute $\phi(m, \theta)$, the corresponding estimate of $\Phi(\boldsymbol{H}_\theta)$. Then we obtain the sequence $\{\phi(m, \theta),\ m = 1, 2, \cdots\}$. If each $\phi(m, \theta)$ is an unbiased estimate of $\Phi(\boldsymbol{H}_\theta)$, the problem (1) can then be reformulated as the following discrete stochastic optimization problem

$$\theta^* = \arg\max_{\theta\in\Theta} \Phi(\boldsymbol{H}_\theta) = \arg\max_{\theta\in\Theta} \mathbb{E}\{\phi(m, \theta)\}. \qquad (3)$$

2.3. Signal Models of Power-controlled CDMA Systems

We consider the power-controlled CDMA systems employing matched-filter receivers in this paper. Suppose that there are K users in a chosen user subset θ. Denote $\boldsymbol{p} = [P_1, \cdots, P_K]^T$ as the transmit powers, $\{h_1, \cdots, h_K\}$ as the path gains, and $[N_1, \cdots, N_K]$ as the spreading gains. Denote η as the noise power in uplink cases, and $\eta = [\eta_1, \cdots, \eta_K]^T$ as the noise levels in downlink cases. Then the received SINR of user k is given by [4]

$$\begin{cases} \mathrm{SINR}_k^{\mathrm{UL}} = \gamma_k^{\mathrm{UL}} = \frac{N_k P_k h_k}{\sum_{l=1,l\neq k}^K P_l h_l + \eta}, \\ \mathrm{SINR}_k^{\mathrm{DL}} = \gamma_k^{\mathrm{DL}} = \frac{N_k P_k h_k}{\sum_{l=1,l\neq k}^K P_l h_k + \eta_k}. \end{cases} \qquad (4)$$

3. ADAPTIVE USER SUBSET SELECTION IN POWER-CONTROLLED CDMA SYSTEMS

3.1. Discrete Stochastic Approximation Algorithm

One method for solving (3) is the exhaustive search of all possible user subsets, which can in principle find the optimum solution. However, it is highly inefficient in the sense that most computations are useless and only those corresponding to the optimal one are eventually useful. Moreover, if the channels are time-varying, such scheme cannot track the time-varying optimal user subset. We now present the discrete stochastic approximation algorithm for solving (3) [8, 9]. which has high computational efficiency in the sense that most of the computational cost is spent close to θ^*. We use the unit vectors $\{\boldsymbol{e}_1, \cdots, \boldsymbol{e}_{|\Theta|}\}$ to denote all $|\Theta|$ possible subsets. Denote $\theta^{(m)}$ as the subset visited at the m-th iteration. We map the subset sequence $\{\theta^{(m)},\ m = 1, 2, \cdots\}$ to the unit vector sequence $\{\boldsymbol{D}(m),\ m = 1, 2, \cdots\}$, where $\boldsymbol{D}(m) = \boldsymbol{e}_j$ if $\theta^{(m)} = \theta_j$. At each iteration m, the algorithm updates the state occupation probability $\pi(m) = [\pi(m, 1), \cdots, \pi(m, |\Theta|)]^T$, where $\pi(m, j) \in [0, 1]$ and $\sum_{j=1}^{|\Theta|} \pi(m, j) = 1$. The discrete stochastic approximation algorithm is then summarized as follows.

Algorithm 1 [User subset selection]

- (a) Initialization: $m \Leftarrow 1$; randomly select $\theta^{(m)} \in \Theta$, and $\hat{\theta}^{(m)} \Leftarrow \theta^{(m)}$; set $\pi(m)$ by $\pi(m, \theta^{(m)}) = 1$, and $\pi(m, \theta) = 0$ for all $\theta \neq \theta^{(m)}$.
- (b) Sampling and evaluation: Given $\theta^{(m)}$, obtain $\hat{\boldsymbol{H}}_{\theta^{(m)}}(m)$; calculate $\phi(m, \theta^{(m)})$; uniformly choose $\tilde{\theta}^{(m)} \in \Theta \setminus \theta^{(m)}$; compute $\phi(m, \tilde{\theta}^{(m)})$.
- (c) Acceptance: If $\phi(m, \tilde{\theta}^{(m)}) > \phi(m, \theta^{(m)})$, then set $\theta^{(m+1)} = \tilde{\theta}^{(m)}$; otherwise set $\theta^{(m+1)} = \theta^{(m)}$.
- (d) Update the state occupation probabilities: $\pi(m+1) = \pi(m) + \mu(m+1)[\boldsymbol{D}(m+1) - \pi(m)]$, where $\mu(m) = \frac{1}{m}$.
- (e) Update the estimate of the optimizer: If $\pi(m+1, \theta^{(m+1)}) > \pi(m+1, \hat{\theta}^{(m)})$, then set $\hat{\theta}^{(m+1)} = \theta^{(m+1)}$; otherwise set $\hat{\theta}^{(m+1)} = \hat{\theta}^{(m)}$.
- (f) $m \Leftarrow m + 1$, and go to step (b).

The sequence $\left\{\theta^{(m)},\ m = 1, 2, \cdots\right\}$ is a Markov chain on the state space Θ, and in general is not expected to converge. In Step (d), $\pi(m) = [\pi(m, 1), \cdots, \pi(m, |\Theta|)]^T$ denotes the empirical state occupation probability of the Markov chain at the m-th iteration, and thus, Step (e) is equivalent to $\hat{\theta}^{(m)} = \arg\max_\theta\ \pi(m, \theta)$. Hence the algorithm essentially chooses the state most frequently visited by the Markov chain. The sequence $\{\hat{\theta}^{(m)},\ m = 1, 2, \cdots\}$ contains the estimates of θ^*. Under certain conditions, $\hat{\theta}^{(m)} \to \theta^*$ almost surely as $m \to \infty$, or equivalently, the Markov chain spends more time in θ^* than in any other state.

3.2. Multiuser Scheduler for Throughput Maximization

Uplink SINR-based Power Control Scheme

Denote $\gamma_k^{\min}$ as the minimal SINR request and $P_{\max}$ as the maximal transmit power for each user k. One optimization strategy for the uplink SBPC is to find the power set to balance the achievable SINR ratios among all users. That is, the SINR balancing problem,

$$\max_{\boldsymbol{p}} \min_{1\leq k\leq K} \frac{\gamma_k^{\mathrm{UL}}(\boldsymbol{p})}{\gamma_k^{\min}}, \quad \text{with } P_k \leq P_{\max},\ \gamma_k^{\mathrm{UL}} \geq \gamma_k^{\min}. \qquad (5)$$

Let $\alpha \triangleq \frac{\gamma_k}{\gamma_k^{\min}},\ 1 \leq k \leq K$, be a balanced SINR ratio. The optimal power set $\boldsymbol{p}^* = [P_1^*, \cdots, P_K^*]^T$ to achieve α can be obtained via an iterative process [6], where each user k adjusts its power at the n-th iteration by $P_k(n) = \min\{P_{\max},\ \gamma_k \frac{P_k(n-1)}{\gamma_k^{\mathrm{UL}}(n-1)}\}$. For solving (5), we start with a small α and obtain $\boldsymbol{p}^*$. If $\max_k\{P_k^*\} < P_{\max}$, increase α by a small factor, and then compute the corresponding $\boldsymbol{p}^*$. The above process is repeated until $|\max_k\{P_k^*\} - P_{\max}|$ is sufficiently small [6].

Downlink SINR-based Power Control Scheme

Denote P_T as the maximal transmit power at the base station. The downlink SINR balancing problem is formulated as

$$\max_{\boldsymbol{p}} \min_{1\le k\le K} \frac{\gamma_k^{\mathrm{DL}}(\boldsymbol{p})}{\gamma_k^{\min}}, \text{ with } \sum_{k=1}^{K} P_k \le P_T,\ \gamma_k^{\mathrm{DL}} \ge \gamma_k^{\min}. \quad (6)$$

Denote $\boldsymbol{p}_{\mathrm{DL}}^*$ and $\alpha_{\mathrm{DL}}^* \triangleq \frac{\gamma_k^{\mathrm{DL}}(\boldsymbol{p}_{\mathrm{DL}}^*)}{\gamma_k^{\min}}$ as the solution to (6). Define $\gamma^{\min} \triangleq \mathrm{diag}(\gamma_1^{\min},\cdots,\gamma_K^{\min})$, $\boldsymbol{N} \triangleq \mathrm{diag}(N_1,\cdots,N_K)$, $\boldsymbol{h} \triangleq \mathrm{diag}(h_1,\cdots,h_K)$, $\mathbf{1} \triangleq [1,\cdots,1]^T$ and

$$\boldsymbol{A} \triangleq \begin{bmatrix} \gamma^{\min}\boldsymbol{N}^{-1}(\boldsymbol{B}-\boldsymbol{I}) & \gamma^{\min}\boldsymbol{N}^{-1}\boldsymbol{h}^{-1}\eta \\ \frac{\mathbf{1}^T\gamma^{\min}\boldsymbol{N}^{-1}(\boldsymbol{B}-\boldsymbol{I})}{P_T} & \frac{\mathbf{1}^T\gamma^{\min}\boldsymbol{N}^{-1}\boldsymbol{h}^{-1}\eta}{P_T} \end{bmatrix},$$

where $\boldsymbol{B}$ is the $K \times K$ matrix whose elements are all one. Let $\tilde{\lambda}$ and $\tilde{\boldsymbol{p}} \triangleq [\tilde{P}_1, \tilde{P}_2, \cdots, \tilde{P}_{K+1}]^T$ be the maximal eigenvalue and the corresponding eigenvector of $\boldsymbol{A}$. Then we have $\boldsymbol{p}_{\mathrm{DL}}^* = \frac{[\tilde{P}_1,\cdots,\tilde{P}_K]^T}{\tilde{P}_{K+1}}$ and $\alpha_{\mathrm{DL}}^* = 1/\tilde{\lambda}$ [5].

Maximal Instantaneous System Throughput Scheduler

Denote $\alpha(\theta)$ as the optimal achievable SINR ratio for θ. For each $k \in \theta$, $X_k \triangleq \log(1+\gamma_k^{\min}\alpha(\theta))$ is defined as its achievable rate. Then $\Phi(\boldsymbol{H}_\theta)$ is the instantaneous weighed system throughput for each θ with $\alpha(\theta) \ge 1$. Note that θ with $\alpha(\theta) < 1$ is an infeasible one. Define $g_k \triangleq 1/(1+N_k/\gamma_k^{\min})$ for each user $k \in \theta$, and $g_\theta \triangleq \sum_{k\in\theta} g_k$ for θ [4]. For the uplink case, θ is feasible if $g_\theta \le \min_{k\in\theta}\{1 - \frac{g_k\eta}{h_k P_{\max}}\}$ [7]. For the downlink case, θ is feasible if $g_\theta \le 1 - \frac{1}{P_T}\sum_{k\in\theta} g_k\eta_k/h_k$ [4]. Note that Algorithm 1 only treats the feasible user subsets as the candidates. Specifically, for each $\tilde{\theta}^{(m)}$ in Step (b), we first evaluate its feasibility, and only if it is feasible, we continue with the other steps.

3.3. Adaptive Algorithm for Time-varying Channels

Under the static channel condition, a decreasing step-size is employed in Algorithm 1, $\mu(m) = 1/m$. With such an approach, the method gradually becomes more and more conservative as the number of iterations increases. Whereas, in the time-varying channel case, we need such a step-size that moving away from a state is permitted when the optimal user subset changes [10]. Hence, Step (d) in Algorithm 1 is replaced by $\pi(m+1) = \pi(m) + \mu\,[\boldsymbol{D}(m+1) - \pi(m)]$, where μ is a fixed step-size satisfying $0 < \mu \le 1$. The fixed step-size introduces an exponential forgetting factor of the past occupation probabilities and allows to track the slowly time-varying optimum user subset.

4. SIMULATION RESULTS

Throughput maximization with fast convergence

We first show the performance of Algorithm 1 in terms of throughput maximization with fast convergence. The simulation conditions are as follows. The number of users is $N = 10$; the spreading gains are $N_k = 128$; the AWGN is 5×10^{-12}W; $P_{\max} = 1$W and $P_T = 5$W; $\gamma_k^{\min} = 10$dB and 6dB for uplink and downlink cases, respectively. The path gain of each user k is $h_k(d_k) = h_k(d_0) - 10n_0\log_{10}(\frac{d_k}{d_0})$ dB, where d_k is its distance from the base station; $h(d_0) = h(100m) = -80$dB; and $n_0 = 4$. We assume the estimated channel as $\hat{h}_k = h_k(1+\triangle h)$, where $\triangle h \sim \mathcal{N}(0, 0.1)$. Note that the weights in $\boldsymbol{w}$ are all set as 1. The path gains for all users are fixed for all simulation runs. Figure 2 shows the total rate of the chosen subset versus the iteration number for the uplink SBPC case, and Fig. 3 shows the similar results for the downlink case. The results obtained over the single simulation run, the averaged results over 100 runs and the optimal results obtained via the exhaustive searches are all shown. It is seen that the algorithm can converge to the best user subset, and can quickly lock on a user subset with high system throughput.

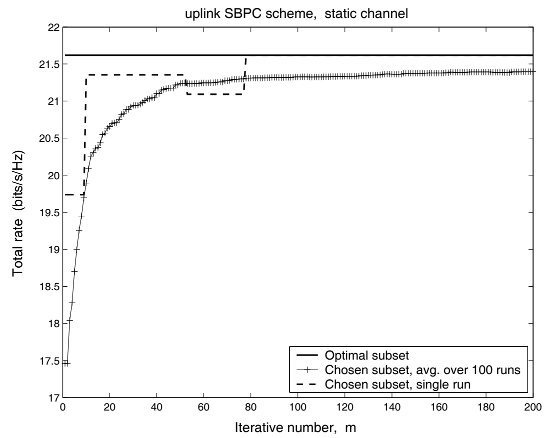

Fig. 2. The total rate of the chosen user subsets versus the iteration number: uplink scheme, fixed channel case.

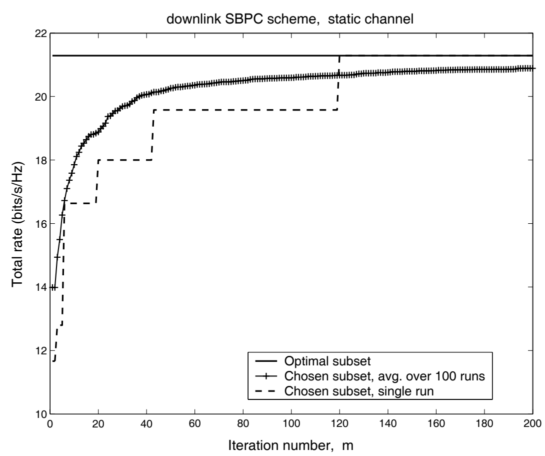

Fig. 3. The total rate of the chosen user subsets versus the iteration number: downlink scheme, fixed channel case.

Time-varying optimal user subset tracking

Next, we show the tracking capability of the algorithm in time-varying channels. Suppose that the channels for all users keep fixed within $\tau = 600$ iterations. The distance for each user k is assumed as $d_k(t) = \beta_1 d_k(t-1) + \beta_2\Delta d_k(t)$, where $\Delta d_k(t) \sim \mathcal{N}(0, \sigma_d^2)$ with $\sigma_d = 100$m; and $\beta_2 = (1-\beta_1^2)^{1/2}$

with $\beta_1 = 0.95$. Note that the radius of the cell in this study is assumed as 1000m. The fixed step-size is set as $\mu = 0.01$. All other conditions are the same as those in Fig. 2. Figure 4 shows the results for the uplink SBPC scheme. It is seen that the time-varying optimal user subset can be closely tracked.

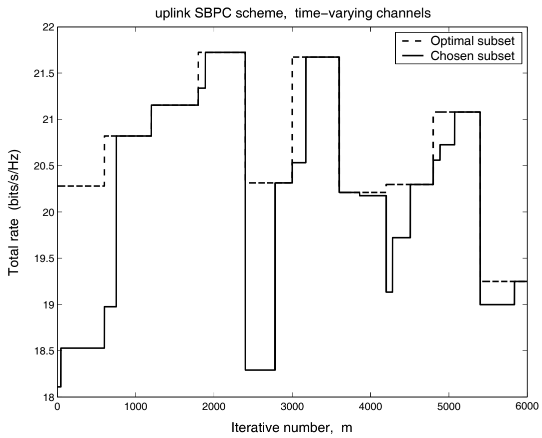

Fig. 4. The total rate of the chosen user subsets versus the iteration number: uplink scheme, time-varying channel case.

Fairness among the users

Finally, we demonstrate the system performance in terms of the fairness. Suppose $\psi_k = \frac{1}{10}$ for each user k. Note that there are 1000 iterations within each interval i, and the similar time-varying channels as those in Fig. 4 are adopted with $\tau = 500$. Figure 5 shows the normalized users' rates versus the interval index for the downlink SBPC case, where $N_k = 32$ and $\eta_k = 10^{-10}$W. It is seen that, by adjusting $\boldsymbol{w}(i)$ as denoted in (2), although the fairness constraints of the users are not satisfied within a short time scale, the long term fairness can be well guaranteed.

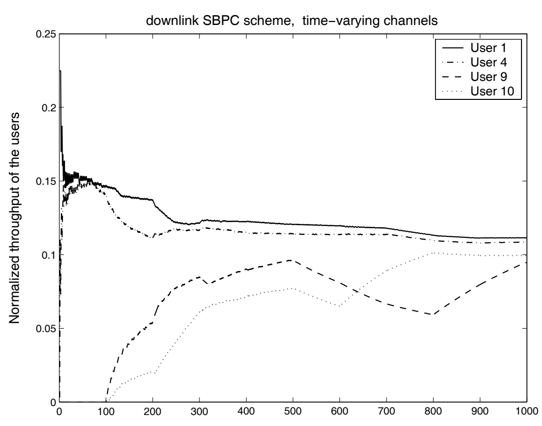

Fig. 5. The normalized throughput of the users versus the scheduling interval index: downlink scheme, time-varying channel case, $N_k = 32$ and $\psi_k = \frac{1}{N},\ k = 1, 2, \cdots, N$.

5. CONCLUSIONS

We have developed a framework for power-controlled CDMA systems to achieve efficient resource allocation while guaranteeing the long term fairness. Given the user subset, the optimal power control scheme can be calculated for different CDMA systems. The multiuser OFS problem then refers to the optimal user subset selection to maximize the instantaneous system throughput subject to the fairness constraints. We have developed the discrete stochastic approximation algorithm to achieve efficient and effective optimal user subset selection to maximize the instantaneous system throughput. We have also extended the algorithm to the time-varying channel case for which our solutions have tracking capabilities. Simulation results demonstrated that the algorithms can effectively select the optimal user subset with good convergence performances, and adaptively track the time-varying optimum in the non-stationary environments. Moreover, the system can guarantee the fairness among all users over large time scales.

6. REFERENCES

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