# MULTIDIMENSIONAL ORTHOGONAL DESIGN FOR ULTRA-WIDEBAND DOWNLINK

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## ABSTRACT

In a traditional time-hopping multiple access ultra-wideband communication system, orthogonal time-hopping sequence can be designed to improve system performance. However, system capacity is restrictive. We propose a new multiple access system model for UWB downlink and deploy orthogonal design in three dimensions: time-hopping code, multiuser code, and pulse waveform, or equivalently the position, polarity, and shape of a basic pulse. Such a system provides more freedoms than a traditional time-hopping system. Thus, it is more robust to multiple access interference and can support more users. Performance of the system is analyzed and verified by simulations.

# 1. INTRODUCTION

Ultra-wideband (UWB) system is found to have a number of appealing features: high multipath diversity, resistant to multipath distortion, low spectral density, high transmission rate and strong penetration ability [1], [2]. These features make UWB an ideal candidate for communication link for wireless local area networks (WLANs) and body area networks (WBANs). For example, UWB can provide reliable high data-rate links between multimedia devices in indoor environments. UWB is also considered for wireless sensor network, where low power consumption, short distance covert communication method is of interest. The potential wide applications of UWB together with recent spectrum mask release by FCC [3] cause a lot of research interests as well as commercial interests in this technology.

A conventional UWB system transmits trains of ultra shortduration pulses [4]. Both pulse position modulation (PPM) and pulse amplitude modulation (PAM) can be used as modulation method [5]. Multiple access is enabled by time-hopping (TH) [6]. Then a RAKE receiver can be used to correlate the received signal with a template signal and information is detected through maximum ratio combining (MRC). However, this conventional receiver is suboptimal in a multiuser communication system. Therefore, multiuser receivers need to be considered for performance improvement [7]. The authors in [8] propose orthogonal time hopping code design with multiuser detection in downlink to improve statistical multiuser interference (MUI) cancellation. But the maximum number of users the system can accommodate is limited. It is also shown in [9] MUI can be deterministically canceled through multistage block-spreading. Yet the proposed zero interleaving scheme reduces system efficiency in long channels.

Here, we further extend the two-dimensional orthogonal design method [8], [9] considering time-hopping codes and multiuser (MU) codes to three-dimensional including pulse shape (PS) in UWB downlink scenarios, for example from the access point to station nodes in WLAN. In a flat channel, our proposed system can remove MUI completely. Even in a multipath channel, it provides extra capability to resist MUI thanks to the additional dimension and the invariant orthogonality of different pulse shapes. Furthermore, if system capacity is of main interest, our multiple access (MA) system can increase number of users significantly when multiple orthogonal pulses are available. In this case, MUI can still be readily dealt with by combining with other techniques, for instance, the zero padding method in [9]. Although we only consider pulse amplitude modulation (PAM) in this paper, the proposed scheme is applicable to pulse position modulation (PPM) because the PPM model can be simply modified to a PAM-like linear model as in [8].

This paper is organized as follows. In Section 2, we introduce the system model, including that of the transmitted signal and the received signal. Then we specifically discuss orthogonal codes and waveform design in Section 3. In Section 4, we design corresponding receivers for the proposed system and analyze their detection performance. Simulation results are presented in Section 5.

#### 2. SYSTEM MODEL

Consider the downlink in a multiple access (MA) UWB system with K users. The transmitted baseband UWB signal for user k can be described by

$$\alpha_k(t) = \sqrt{\mathcal{P}} \sum_{i=-\infty}^{\infty} b_k(\lfloor i/N_f \rfloor) d_{m_2}(i) w_{m_3}(t - iT_f - c_{m_1}(i)T_c)$$
<sup>(1)</sup>

where  $\mathcal{P}$  is the transmission power,  $T_f$  is the frame duration, and  $T_c$  is the chip duration.  $b_k(\lfloor i/N_f \rfloor)$  is the kth user's information bearing symbol during the *i*th frame. It modulates the transmitted pulse train using PAM. In a binary anitpodal system,  $b_k = \pm 1$ .  $N_f$  is the number of frames over which each symbol repeats.  $c_{m_1}(i)$ ,  $d_{m_2}(i)$ , and  $w_{m_3}(t)$  are three user-related signature components. We shall describe them in detail. As in a conventional TH system,  $c_{m_1}(i) \in [0, N_c - 1]$  is a periodic TH sequence used to separate signal from different users within each frame. It adds an additional time shift of  $c_{m_2}(i)T_c$  to each pulse in the transmitted pulse train to avoid catastrophic collisions. We will choose its period as one symbol interval.  $d_{m_2}(i) \in \{1, -1\}$  is a periodic user-dependent multiuser code. It repeats every symbol period and resembles the signature code in a code-division multiple access (CDMA) com-

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munication system.  $w_{m_3}(t)$  represents the basic monopulse used to convoy information in UWB communications. In a conventional system, one pulse shape is used for all users. But in the above model, several different waveforms are employed and we assign them among all users. Therefore, the pulse waveform also becomes user dependent. Here,  $m_1, m_2, m_3$  are respectively indices of the TH code, multiuser code and pulse waveform assigned to the kth user. Because, with a certain assignment rule, we can uniquely determine these three indices from a user index k, we will use index k and its associated three-tuple set  $(m_1, m_2, m_3)$ interchangeably to refer to the kth user.

We can see this UWB signal model provides us three freedoms to design multiple access schemes and thus more flexibility. With a properly designed scheme, MUI can be cancelled effectively. Questions about how to design codes and waveforms and how to assign them to users will be addressed in the next section.

The TH sequence  $\{c_{m_1}(i)\}\$  specifies in which chip a monopulse will appear within one frame. If we consider a periodic waveform repeated every chip duration  $w_{m_3}(t - iT_c)$ , we can define a new equivalent sequence  $\{\tilde{c}_{m_1}(i)\}\$  from  $\{c_{m_1}(i)\}\$  uniquely.  $\{\tilde{c}_{m_1}(i)\}\$  consists of only zeros and ones with one indicating a pulse appears in that chip position and zero otherwise. So, (1) can be expressed as

$$\alpha_k(t) = \sqrt{\mathcal{P}} \sum_{i=-\infty}^{\infty} b_k(\lfloor i/N_c N_f \rfloor) s_{m_1,m_2}(i) w_{m_3}(t-iT_c)$$
(2)

where  $s_{m_1,m_2}(i) \stackrel{\Delta}{=} \widetilde{c}_{m_1}(i) d_{m_2}(\lfloor i/N_c \rfloor)$  and we have changed *i* from frame index in (1) to chip index. The sequence  $\{s_{m_1,m_2}(i)\}$  repeats from symbol to symbol because both the TH sequence and the multiuser sequence have period equal to one symbol interval. If we arrange all codes within one period in vectors, i.e.  $s_{m_1,m_2} = [s_{m_1,m_2}(0), \cdots, s_{m_1,m_2}(N_cN_f - 1)]^T$ ,  $\widetilde{c}_{m_1} = [\widetilde{c}_{m_1}(0), \cdots, \widetilde{c}_{m_1}(N_cN_f - 1)]^T$ ,  $d_{m_2} = [d_{m_2}(0), \cdots, d_{m_2}(N_f - 1)]^T$ , we can easily find relation among them:

$$\boldsymbol{s}_{m_1,m_2} = \widetilde{\boldsymbol{c}}_{m_1} \odot (\boldsymbol{d}_{m_2} \otimes \boldsymbol{1}) \tag{3}$$

where  $\odot$  and  $\otimes$  represent Hadamard product and Kronecker product respectively, **1** is a vector of length  $N_c$  with all elements equal to one.

It is clear according to (2) that input  $b_k(i)$  is modulated by chip-rate waveform  $w_{m_3}(t - iT_c)$  and  $s_{m_1,m_2}(i) \in \{-1, 0, 1\}$ changes the position and polarity of each basic pulse  $w_{m_3}(t)$ . This interpretation can give us better insight on our proposed system. That is, the aforementioned three freedoms for multiple access scheme are essentially achieved by fully utilizing three aspects of a monopulse, namely position, polarity and shape.

The transmitted signal  $\alpha_k(t)$  propagates through a linear channel with impulse response  $\bar{g}(t)$ . At the receiver, the channel output is first passed through a matched filter matched to the monopulse waveform  $w_{m'_3}(t)$  of the desired user k'. We can define a frontend effective channel including effects from modulated pulse at the transmitter, propagation channel, matched filter at the receiver and power factor by  $g_{m_3,m'_3}(t) = \sqrt{\mathcal{P}} w_{m_3}(t) \star \bar{g}(t) \star w_{m'_3}(-t)$  where  $\star$  denotes convolution. Considering additive white Gaussian noise (AWGN) v(t), the output of the matched filter at the k'th receiver becomes

$$y_{k'}(t) = \sum_{k,i} b_k(\lfloor i/(N_c N_f) \rfloor) s_{m_1,m_2}(i) g_{m_3,m'_3}(t-iT_c) + v(t).$$
(4)

Then  $y_{k'}(t)$  is sampled at chip rate to yield a discrete-time output  $y_{k'}(n) = y_{k'}(t)|_{t=nT_c}$ . Assuming the effective channel has length  $qT_c$  and using its discrete-time counterpart, we obtain a discrete-time chip-rate model

$$y_{k'(n)} = \sum_{k,i} \sum_{i=0}^{q} b_k(\lfloor i/(N_c N_f) \rfloor) s_{m_1,m_2}(i) g_{m_3,m'_3}(n-i) + v(n)$$
(5)

Consider P symbol intervals of data samples with corresponding time instants  $nN_cN_f + p$  for  $p = 1, \dots, PN_cN_f$  and collect them in a big vector  $\boldsymbol{y}_n$  of length  $\nu = PN_cN_f$ . Then a vector form data model follows

$$\boldsymbol{y}_{n,k'} = \sum_{k,l} \boldsymbol{S}_{m_1,m_2,l} \boldsymbol{g}_{m_3,m'_3} b_k(n+l) + \boldsymbol{v}_n \qquad (6)$$

where symbol index *l* takes all integers  $-\lceil q/(N_cN_f) \rceil, \dots, P-1$ .  $g_{m_3,m'_3}$  is the channel vector which contains channel coefficients at the chip rate.  $S_{m_1,m_2,l}$  is a Toeplitz matrix constructed by having  $s_{m_1,m_2}$  and its downshifted versions as its columns.

## 3. MULTIPLE ACCESS ORTHOGONAL DESIGN

We have seen the above system provides us more freedoms in implementing a multiple access system. The way we choose time hopping code, multiuser code and pulse waveform will result in different multiple access schemes. The special case that all users share an identical multiuser code and pulse waveform is the conventional time-hopping system. Here, we design an orthogonal system by applying orthogonal design at all these three dimensions to increase system capacity and thus improve system performance in a multiuser environment. That is, we choose  $\tilde{c}_{m_1}(i)$ ,  $d_{m,2}(i)$ and  $w_{m_3}(t)$  so that

$$\widetilde{\boldsymbol{c}}_{m_{1}}^{T}\widetilde{\boldsymbol{c}}_{m_{1}'} = 0, \ \forall m_{1} \neq m_{1}'$$

$$\boldsymbol{d}_{m_{2}}^{T}\boldsymbol{d}_{m_{2}'} = 0, \ \forall m_{2} \neq m_{2}'$$

$$\int_{0}^{T_{c}} w_{m_{3}}(t)w_{m_{3}'}(t)dt = 0, \ \forall m_{3} \neq m_{3}'$$
(7)

The first condition is equivalent to  $c_{m_1}(i) \neq c'_{m_1}(i) \forall i \in [0, N_f - 1]$ . We can generate the sequence user after user, by avoiding same code as previous users. We can find maximum  $N_c$  such orthogonal sequences. For the second condition, we choose Hadamard code for  $d_{m_2}$ . There are totally  $N_f$  such code vectors with length  $N_f$ . From [10], we know a set of pulse waveforms that satisfy the third condition are the so-called modified Hermite polynomials (MHP), given by

$$w_{m_3}(t) = (-1)^{m_3} e^{-\frac{t^2}{4}} \frac{d^{m_3}}{dt^{m_3}} \left( e^{-\frac{t^2}{2}} \right) \quad (m_3 = 0, 1, 2, \cdots).$$
(8)

Assume we employ  $N_w$  such orthogonal waveforms. The maximum number of users our system can support is  $N_c N_f N_w$ , which is greater than the maximum number of user  $N_c N_f$  in [9]. Then we can assign these codes and waveforms to different users according to a certain rule. For example, we can adopt the following simple rule:

- 1. We assign each user with a different multiuser code;
- 2. If the number of users is greater than  $N_f$ , we group users based on pulse waveforms. Within each group, we assign orthogonal MU codes to different users.

3. If the number of users is greater than  $N_w N_f$ , we further consider orthogonal TH codes to differentiate user groups.

Then the index set  $(m_1, m_2, m_3)$  for user k is:  $m_1 = \lfloor k/N_f N_w \rfloor$ ,  $m_2 = k \mod N_f$ , and  $m_3 = (\lfloor k/N_f \rfloor) \mod N_w$ .

In a flat channel, we can achieve orthogonality between signals for two different users by applying the above design. Then MUI for undesired users can be successfully cancelled.

*Proposition:* If we design time hopping code, multiuser code and pulse waveform as (7), signal for kth user can be completely removed at k'th receiver in a flat downlink environment if  $k \neq k'$ .

*Proof:* For both k and k', we can uniquely find their associated sets  $(m_1, m_2, m_3)$  and  $(m'_1, m'_2, m'_3)$  respectively. From (6), the received kth user's signal at receiver k' in a flat channel reduces to:

$$\boldsymbol{y}_{n,k,k'} = \boldsymbol{s}_{m_1,m_2} g_{m_3,m'_2} b_k(n), \tag{9}$$

because l = 0 now. Matrix  $S_{m_1,m_2,l}$  has degraded to a vector and channel vector to a scalar. Using (3) and the property of Kronecker product, we have:

$$s_{m'_{1},m'_{2}}^{T}s_{m_{1},m_{2}} = \left(\widetilde{c}_{m'_{1}}\odot(d_{m'_{2}}\otimes\mathbf{1})\right)^{T}(\widetilde{c}_{m_{1}}\odot(d_{m_{2}}\otimes\mathbf{1}))$$
$$= (\widetilde{c}_{m'_{1}}^{T}\widetilde{c}_{m_{1}})\odot\left((d_{m'_{2}}\otimes\mathbf{1})^{T}(d_{m_{2}}\otimes\mathbf{1})\right)$$
$$= (\widetilde{c}_{m'_{1}}^{T}\widetilde{c}_{m_{1}})\odot\left((d_{m'_{2}}^{T}d_{m_{2}})\otimes(\mathbf{1}^{T}\mathbf{1})\right) \quad (10)$$

From (7), we know if  $m_1 \neq m'_1$ ,  $\tilde{c}_{m'_1}^T \tilde{c}_{m_1} = 0$ , or if  $m_2 \neq m'_2$ , it is easy to show  $d_{m'_2}^T d_{m_2} = 0$ . If  $m_3 \neq m'_3$ , noticing flat channel, we have

$$g_{m_3,m_3'} = \int w_{m_3}(t)\bar{g}(t)w_{m_3'}(t)dt = 0.$$
(11)

Because  $k \neq k'$ , at least one element in  $(m_1, m_2, m_3)$  is different from corresponding element in  $(m'_1, m'_2, m'_3)$ , with the above results, we obtain

$$\boldsymbol{s}_{k'}^T \boldsymbol{y}_{n,k,k'} = 0, \ \forall k \neq k'.$$

$$(12)$$

This implies we can completely remove signal for user k at the k'th receiver.  $\Box$ 

In a multipath channel, orthogonality between  $S_{m_1,m_2,l}$  and  $S_{m'_1,m'_2,l}$  is destroyed. Thus, MUI cannot be cancelled completely as above. But it can be easily shown that MUI from user groups with different pulse shapes can still be cancelled if path delay differentials are multiples of  $T_c$ . This may not be the case in a real environment. We will assess the system performance due to the modeling error later by simulation. If MUI needs to be removed completely for an arbitrary multipath channel, we can incorporate the block spreading and interleaving technique in [9].

#### 4. SYMBOL DETECTION

If channel information is known or estimated at the receiver [11], we can apply a linear receiver  $u_{k'}$  to estimate information symbols:

$$\hat{\boldsymbol{b}}_{k'}(n) = \boldsymbol{u}_{k'}^T \boldsymbol{y}_{n,k'}.$$
(13)

Receivers can be a RAKE receiver, or an MMSE receiver:

$$u_{k',RAKE} = S_{m'_1,m'_2,0}g_{m'_3,m'_3},$$
  
$$u_{k',MMSE} = R_{k'}^{-1}S_{m'_1,m'_2,0}g_{m_3,m'_3},$$
 (14)

where  $R_{k'}$  is the autocorrelation matrix of data vector  $y_{n,k'}$ :

Without loss of generality, we study the average bit error rate (BER) of user one. Assume its index set is  $\{m_1 = 0, m_2 = 0, m_3 = 0\}$ . Let us first consider symbol "1" is transmitted. We can express the received data vector (6) as a sum of desired signal, interference (MUI and intersymbol interference) and noise:

$$y_{n,k'} = S_{0,0,0}g_{0,0} + \eta, \qquad (15)$$

where  $\eta$  represents interference plus noise. We assume  $\eta$  is a Gaussian random vector. It has zero mean and variance  $E\{\eta\eta^H\}$ . Choose the detection criterion for the *n*th symbol as  $u_1^H y_{n,1} > 0$ . Using (15), it becomes

$$\boldsymbol{u}_{1}^{H}\boldsymbol{\eta} > -\boldsymbol{u}_{1}^{H}\boldsymbol{S}_{0,0,0}\boldsymbol{g}_{0,0}.$$
(16)

Here,  $u_1^H \eta$  is also a Gaussian vector with zero mean and variance

$$\sigma^{2} = u_{1}^{H} \sum_{k \neq 1} S_{k,l} g_{0,0} g_{0,0}^{H} S_{k,l}^{H} u_{1} + u_{1}^{H} \sum_{l \neq 0} S_{k,l} g_{0,0} g_{0,0}^{H} S_{k,l}^{H} u_{1} + \sigma_{v}^{2} u_{1}^{H} u_{1}.$$
 (17)

Then we can obtain the conditional error probability when "1" is transmitted

$$P_{E,1} = Q(\frac{\boldsymbol{u}_1^H \boldsymbol{S}_{0,0,0} \boldsymbol{g}_{0,0}}{\sigma}), \qquad (18)$$

where  $Q(x) \stackrel{\Delta}{=} \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$ . We can similarly find  $P_{E,-1}$ . If two information symbols are equally probable, average BER is  $P_E = 1/2 (P_{E,1} + P_{E,-1})$ .

### 5. NUMERICAL EXAMPLES

In our simulation, we adopt two different pulse waveforms: the normalized first derivative of Gaussian pulse and the second derivative of Gaussian pulse, both with pulse duration  $D_g = 0.7ns$ . Other system parameters are chosen as  $N_c = 4$ ,  $N_f = 4$  and  $T_c = D_g$ . Thus the maximum capacity of our system is 32. The multipath channel has seven paths with two neighboring paths separated by a random delay uniformly distributed between  $T_c$  and  $1.5T_c$ . Path gains are modeled as independent Gaussian random variables and weighted by linearly decreasing weights [9]. We assume the receiver is synchronized to the first path.

First, we compare the average BER performance of a conventional TH system and that of a system using TH code and pulse shape (PS) in Fig. 1. The results are obtained using a subspace MMSE receiver [12], assuming infinite data length. It is clear that for a four-user system where a conventional time-hopping design method reaches the system capacity, more than 3dB gain is achieved for a fixed BER level  $10^{-3}$  by using both TH and PS instead of only TH. For lower BER, the gain becomes more significant. When the number of user increases to six, the system utilizing PS still performs better than the conventional system with only four users. To see the result for a TH system with eight users, we have used random codes instead of orthogonal codes. Corresponding analytical curves under two dimensional design are plotted in dash-dotted lines in Fig. 1. We can see they match with the experimental results very well.

Similarly, we compare performance of a one-dimensional design method based only on multiuser code and a two-dimensional design method based on multiuser code and pulse shape in Fig. 2. Up to SNR of 15dB, the difference is not remarkable. When SNR is above 15dB, the advantage of the two-dimensional design method becomes apparent. Because of this consistency, we will present only the analytical curves in the following tests.

In the last example, we compare a two-dimensional design method with TH and MU codes [9] and our three-dimensional design method. As we have observed high consistency between analytical and experimental curves in above two figures, we only present analytical results in the current plot. Although there is significant performance loss with 32 users and multipath effect, we can still observe that the three-dimensional design is superior to the two-dimensional design with 16 users.

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**Fig. 1.** Performance comparison between conventional timehopping system and a system using both time-hopping code and pulse shape (PS).



Fig. 2. Performance comparison of a system using multiuser code and a system using both multiuser code and pulse shape.



Fig. 3. Performance comparison of a system using multiuser code and time-hopping code and a system using three-dimensional design method.