HIGH-SPEED A/D CONVERSION FOR ULTRA-WIDEBAND SIGNALS BASED ON SIGNAL PROJECTION OVER BASIS FUNCTIONS

Sebastian Hoyos†, Brian M. Sadler‡, Gonzalo R. Arce†

†Department of Electrical and Computer Engineering
University of Delaware, Newark, DE 19716-3130

‡Army Research Laboratory, Adelphi, MD 20783-1197

ABSTRACT

This paper introduces techniques to perform analog to digital (A/D) conversion, based on the quantization of the coefficients obtained by the projection of a continuous-time signal over an orthogonal space. This framework for A/D conversion is motivated by the sampling of an input signal in domains which may lead to lower levels of signal distortion and significantly less demanding A/D conversion characteristics. The A/D conversion distortion is reduced by assigning optimal bit rates according to the variance distribution of the coefficients. Moreover, since the quantization of the coefficients is realized at the end of a time window during which the signal is projected, the speed of the quantizers can be much lower than the one needed in conventional time-domain ADCs. In particular, we study ADC in the frequency domain which overcomes some of the difficulties encountered with conventional time-domain A/D conversion of signals with very large bandwidths, such as ultra-wideband (UWB) signals.

1. INTRODUCTION

Analog to digital conversion is one of the key elements that has enabled the development and implementation of digital signal processing systems. Many of these systems have been successfully deployed thanks to the advances in analog to digital converters (ADCs) which convert continuous-time signals to discrete-time binary-coded words. However, while the transmission and reception rates and the bandwidths of the signals used in today's high-speed applications continue to increase, the progress in the development of faster and higher resolution ADCs faces technological barriers. A recent survey in A/D conversion [1] indicates that one bit of resolution is lost for every doubling of the sampling rate and the highest Nyquist rate attained by current technology is roughly 8 giga samples per second (Gs/s) with 3 bits of resolution. Among the difficulties that have slowed the evolution of ADCs is the aperture jitter or aperture uncertainty, which is the sample-to-sample variation of the instant in time at which sampling occurs. Moreover, the speed of sampling is limited by the frequency f_T of the device used in the design, which limits the ability of the comparators to make an unambiguous decision about the input voltage.

In order to overcome these problems, techniques that aim to relax the operational conditions of the ADCs have been proposed. In general, these techniques perform multi-band signal processing [2–4] in which the spectrum of the signal is channelized into several bands by means of a bank of bandpass filters. A/D conversion thus occurs at a much reduced speed for each one of the resultant bandpass signals. Unfortunately, the implementation of the bank of bandpass filters needed in multi-band methods can be potentially troublesome; problems such as spectrum sharing due to the nonideal characteristics of the bandpass filters will affect the overall system performance.

This paper introduces approaches to the analog to digital conversion problem that potentially overcome some of the limitations encountered in the implementation of time-domain A/D converters. These approaches exploit the signal representations in domains other than the classical time domain, leading to reduced speed at the comparators that perform the quantization of the sampled signal, and potentially improves the distortion versus average bit rates of the A/D conversion. Firstly, the reduction of the quantizer's speed is achieved because the quantization of the coefficients is carried out at the end of a time window during which the signal is projected over the orthogonal basis. Thus, the quantizers run at a speed that is inversely proportional to the time-window duration, which can be properly designed to meet the speeds allowed by the technology used in the implementation. Secondly, the potential improvement in the distortion of the orthogonal space ADCs is achieved by op-

Prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011.

timally allocating bit rates in the quantization of the coefficients obtained through the projection of the continuous-time signal over the set of basis functions. The possibility of efficiently allocating the available resources in terms of number of bits per sample is a powerful new feature in A/D conversion that is not available in conventional time-domain ADC. Optimal bit rate allocation is possible in the proposed A/D conversion scheme since some signal characteristics such as power distribution that are hidden in the time-domain can now be explored by projecting the signal in orthogonal spaces.

2. ANALOG TO DIGITAL CONVERSION IN ORTHOGONAL SPACES

Figure 1 shows the basic orthogonal expansion principle of the proposed A/D conversion. The received analog bandlimited signal $s(t)$ is decomposed every T_c seconds into N components which are obtained through the projection over an orthogonal basis $\Phi_l(t)\ |_{l=0}^{N-1}$. The coefficients $a_l^{(m)}\ |_{l=0}^{N-1}$ are found as

$$a_l^{(m)} = \langle s(t), \Phi_l(t) \rangle_{m,T_c} = \int_0^{T_c} s(t+mT_c)\Phi_l^*(t)\ \mathrm{d}t, \quad (1)$$

where the mean square error criterion is used to approximate the received signal $s(t)$ in the interval $mT_c \leq t \leq (m+1)T_c$, as follows

$$\hat{s}^{(m)}(t) = \sum_{l=0}^{N-1} a_l^{(m)} \Phi_l(t), \qquad 0 \leq t \leq T_c, \quad (2)$$

where the signal $\hat{s}^{(m)}(t),\ 0 \leq t \leq T_c$ is the best MSE approximation of the input signal $s(t),\ mT_c \leq t \leq (m+1)T_c$. At the end of the conversion time T_c, the coefficients $a_l\ |_{l=0}^{N-1}$ are fed to a set of quantizers $Q_l\ |_{l=0}^{N-1}$, one for each coefficient, which return the digital words $\bar{a}_l\ |_{l=0}^{N-1}$. These values represent the output of the analog to digital converter for the input signal in a T_c second interval. Notice that the signal $s(t)$ is being segmented by a rectangular window for simplicity, however windows with preferable characteristics can instead be used. The number of coefficients N used in the A/D conversion is intimately related to the conversion time T_c, and will affect the degree of the approximation pointed out in (2), up to the point where the signal $s(t)$ is represented with zero error energy given that a sufficient number of coefficients N_* is used. When fewer than N_* coefficients are used in the A/D signal conversion, some distortion is introduced. This distortion, plus the distortion introduced in the quantization process, constitute the overall distortion of the proposed A/D conversion.

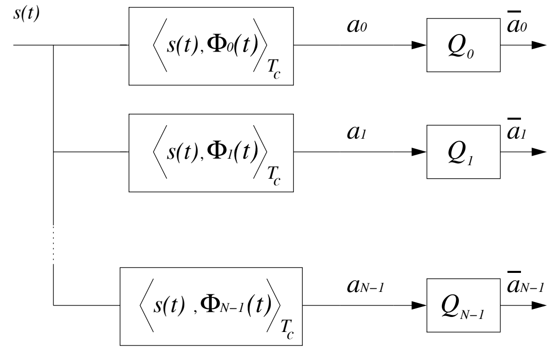

Fig. 1. Block diagram of the analog to digital converter, that expands the received signal using a set of orthogonal basis functions.

3. A/D CONVERSION DISTORTION

Without loss of generality consider the interval $0 \leq t \leq T_c$, in which the coefficients $\bar{a}_l = Q_l(a_l)\ |_{l=0}^{N-1}$ at the output of the A/D converter provide a representation of the analog input signal in the conversion time T_c. The reconstructed signal is expressed as

$$\tilde{s}(t) = \sum_{l=0}^{N-1} \bar{a}_l \Phi_l(t), \qquad 0 \leq t \leq T_c. \quad (3)$$

The total distortion is due to the potentially limited number of coefficients N, which will be denoted as D_N, and by the finite number of bits used in the quantization of the coefficients $a_l\ |_{l=0}^{N-1}$, which will be denoted as D_Q. Using the MSE criterion, the total distortion D can be expressed as

$$D = E\{|s(t) - \tilde{s}(t)|^2\}. \quad (4)$$

In order to eliminate the time dependence in the distortion D, we proceed to take the time average as follows

$$\begin{aligned} \bar{D} &= \frac{1}{T_c}\int_0^{T_c} E\{|s(t) - \tilde{s}(t)|^2\}\ \mathrm{d}t \\ &= \bar{E}\left\{\left|\sum_{l=N}^{N_*-1} a_l \Phi_l(t)\right|^2\right\} + \bar{E}\left\{\left|\sum_{l=0}^{N-1} (a_l - \bar{a}_l)\Phi_l(t)\right|^2\right\} \\ &= D_N + D_Q. \end{aligned} \quad (5)$$

where $\bar{E}\{\cdot\} = 1/T_c \int_0^{T_c} E\{\cdot\}\ \mathrm{d}t$. Equation (5) follows from the fact that the functions $\Phi_l(t)\ |_{l=0}^{N_*-1}$ are orthogonal in the interval $0 \leq t \leq T_c$.

It is possible to show that the distortion D_N is given by:

$$D_N = \frac{1}{T_c}\left(E_{s,T_c} - \sum_{l=0}^{N-1} \sigma_l^2\right), \quad (6)$$

where E_{s,T_c} is the energy of the signal in the conversion interval T_c, σ_l^2 is the variance of the coefficient a_l (it is assumed $E\{a_l\} = 0$ without loss of generality) and the distortion D_N is nonnegative by definition. When the number of coefficients is N_*, the distortion reaches the zero value and the received signal $s(t)$ can be represented as:

$$s(t) = \sum_{l=0}^{N_*-1} a_l \Phi_l(t), \qquad 0 \leq t \leq T_c. \tag{7}$$

The average quantization distortion $\bar{D}_Q$ can also be found as

$$\bar{D}_Q = \frac{1}{NT_c} \sum_{l=0}^{N-1} \epsilon_l^2 \sigma_l^2 2^{-2R_l}, \tag{8}$$

where the bit rates $R_l \,|_{l=1}^{N}$ are the number of bits used in the quantizers $Q_l \,|_{l=1}^{N}$ and the division by N is used to average across the coefficients.

3.1. Optimum Bit Rate Allocation

Since each coefficient a_l exhibits distinct second order statistics, it is possible to find the optimal bit allocation among the N coefficients, i.e., we want to find the set of rates $R_l \,|_{l=0}^{N-1}$ constrained to $\sum_{l=0}^{N-1} R_l = RN$ such that the distortion in (8) is minimized. This classical optimization problem can be solved using Lagrange multipliers leading to the following result

$$R_l = R + \frac{1}{2} \log_2 \left(\frac{\epsilon_l^2 \sigma_l^2}{\prod_{l=0}^{N-1} \epsilon_l^2 \sigma_l^2} \right). \tag{9}$$

The optimum solution assigns more bits to the coefficients with larger variance such that the distortion of all the coefficients is uniform and equal to

$$\frac{1}{T_c} D_{Q_l} = \bar{D}_Q = \frac{1}{T_c} \left(\prod_{l=0}^{N-1} \epsilon_l^2 \sigma_l^2 \right)^{1/N} 2^{-2R}. \tag{10}$$

This bit allocation resembles the concept of reverse *water-filling* found in rate distortion theory [5].

3.2. Comparison between the ADC based on signal projections and conventional ADC

It is interesting to compare the performance of the A/D conversion based on signal projection with the conventional pulse code modulation (PCM) technique in which each time-domain sample is quantized with a constant number of bits R. The distortion incurred in PCM, assuming that N samples are taken in a T_c window using the Nyquist criteria, is simply

$$D_{PCM} = \frac{1}{T_c} \epsilon_t^2 \sigma_t^2 2^{-2R}, \tag{11}$$

where the sub-index t stands for time, ϵ_t depends on the pdf of a sample of the time-domain signal which is assumed stationary, σ_t^2 is the sample's variance, and R is large so this expression holds in general.

Now, we define a figure of merit of the proposed A/D conversion in orthogonal spaces, the orthogonal space A/D conversion gain (G). This figure of merit compares the performance of the proposed A/D method with the performance of conventional A/D conversion with PCM, defined as

$$G = \frac{D_{PCM}}{\bar{D}_Q + D_N}. \tag{12}$$

Substituting (6), (8) and (11) into (12), we have

$$G = \frac{\epsilon_t^2 \sigma_t^2 2^{-2R}}{\left(\prod_{l=0}^{N-1} \epsilon_l^2 \sigma_l^2 \right)^{1/N} 2^{-2R} + E_{s,T_c} - \sum_{l=0}^{N-1} \sigma_l^2}. \tag{13}$$

where the desired result $G > 1$ depends on N and the variance distribution as it is illustrated in example 4.1.

4. ADC IN THE FREQUENCY DOMAIN

The complex exponential functions that constitute an orthogonal basis allow sampling of the continuous-time signal spectrum at the frequencies $F_l \,|_{l=0}^{N-1}$, leading to the set of frequency coefficients

$$c_l = \int_0^{T_c} s(t) e^{-j2\pi F_l t} \, \mathrm{d}t. \qquad l = 0, \cdots, N-1. \tag{14}$$

These coefficients are then quantized by a set of quantizers $Q_l \,|_{l=0}^{N-1}$, which in turn produce the ADC output digital coefficients $\bar{c}_l \,|_{l=0}^{N-1}$. The frequency sample spacing $\Delta F = F_l - F_{l-1}$ complies with $\Delta F \leq \frac{1}{T_c}$ in order to avoid aliasing in the discrete-time domain. Thus, the optimal number of coefficients N_* necessary to fully sample the signal spectrum with bandwidth W, without introducing time aliasing, is proportional to the time-bandwidth product

$$N_* = \left\lceil \frac{W}{\Delta F} + 1 \right\rceil \geq \lceil WT_c + 1 \rceil, \tag{15}$$

where the operator $\lceil \; \rceil$ is used to ensure that N_* is the closest upper integer that avoids discrete-time aliasing. Assuming that the input signal $s(t)$ is real-valued, the coefficients $c_l \,|_{l=0}^{N-1}$ are related to the discrete Fourier transform (DFT) coefficients as follows

$$\begin{aligned} \mathbf{S}(k) &= [S(0), S(1), \cdots, S(K-1)] \\ &= F_s \left[c_0, c_1, \cdots, c_{N-1}, c_{N-2}^*, \cdots, c_1^* \right], \end{aligned} \tag{16}$$

where it is assumed that samples are taken from 0 Hz, $F_s = K/T_c$ is the time sampling frequency which is the Nyquist rate at which a conventional time-domain ADC would operate, and $k = 0, \cdots, K-1$. So, if N samples are taken from the spectrum of the real-valued continuous-time signal, $K = 2(N_*-1)$ DFT coefficients are obtained and these

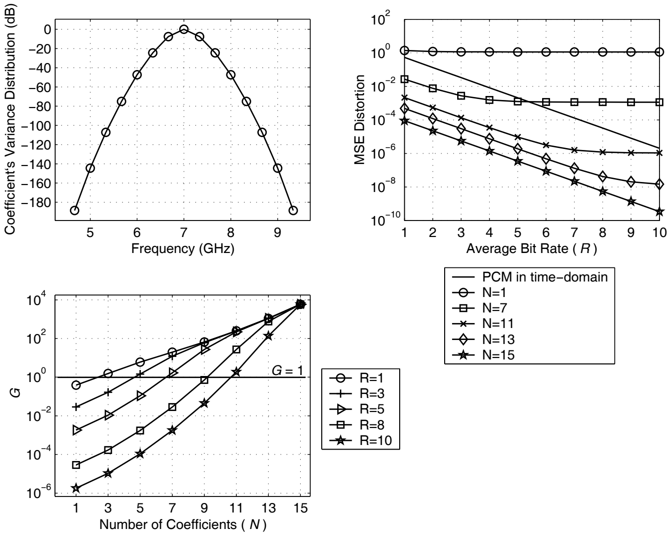

Fig. 2. (a) Signal power spectrum density. (b) MSE distortion. (c) The A/D conversion gain (G) vs. the number of coefficients (N).

coefficients constitute the digital representation of the time-domain signal. The relationship in (16) is valid only if the K samples of the continuous time signal in the interval T_c comply simultaneously with Nyquist rate ($F_s \geq 2W$) and Eqn. (15). Otherwise, the samples $S(k) \mid_{k=0}^{N-1}$ provide an approximate representation of the signal, as was indicated in (2).

4.1. Example

Let us consider a stationary zero-mean Gaussian continuous source with variance σ_t^2, bandwidth $W = 5$ GHz @ -185 dB, central frequency $F_c = 7$ GHz and a power spectrum density (PSD) shown in Fig. 2 (a). The conversion time is $T_c = 3$ ns ($\Delta F = 333.33$ MHz) and the bit rates are optimally distributed among the coefficients as indicated by Eqn. (9), leading to the set of curves of MSE distortion vs. average bit rate R plotted in Fig. 2 (b). The MSE distortion for PCM is also shown for comparison purposes. The orthogonal space A/D conversion gain (G) is plotted in Fig. 2 (c) against the number of coefficients N ($N_* = 15$) for several values of average bit rates R. These figures show the potential gain of performing the A/D conversion in the frequency domain together with optimal bit rate allocation, specially when the target average bit rate is low. For this example, a gain of up to $5.9 * 10^3$ (37.7 dB) can be achieved when $N_* = 15$ coefficients are implemented.

Furthermore, assume that a mono-bit (i.e., $R = 1$) implementation is desired and lowering the sampling rate is the main concern in the design. So, we would like to trade distortion gain for a lower operational speed of the quantizers. Figure 2 (c) shows that a mono-bit implementation with $N = 3$ coefficients (6 real-valued quantizers) achieves the same distortion rate of a time-domain ADC with PCM. However, the orthogonal space ADC operates at $1/T_c = 333.33$ MHz whereas the time-domain ADC requires a sampling rate of 10 GHz to meet Nyquist criteria. If a time-interleaved architecture is implemented in the time domain ADC to reduce the speed of the comparators to 333.33 MHz, a total of 30 ADCs would have to be used, leading to an implementation that requires 24 more quantizers than the orthogonal space ADC implementation.

5. CONCLUSIONS

This paper introduces analog to digital conversion in orthogonal spaces, where instead of sampling the signal in the time domain as it has been conventionally done in A/D conversion, the input analog signal is projected over an orthogonal space before quantization takes place. Quantization is then carried out over the coefficients obtained from this projection. This A/D conversion technique provides a potential gain over time-domain ADCs when optimal bit rate allocation is used in the quantization process of the coefficients. Additionally, a reduction of the quantizer's operational speed is achieved, at cost of using more ADCs, as the A/D conversion is performed at the end of a properly chosen time window of T_c seconds during which the signal is projected. Furthermore, the new technique possesses some degree of flexibility in the design as trading between speed and distortion can be obtained by properly choosing the conversion time T_c, the number of coefficients N, and the bit rates $R_l \mid_{l=0}^{N-1}$.

6. REFERENCES

[1] R. H. Walden, "Analog-to-digital converter survey and analysis," *IEEE Journal on Selected Areas in Communications*, vol. 17, pp. 539–550, April 1999.

[2] S. R. Velazquez, T. Q. Nguyen, and S. R. Broadstone, "Design of hybrid filter banks for analog/digital conversion," *IEEE Transactions on Signal Processing*, vol. 4, pp. 956–967, April 1998.

[3] W. Namgoong, "A channelized digital ultra-wideband receiver," *IEEE Transactions of Wireless Communications*, vol. 3, pp. 502–510, May 2003.

[4] P. Aziz, H. Sorensen, and J. Van der Spiegel, "Multiband sigma-delta modulation," *Electronics Letters*, vol. 29 Issue: 9, pp. 760–762, Apr. 1993.

[5] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, John Wiley & Sons, Inc, 1991.