# CAPACITY-MAXIMIZING RESOURCE ALLOCATION FOR DATA-AIDED TIMING AND CHANNEL ESTIMATION IN ULTRA-WIDEBAND RADIOS

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## ABSTRACT

The overall system performance of data-aided ultra-wideband (UWB) communications relies critically on the accuracy of synchronization and channel estimation during the training phase. The total transmission resources should be properly allocated between training and information symbols in order to strike a desired balance between performance and information rate. To this end, this paper derives optimum transmission schemes that judiciously allocate the limited transmit power and pulse numbers for signal reception tasks performing optimal timing acquisition, channel estimation, as well as symbol detection. The resulting selection of transmitter parameters not only enables both the timing and channel estimators to attain the minimum mean-square estimation errors, but also maximizes the average system capacity.

# 1. INTRODUCTION

Ultra wide-band (UWB) technology has attracted great interest in both academia and industry for its promising future in short-range, high-data-rate indoor wireless communications. Conveying information over repeated ultra-short pulses, UWB signaling entails ample diversity inherent in its enormous bandwidth. In order to collect the diversity gain, an optimal correlation based receiver requires accurate timing-offset and channel estimates, both of which are challenging to obtain due to the unique UWB transmission structure.

In a data-aided mode, the generalized likelihood ratio test (GLRT) derived in [1] suggests that training symbols can be partitioned into two non-overlapping subsets to separately perform channel estimation and timing acquisition without loss of optimality. To optimize the overall timing acquisition performance that is affected by both subsets, optimum training sequence (TS) design is investigated in [2] under the constraints on total training resources (in terms of power and pilot symbol numbers). Under perfect timing, transmission resource allocation for channel estimation has been investigated using the pilot waveform assisted modulation (PWAM) framework [3]. In this paper, we jointly consider the demands on transmission resources that are imposed from all three key elements in a UWB system: timing acquisition, channel estimation, and detection of information-bearing symbols. Based on the two-subset TS structure in [2], we use one of the subsets to perform PWAM-type channel estimation [3], yielding low-complexity yet near-optimum receiver design. Design tradeoffs reflect in the fact that more training symbols improve

both timing-offset, channel, and symbol estimation, but also reduce transmission rate. To strike a desired performancethroughput tradeoff at the system level, we derive optimum energy and number allocation between training and information bearing symbols to jointly maximize the average system capacity, which is shown to be equivalent to minimizing the mean-square timing and channel estimation errors. Recognizing the impact of nonlinear amplifiers on system implementations, we also consider the optimum transmission resource allocation under a constant-envelope constraint.

# 2. SYMSTEM MODEL

In a UWB peer-to-peer communication system, every information symbol  $s(n) \in \{\pm 1\}$  is conveyed over  $N_f$  repeated pulses p(t), with one pulse per frame of frame duration  $T_f$ . Each p(t) has unit energy and ultra-short duration  $T_p$  ( $T_p \ll T_f$ ) at the nanosecond scale. During the training phase, each training symbol has energy  $\mathcal{E}_{t,0}$  spread over  $N_f$  frames. The transmitted signal is  $u(t) := \sqrt{\mathcal{E}_{t,0}/N_f} \sum_{k=0}^{\infty} s(k)p_s(t-kT_s)$ , where  $p_s(t) := \sum_{j=0}^{N_f-1} p(t-jT_f-c_jT_c)$  is the transmit symbolwaveform of duration  $T_s := N_fT_f$ , and  $\{c_j\}_{j=0}^{N_f-1}$  is a pseudorandom time-hopping code with  $c_jT_c < T_f, \forall j$ .

The signal u(t) passes through a L-path dense multipath channel with impulse response  $g(t) := \sum_{l=0}^{L-1} \alpha_l \delta(t-\tau_{l,0})$  [4], where  $\alpha_l$  and  $\tau_{l,0}$  denote the attenuation and delay of the lth path, respectively. The overall channel is given by h(t) := $(g*p)(t) = \sum_{l=0}^{L-1} \alpha_l p(t-\tau_{l,0})$  (with \* denoting convolution), and the composite received symbol-waveform is  $h_s(t) := (g*$  $p_s)(t) = \sum_{j=0}^{N_f-1} h(t-jT_f-c_jT_c)$ . Denoting the receiver timing-offset as  $\tau_0$ , the received signal can be expressed as

$$r(t) = \sqrt{\frac{\mathcal{E}_{t,0}}{N_f}} \sum_{k=0}^{\infty} s(k) h_s(t - nT_s - \tau_0) + w_t(t), \quad (1)$$

where  $w_t(t)$  is a zero-mean Gaussian noise with PSD  $\sigma_w^2$ .

We suppose h(t) and  $\tau_0$  remain invariant over a burst of duration  $NT_s$ , but may change from burst to burst in a slowvarying channel. The timing-offset  $\tau_0$  can be written as  $\tau_0 :=$  $n_f T_f + \epsilon$ , where  $n_f$  is an integer representing the framelevel acquisition error, and  $\epsilon \in [0, T_f)$  denotes the small-scale tracking error. Mis-timing is confined to be within a symbol after energy detection, thus  $n_f \in [0, N_f - 1]$ . In every burst of N symbols, we use a total of  $N_t$  training symbols to estimate both the acquisition error  $n_f$  and the channel h(t), while treating  $\epsilon$  as an estimation error. Increasing  $N_t$ , or alternatively, the total training energy  $\mathcal{E}_t := \mathcal{E}_{t,0}N_t$ , can improve the accuracy of both channel and timing estimates  $\hat{h}(t)$  and  $\hat{n}_f$ . While the resulting symbol detection performance improves, the information rate drops. To balance this paradox, we strive to judiciously assign energy  $\mathcal{E}_s$  and number  $N_s$  of information symbols under the constraint of total energy  $\mathcal{E} := \mathcal{E}_t + \mathcal{E}_s$  and burst size  $N = N_t + N_s$ . The goal is to maximize the system capacity C given the fixed transmission resources per burst.

## 3. TRANSMISSION RESOURCE ALLOCATION

#### 3.1. Training Sequence Design and Timing Acquisition

The training sequence (TS) pattern plays a critical role in developing timing and channel estimators, and directly affects the estimation performance. Depending on whether two consecutive symbols have the same or opposite signs, the  $N_t$ training symbols can be grouped into two subsets, that is,  $S_+ := \{s(n) : s(n) = s(n-1)\}$  and  $S_- := \{s(n) : s(n) = s(n)\}$ -s(n-1). As observed in [1], the received signal resulting from  $S_+$  does not experience any symbol sign transition, therefore does not contain timing information, but is useful for channel estimation as if in a no-data-modulation mode. Timing offset parameters can be extracted from the symbol transition in  $S_{-}$ , using the channel estimate obtained from  $S_{+}$ . Such an observation is rigorized in [1] to suggest that estimation of channel amplitudes and timing errors can be carried out separately from the non-overlapping subsets  $S_+$  and  $S_-$ , without loss of optimality.

Based on this separability property [1], it is convenient to place  $N_t^+$  symbols in  $\mathcal{S}_+$  at the first segment of TS followed by  $N_t^- = N_t - N_t^+$  symbols in  $\mathcal{S}_-$ . With such a TS placement, a high-performance maximum-likelihood digital synchronizer can be constructed. Sampling at a low rate of one sample per symbol, an effective way to collect sufficient multipath energy while bypassing the unknown channel h(t)is to select a noisy template  $p_r(t) := r(t), t \in [N_f T_f, (N_f + 1)T_f)$  as the receiver correlation template [2]. The symbolrate samples at the correlator output is thus given by y(n) := $\int_{nT_s}^{(n+1)T_s} r(t) \sum_{j=0}^{N_f-1} p_r(t-nT_s-jT_f) dt$ . Note that the noisefree version of  $p_r(t)$  matches exactly to the unknown channel thanks to the TS structure; thus y(n) enjoys near-optimum energy capture via maximum-ratio combining, without resorting to channel estimation. Due to mis-timing, each sample y(n)may contain two consecutive input symbols. Let us define amplitude  $A_{\epsilon} := \sqrt{\mathcal{E}_{t,0}} \mathcal{A}_{\epsilon}$  and a tracking-induced energy loss factor  $\lambda_{h,\epsilon} := (1/\mathcal{R}_h) \int_{T_f-\epsilon}^{T_f} h^2(t) dt$ , where  $\mathcal{R}_h := \int_0^{T_f} h^2(t) dt$ is the total channel energy. Accordingly y(n) can be expressed as [1]

$$y(n) = A_{\epsilon} \left( s(n) \left( 1 - \frac{n_f + \lambda_{h,\epsilon}}{N_f} \right) + s(n-1) \frac{n_f + \lambda_{h,\epsilon}}{N_f} \right) + w_t(n)$$
(2)

where the noise sample  $w_t(n)$  is a Gaussian random variable with zero mean and variance  $\sigma_t^2 := \sigma_w^2 \mathcal{R}_h N_f$ . Observe from (2) that  $n'_f := n_f + \lambda_{h,\epsilon}$  represents the por-

Observe from (2) that  $n'_f := n_f + \lambda_{h,\epsilon}$  represents the portion of signal waveform collected in the symbol-long correlation template due to mis-timing. Knowing  $\{s(n), y(n)\}$ , this observation can be exploited optimally to obtain the maximum likelihood estimate (MLE) of the unknown  $n'_f$  [2], which approximates the true timing offset  $n_f$  subject to a bounded timing ambiguity  $\lambda_{h,\epsilon} \in [0, 1]$  caused by the unknown  $\epsilon$ . The ML formulation in [2] achieves the Cramer-Rao lower bound CRB $(n'_f)$ , which can be exploited to optimally allocate the subset energies  $\mathcal{E}_t^+ := N_t^+ \mathcal{E}_{t,0}$  and  $\mathcal{E}_t^- := N_t^- \mathcal{E}_{t,0}$ . These results are summarized below [2]:

**Result 1 (Training Resource Allocation and CRB)** The MLEs of  $A_{\epsilon}$  and  $n_{f}$  can be obtained from symbol-rate samples  $\{y(n)\}$ . For a fixed total training energy  $\mathcal{E}_{t}$ , the optimal energy allocation to minimize the  $CRB(n'_{f})$  is given by  $(\mathcal{E}_{t}^{+})_{opt} = \mathcal{E}_{t}(N_{f} - 2n_{f})/(2(N_{f} - n_{f}))$ , and the resulting  $CRB(n'_{f})$  is upper bounded by  $CRB(n'_{f})_{opt} = N_{f}^{2}\sigma_{w}^{2}/A_{\epsilon}^{2}$ .

Having acquired  $\hat{n}_f$ , the frame-level residual timing error  $\tilde{n}_f$  becomes  $\tilde{n}_f := \lfloor \hat{n}_f - n_f \rfloor_{N_f}$ , where  $\lfloor \cdot \rfloor$  represents the modulo operation with based  $N_f$ . Recalling the definition  $\hat{n}'_f := \hat{n}_f + \lambda_{h,\epsilon}$ , and noting that  $\hat{n}_f$  is an unbiased estimate, the mean-square estimation error (MSE) of  $\tilde{n}_f$  becomes  $\sigma^2_{\tilde{n}_f} = \sigma^2_{\tilde{n}_f} := E\{\lvert \hat{n}_f - n_f \rvert^2\} = N_f \sigma^2_w / (\mathcal{E}_t \mathcal{R}_h) + \sigma^2_\epsilon$ . Note that  $\sigma^2_{\tilde{n}_f}$  is inversely proportional to the total training energy  $\mathcal{E}_t$  and the energy capture index  $\mathcal{R}_h$ . In addition, an additive noise floor  $\sigma^2_\epsilon$  arises from  $\lambda_{h,\epsilon}$ , whose mean  $\lambda_{h,\epsilon}$  and MSE  $\sigma^2_\epsilon$  are both bounded in [0, 1]. The residual acquisition error  $\tilde{n}_f$  will affect the ensuing channel estimation and symbol detection quality, while its MSE  $\sigma^2_{\tilde{n}_f}$  will be useful in transmission resource allocation: a comparatively smaller  $\sigma^2_{\tilde{n}_f}$  suggests less resources to be allocated for synchronization purpose.

## **3.2.** Channel Estimation

Channel estimation can be carried out during the first  $N_t^+$  symbol intervals, since the corresponding TS subset is equivalent to having no data modulation. Subject to the residual mis-timing  $\tilde{\tau}_0 := \tilde{n}_f N_f + \epsilon$ , the effective symbol-long channel  $h_r(t)$  is related to  $h_s(t)$  by

$$h_r(t) = \begin{cases} h_s(t - \tilde{\tau}_0 + T_s), & t \in [0, \tilde{\tau}_0); \\ h_s(t - \tilde{\tau}_0), & t \in [\tilde{\tau}_0, T_s). \end{cases}$$
(3)

In fact,  $h_r(t)$  is nothing but a circularly-shifted (by  $\tilde{\tau}_0$ ) version of  $h_s(t)$  bounded within  $[0, T_s)$ . The received signal in the *n*-th symbol interval is given by  $r_n(t) := r(t + nT_s) = \sqrt{\mathcal{E}_{t,0}/N_f}h_r(t) + w_n(t)$ , where  $t \in [0, T_s)$ , and  $n \in [0, N_t^+ - 1]$ . The segments  $\{r_n(t)\}$  can be summed up and scaled by  $\mu := N_t^+ \sqrt{\mathcal{E}_{t,0}/N_f}$  to yield an unbiased least-square (LS) channel estimate of  $h_r(t)$ :

$$\hat{h}_r(t) := \mu^{-1} \sum_{n=0}^{N_t^+ - 1} r_n(t) = h_r(t) + \mu^{-1} \sum_{n=0}^{N_t^+ - 1} w_n(t).$$
(4)

It can be shown that this LS estimator achieves the estimation CRB lower bound [3]. Let us define the residual channel estimation error as  $\tilde{h}_r(t) := \hat{h}_r(t) - h_r(t)$ . The MSE of  $\tilde{h}_r(t)$  is the same as the variance of  $\hat{h}_r(t)$ , both of which can be deduced from (4) as  $\sigma_{\hat{h}_r}^2 = \sigma_{\hat{h}_r}^2 := E\{\tilde{h}_r^2(t)\} = 2N_f \sigma_w^2 / \mathcal{E}_t$ .

#### 3.3. System-Level Resource Allocation

Both the timing and channel estimation MSEs  $\sigma_{\tilde{n}_f}^2$  and  $\sigma_{\tilde{h}_r}^2$  decrease monotonically as the training energy  $\mathcal{E}_t$  increases. On the other hand, for fixed total energy per burst, the energy of information symbols  $\mathcal{E}_s$  decreases as  $\mathcal{E}_t$  increases. To balance performance versus information rate, we seek optimal resource allocation between training and information symbols to jointly maximize the average system capacity C.

The received information-conveying r(t) can be easily obtained by replacing  $\mathcal{E}_{t,0}$ ,  $\tau_0$ , and  $w_t(t)$  in (1) by  $\mathcal{E}_{s,0} := \mathcal{E}_s/N_s$ ,  $\tilde{\tau}_0$ , and  $w_s(t)$ . Subject to  $\tilde{\tau}_0$ , each  $T_s$ -long segment of r(t), denoted by  $r_n(t) := r(t + nT_s), t \in [0, T_s)$ , is given by

$$r_n(t) = \sqrt{\frac{\mathcal{E}_{s,0}}{N_f}} \left( s(n)h_s(t - \tilde{\tau}_0) + s(n-1)h_s(t - \tilde{\tau}_0 + T_s) \right) + w_n(t).$$
(5)

Subject to imperfect channel estimation, an optimum correlator uses the  $T_s$ -long channel estimate  $\hat{h}_r(t)$  obtained in Section 3.2 as the correlation template to yield decision statistic  $y_s(n) := \int_0^{T_s} r_n(t)\hat{h}_r(t)dt$  for detecting s(n). Similar to (2),  $y_s(n)$  can be derived as

$$y_s(n) = \sqrt{\frac{\varepsilon_{s,0}}{N_f}} \mathcal{R}_h\left(s(n)(N_f - \tilde{n}_f') + s(n-1)\tilde{n}_f'\right) + \zeta(n),$$
(6)

where  $\tilde{n}'_f = \tilde{n}_f + \lambda_{h,\epsilon}$  is the residual acquisition error subject to  $\epsilon$ -induced ambiguity, and  $\zeta(n)$  is the composite noise term resulting from not only the ambient noise, but also the random timing and channel estimation errors as well. Denoting the signal component of  $r_n(t)$  as  $r_{ns}(t) := r_n(t) - w_n(t)$ ,  $\zeta(n)$  comprises the following three terms:  $\zeta_1(n) := \int_0^{T_s} w_n(t)h_r(t)dt$ ,  $\zeta_2(n) := \int_0^{T_s} r_{ns}(t)\tilde{h}_r(t)dt$ , and  $\zeta_3(n) := \int_0^{T_s} w_n(t)\tilde{h}_r(t)dt$ . It is shown that these three terms can be approximated to be Gaussian random variables with zero mean [3], so is  $\zeta(n)$ . All the contributing factors to  $\zeta(n)$ , including the auto- or cross-correlations among  $w_n(t)$ ,  $h_r(t)$ ,  $\tilde{h}_r(t)$ , are statistically known via  $\sigma_w^2$ ,  $\mathcal{R}_h$ ,  $\sigma_{\tilde{h}_r}^2$ , subject to the impact of residual mis-timing. After some tedious derivations and defining  $N_c := T_f/T_c$ , we reach the variance  $\sigma_{\zeta}^2$  of  $\zeta(n)$  as (proof omitted for space limit):

$$\sigma_{\zeta}^{2} = \sigma_{\zeta_{1}}^{2} + \sigma_{\zeta_{2}}^{2} + \sigma_{\zeta_{3}}^{2}$$

$$= \sigma_{w}^{2} \Big( N_{f} \mathcal{R}_{h} + \sigma_{\hat{h}_{r}}^{2} N_{c} \Big) + \frac{\varepsilon_{s,0}}{N_{f}^{2}} \mathcal{R}_{h} \sigma_{\hat{h}_{r}}^{2} \Big( N_{f}^{2} + 2\sigma_{\tilde{n}_{f}}^{2} - 2(N_{f} - 1)\bar{\lambda}_{h,\epsilon} \Big) .$$
(7)

To get the system capacity C, let us exploit the effective SNR  $\rho_{eff}$  of each information symbol. We partition the discrete signal model in (6) into its signal component  $\tilde{s}(n) := \sqrt{\mathcal{E}_{s,0}/N_f}\mathcal{R}_h(N_f-\tilde{n}'_f)s(n)$  and its noise component  $\tilde{w}(n) := \sqrt{\mathcal{E}_{s,0}/N_f}\mathcal{R}_h\tilde{n}'_fs(n-1)+\zeta(n)$ . The power of those two components are denoted by  $E_{\tilde{n}_f,\epsilon} \{|\tilde{s}(n)|^2\}$  and  $E_{\tilde{n}_f,\epsilon} \{|\tilde{w}(n)|^2\}$ respectively. Consequently, we obtain the effective SNR per information symbol as:

$$\rho_{eff} := \frac{E_{\tilde{n}_f,\epsilon} \left\{ |\tilde{s}(n)|^2 \right\}}{E_{\tilde{n}_f,\epsilon} \left\{ |\tilde{w}(n)|^2 \right\}} = \frac{\mathcal{E}_s \mathcal{R}_h^2 \left( \sigma_{\tilde{n}_f}^2 + \sigma_\epsilon^2 + (N_f - \bar{\lambda}_{h,\epsilon})^2 \right)}{\mathcal{E}_s \mathcal{R}_h^2 \left( \sigma_{\tilde{n}_f}^2 + \sigma_\epsilon^2 + \bar{\lambda}_{h,\epsilon}^2 \right) + N_f N_s \sigma_\zeta^2} \tag{8}$$

To link the system capacity C with  $\rho_{eff}$ , we treat the overall UWB system, including the transmitter, the channel, the correlator, and the symbol detector, as a binary symmetric channel (BSC) with transition probability  $p = Q(\sqrt{\rho_{eff}})^1$  [3]. Inspection on the mutual information of the BSC reveals that the average channel capacity is

$$C = \frac{N_s}{N} E_h \{ p \log_2 p + (1-p) \log_2(1-p) + 1 \}.$$
 (9)

It is shown from (8) that when the channel coefficients is perfectly estimated and timing is known, i.e.,  $\hat{h}(t) = h(t)$ ,  $\sigma_{\tilde{n}_f}^2 = 0$ , and  $\lambda_{h,\epsilon} = 0$ , the effective SNR becomes  $\rho_{eff} = \mathcal{E}_s \mathcal{R}_h / (N_s \sigma_w^2)$ , which provides the upper bound for the effective SNR. Its associated capacity *C* also gives an upper bound for the average capacity in the presence of imperfect channel and timing estimation.

Having obtained the expression of C, we can start deriving system-level resource allocation. Let us define an energy allocation factor  $\beta := \mathcal{E}_s / \mathcal{E} \in (0, 1)$ , which leads to  $\mathcal{E}_t = (1 - \beta)\mathcal{E}$ . For convenience we also define the nominal transmit-SNR  $\rho := \mathcal{E}/(N\sigma_w^2)$  and the nominal receive-SNR  $\rho_r := \mathcal{R}_h \rho$ , which simply scales  $\rho$  by the energy capture  $\mathcal{R}_h$ . Putting all together, we express the effective SNR in (8) with respect to  $\beta$  as follows:

$$\rho_{eff}(\beta) = \frac{\frac{N_f^2}{(1-\beta)\rho_r N} + A}{\frac{N_f^2 + 2D}{(1-\beta)\rho_r N} + \frac{N_f^2 N_s}{\beta N \rho_r} + \frac{4N_f^2}{(1-\beta)^2 \rho_r^2 N^2} + \frac{2N_f^2 N_c N_s}{\beta (1-\beta) \rho_r^2 N^2} + B},$$
(10)

where the three constants A, B, and D are defined as  $A := 2\sigma_{\epsilon}^2 + (N_f - \bar{\lambda}_{h,\epsilon})^2$ ,  $B := 2\sigma_{\epsilon}^2 + \bar{\lambda}_{h,\epsilon}^2$  and  $D := N_f^2 + 2\sigma_{\epsilon}^2 - 2(N_f - 1)\bar{\lambda}_{h,\epsilon}$ , respectively. Fixing  $N_s$  and N, the capacity C increases monotonously with  $\rho_{eff}$ , therefore the optimal  $\beta^*$  that maximizes  $\rho_{eff}$  also maximizes C. When seeking a closed-form  $\beta^*$ , we find  $\partial \rho_{eff} / \partial \beta$  to be a third-order polynomial in  $\beta$ . Solutions to  $\beta^*$  can be either obtained from the roots of  $\partial \rho_{eff} / \partial \beta$ , or directly sought from (10) numerically.

**Proposition 1 (Energy Allocation per Burst)** For any given burst size N, information symbol number  $N_s$ , and total energy  $\mathcal{E}$  per burst, the energy allocation factor  $\beta^*$  maximizing the capacity C can be numerically solved by maximizing (10).

Proposition 1 implies that when the optimal  $\beta^*$  is selected, the SNRs for training and information symbols are generally not equal. Transmissions with uneven instantaneous power level may reduce the efficiency of nonlinear amplifiers in system implementation. To avoid this problem, we consider a more practical situation where training and information symbols have the same energy per symbol, i.e.,  $\mathcal{E}_{t,0} = \mathcal{E}_{s,0}$ . In this case, we introduce a number allocation factor  $\alpha := N_s/N \in$ (0, 1). Accordingly, the number of training symbols is  $N_t =$  $(1 - \beta)N$ , and the per-symbol energy is  $\mathcal{E}_{t,0} = \mathcal{E}_{s,0} = \mathcal{E}/N$ . Similar to (10), the effective SNR in terms of  $\alpha$  is given by

$$\rho_{eff}(\alpha) = \frac{\frac{N_f^2}{(1-\alpha)\rho_r N} + A}{\frac{N_f^2 + 2D}{(1-\alpha)\rho_r N} + \frac{N_f^2}{N\rho_r} + \frac{4N_f^2}{(1-\alpha)^2\rho_r^2 N^2} + \frac{2N_f^2 N_c}{(1-\alpha)\rho_r^2 N} + B}.$$
(11)
$$^{-1}Q(x) := (1/\sqrt{2\pi}) \int_x^{\infty} \exp(-y^2/2) dy.$$

Therefore, the average capacity becomes a function of  $\alpha$ , that is,

$$C(\alpha) = \alpha E_h \left[ p(\alpha) \log_2 p(\alpha) + (1 - p(\alpha)) \log_2 (1 - p(\alpha)) + 1 \right]$$
(12)

where the transition probability is  $p(\alpha) := Q(\sqrt{\rho_{eff}(\alpha)})$ . Substituting (11) into (12), we obtain the optimum  $\alpha^*$  by maximizing the average capacity C via numerical search.

**Proposition 2 (Number Allocation per Burst)** For any given burst size N and with equal per-symbol energy, the optimum number allocation factor  $\alpha^*$  that maximizes the average capacity C can be numerically obtained by maximizing (12).

# 4. SIMULATIONS

Simulation results are compared under both optimum resource allocation and non-optimal cases to validate our analyses and designs. In all test cases, the transmission parameters are selected as  $T_p = T_c = 1ns$ ,  $T_f = 100ns$ ,  $N_f = 50$  and N = 100, and the random channel parameters are generated according to [4], with  $\Gamma = 30ns$ ,  $\gamma = 5ns$ ,  $1/\Lambda = 2ns$  and  $1/\lambda = 0.5ns$ .

For energy allocation per burst, Figure 1(a-b) depict the effective SNR  $\rho_{eff}$  and the corresponding average capacity vs.  $N_s/N$ , for various  $\beta$  values. It is observed that the optimum  $\beta^*$  offers the maximum effective SNR and system capacity when  $N_s$  is fixed. The results for  $\beta = 0.5$  are very close to the best case especially at a low nominal SNR  $\rho$ , which provides a reasonable near-optimum parameter in the implementation.

For number allocation per burst under the equal-power constraint, Figure 2 depicts the average system capacity over 500 random channel realizations vs. the nominal SNR  $\rho$ , for various number allocation factors  $\alpha$ . It has been confirmed by Figure 1 that C increases monotonously as  $N_s$  increases, for a fixed  $\beta$  when unequal power transmission is allowed. As a result, the optimum  $N_s^*$  should be  $N_s^* = N - 3$ , where  $N_t = 3$  is the minimum number of training symbols needed for both timing and channel estimation, as can be deduced from the two-subset TS structure. Hence, we can get the maximum  $C(\beta^*)$  at  $N_s^*$  for reference. It is shown that the optimum number allocation  $\alpha^*$  using Proposition 2 subject to equal-power transmission offers system capacity that is very close to the maximum  $C(\beta^*)$  at  $N_s^*$ .

#### 5. SUMMARY

In this paper, system-level resource allocation results are derived to facilitate transmitter design in trading off performance and information rate. Because of the use of near-optimum, low-complexity receiver design for timing, channel estimation, and symbol detection, our optimum allocation strategies not only attain the maximum system capacity, but also minimize the mean-square channel and timing estimation errors, and thereby achieve the CRBs.

# 6. REFERENCES

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**Fig. 1.** Energy allocation per burst using Proposition 1: (a)  $\rho_{eff}$  vs.  $N_s/N$  (b) average capacity vs.  $N_s/N$ .



Fig. 2. Number allocation per burst using Proposition 2