OPTIMAL WAVEFORM DESIGN FOR UWB RADIOS

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ABSTRACT

Realizing the benefits of ultra-wideband (UWB) communications hinges critically on judicious pulse shape design to enable UWB spectral mask compatibility, and co-existence with and adaptation to other wireless devices. To this end, we propose a convex optimization based waveform design method for UWB radios. By casting the pulse design problem as a (convex) semidefinite program (SDP) over the pulse autocorrelation, globally optimal waveform designs can be efficiently obtained. While the focus of this paper is on the design of waveforms that optimally utilize the bandwidth and power allowed by the spectral mask, the flexibility of the SDP framework also allows the optimization of several other system objectives.

1. INTRODUCTION

With the release of FCC spectral masks in 2002, ultra wideband (UWB) radios have attracted increasing interest for their potential applications in short-range high-data-rate wireless communications. By conveying information over ultra-short pulses (at a nanosecond scale), UWB signals expose the fine time resolution offered by their enormous bandwidth. The resulting temporal diversity makes UWB technology a promising alternative for robust wireless indoor communications. However, the benefits of UWB signaling may be offset by the interference to and from existing systems operating in overlapping frequency bands. For spectrum overlay considerations, FCC regulations imposed a spectral mask that strictly limits the power spectrum of a UWB signal to be well below the noise floor. On the other hand, the transmission quality of a UWB system is determined by the received signal-to-noise ratio (SNR). Given the stringent transmit power limitations, maximization of the received SNR requires efficient utilization of the bandwidth and power allowed by the FCC spectral mask. The goal is to design the underlying UWB pulse shape so as to optimize the spectral shape of the transmitted signal.

Unfortunately, the widely adopted Gaussian monocycle [1] exhibits a poor fit to the FCC spectral mask and is not desirable for practical usage. Recently, a new pulse [2] is designed corresponding to the dominant eigenvector of a channel matrix that is constructed by sampling the spectral mask. Although the generated pulse [2] conforms to the spectral mask, it does not achieve the most efficient spectral utilization, and requires a high sampling rate (64GHz) that could lead to implementation difficulties. Digital filter design based on the Parks-McClellan algorithm has also been exploited for shaping UWB pulses under mask-fitting requirements [3].

In this paper, a new pulse design method based on semidefinite programming (SDP) is introduced to achieve optimal spectral utilization at a relatively low sampling rate. This pulse design framework capitalizes on the fact that many of the desired properties of a waveform, including the optimum spectral utilization, can be expressed as properties of the auto-correlation of the waveform [6]. By reformulating the design problem in terms of the autocorrelation sequence of the “pulse-shaping” filter, many of the design constraints such as the spectral mask constraints become linear, hence the design problems become linearly constrained convex optimization problems. The transformed convex SDP problems can be efficiently solved for globally optimum solutions via off-the-shelf interior point methods, as we will demonstrate in the ensuing design examples.

2. SIGNAL MODEL & PROBLEM STATEMENT

2.1. Signal Model

In a UWB system, every information symbol is conveyed over a train of N_f repeated basic pulses, with one pulse per frame of frame duration T_f. Each pulse $p(t)$ is limited to an ultra-short duration T_p of the nanosecond scale ($T_p \ll T_f$), and hence occupies an ultra-wide bandwidth. The equivalent symbol signature waveform is $p_s(t) := \sum_{n=0}^{N_f-1} p(t - c_n T_c - nT_f)$ of symbol duration $T_s := N_f T_f$, where the sequence $\{c_n\}_{n=0}^{N_f-1}$ represents the user-specific pseudo-random time-hopping (TH) code with $c_n T_c \ll T_f, \forall n \in [0, N_f - 1]$. Let $b_k \in \{\pm 1\}$ be independent and identically distributed (*i.i.d*) binary data symbols with energy $\mathcal{E}_s$ spread over N_f frames. Focusing on pulse amplitude modulation (PAM), we express the transmitted PAM UWB waveform as:

$$u(t) = \sqrt{\mathcal{E}_s/N_f} \sum_k b_k p_s(t - kT_s). \tag{1}$$

The power spectral density (PSD) of $u(t)$ is then given by

$$\phi_{uu}(f) = \frac{\mathcal{E}_s}{N_f} \cdot \frac{1}{T_s} |P_s(f)|^2 \quad (2)$$

where $P_s(f)$ is the Fourier Transform (FT) of $p_s(t)$, whose spectrum depends not only on $p(t)$, but also on the TH code $\{c_n\}_{n=0}^{N_f-1}$. Specifically, $P_s(f)$ can be expressed as

$$P_s(f) = P(f) \sum_{n=0}^{N_f-1} e^{-j2\pi f n T_f} e^{-j2\pi f c_n T_c} \quad (3)$$

where $P(f)$ is the FT of $p(t)$. Eq. (2) now becomes

$$\phi_{uu}(f) = \frac{\mathcal{E}_s}{T_s N_f} |P(f)|^2 \left| \sum_{n=0}^{N_f-1} e^{-j2\pi f(nT_f + c_n T_c)} \right|^2. \quad (4)$$

When the TH code $\{c_n\}_{n=0}^{N_f-1}$ is independent and uniformly distributed over $[0, N_c - 1]$ with integer values, $\phi_{uu}(f)$ can be approximated as [3]:

$$\phi_{uu}(f) \approx \mathcal{E}_s \frac{|P(f)|^2}{T_f}. \quad (5)$$

It is observed that the spectral shape of $p(t)$ determines the power spectrum of a UWB transmitter. Hence the UWB pulse design problem is equivalent to designing the basic pulse $p(t)$ to meet all the system requirements.

2.2. Problem Statement

Based on the FCC spectral mask $S(f)$ illustrated in Fig. 1, we observe that most of UWB signal power should be allocated to the 3.6–10.1GHz band, while considerable attenuation is imposed in other regions of the spectrum, especially for frequencies up to 3.1GHz. These constraints are designed to avoid interference to existing systems. Accordingly, we define $\mathcal{F}_p := \{f | f \in [3.1, 10.6]\text{GHz}\}$ as the UWB passband, and $\mathcal{F}_s := \{f | f \in \{[0, +\infty] - \mathcal{F}_p\}\}$ as the UWB stopband. To maximize the received SNR, the spectral shape of a UWB pulse should optimally utilize the allowed bandwidth and power spectra within the passband $\mathcal{F}_p$, while at the same time respecting the spectral mask $S(f)$. The spectrum utilization efficiency can be measured by the normalized effective signal power (NESP) ψ:

$$\psi = \frac{\int_{\mathcal{F}_p} |P(f)|^2 \, df}{\int_{\mathcal{F}_p} S(f) \, df} \times 100\%. \quad (6)$$

The objective of our optimum pulse design problem is to find $p(t)$ that maximizes the NESP under the spectral mask constraint. This can be mathematically formulated as follows:

$$\max_{p(t)} \psi \quad \text{subject to } |P(f)|^2 \leq S(f), \quad \forall f. \quad (7)$$

3. SEMIDEFINITE PROGRAMMING

In the next section we will transform (7) to a semidefinite programming problem (SDP). Semidefinite programs can be written in the form [4]:

$$\min_{\mathbf{X}} \operatorname{tr}(\mathbf{CX}) \quad \text{subject to } \operatorname{tr}(\mathbf{A}_k \mathbf{X}) = b_k, \ \mathbf{X} \succeq \mathbf{0}, \quad (8)$$

where $\mathbf{X}$ is the (symmetric) matrix variable, $\mathbf{C}$ and $\mathbf{A_k}$ are symmetric matrices describing the objective and the kth linear constraint, respectively, and there is a finite number of linear equality constraints. (Recall that $\operatorname{tr}(\mathbf{CX}) = \sum_{i,j} [\mathbf{C}]_{ij} [\mathbf{X}]_{ij}$, where $[\cdot]_{ij}$ denotes the (i,j)-th element of a matrix.) The constraint $\mathbf{X} \succeq \mathbf{0}$ constrains $\mathbf{X}$ to be (symmetric and) positive semidefinite; i.e., $\mathbf{z}^T \mathbf{X} \mathbf{z} \geq 0, \forall \mathbf{z} \in R^{n \times 1}$. Semidefinite programming problems (SDPs) are convex and can be efficiently solved in polynomial time using interior-point methods [5]. We now discuss the formulation of UWB pulse design as an SDP over the autocorrelation of the pulse shape.

4. OPTIMAL UWB PULSE DESIGN

Consider a DSP-based pulse implementation scheme. Building upon the Gaussian monocycle $q(t)$ that is readily available from a UWB transmitter, our synthesized pulse $p(t)$ can be written as [3]

$$p(t) = \sum_{i=0}^{L-1} g_i q(t - iT_q), \quad (9)$$

where T_q is the sampling interval, and the set $\{g_i\}_{i=0}^{L-1}$ contains the L pulse coefficients to be designed according to (7). The power spectrum $S_p(f) := |P(f)|^2$ of $p(t)$ is

$$S_p(f) = |Q(f)|^2 \left| \sum_{i=0}^{L-1} g_i e^{-j2\pi i f T_q} \right|^2 \approx \left| \sum_{i=0}^{L-1} g_i e^{-j2\pi i f T_q} \right|^2 \quad (10)$$

where $Q(f)$ is the FT of $q(t)$ which is sufficiently flat over the bandwidth of our interest. The sampling frequency $F_q := 1/T_q$ and the pulse duration T_p of $p(t)$ are approximately related by $T_p = LT_q$.

In addition to complying with the FCC spectral mask, we impose a tighter spectral mask in the stop band in order to reduce interference to other services operating in that band. We adjust $\mathcal{E}_s$ to normalize the spectra components in the passband to be 1 (0 dB), and impose a new mask $S_a(f)$ given as below (Fig. 1):

$$S_a(f) = \begin{cases} 0 \text{ dB}, & 3.1\text{GHz} \leq f \leq 10.6\text{GHz}; \\ -40 \text{ dB}, & 0 \leq f < 3.1\text{GHz}; \\ -15 \text{ dB}, & f > 10.6\text{GHz}. \end{cases} \quad (11)$$

Now our pulse design problem can be reformulated as:
Design Problem: Given L and $S_a(f)$, find a vector $\vec{\mathbf{g}} = (g_0, g_1, \ldots, g_{L-1}) \in R^{L \times 1}$ that maximizes the normalized effective signal power ψ subject to $S_p(f) \leq S_a(f)$, or show that $\vec{\mathbf{g}}$ does not exist.

Generally, this problem is not convex in $\vec{\mathbf{g}}$ and hence algorithms for solving it must deal with the potential for local solutions. Indeed, the filter design approach in [3] can become trapped in a local optimum. In order to avoid these problems, let us define (part of) the autocorrelation sequence $\vec{\mathbf{r}} = (r_0, r_1, \ldots, r_{L-1})$ of $\vec{\mathbf{g}}$ as:

$$r_k = \sum_{i=0}^{L-1} g_i g_{i+k}, \;\; k = 0, 1, \ldots, L-1. \tag{12}$$

With $r_{-k} = r_k$, $S_p(f)$ becomes a linear function of $\vec{\mathbf{r}}$:

$$S_p(f) = r_0 + 2\sum_{k=1}^{L-1} r_k \cos(2\pi k f). \tag{13}$$

Using (13), the NESP ψ can be written as

$$\psi = r_0 + \sum_{k=1}^{L-1} \frac{\sin(2\pi k\beta) - \sin(2\pi k\alpha)}{k\pi(\beta-\alpha)} r_k, \tag{14}$$

where $\alpha = T_q \cdot 3.1GHz$ and $\beta = T_q \cdot 10.6GHz$. The metric in (14) is also linear in $\vec{\mathbf{r}}$. The coefficients form a metric vector $\vec{\mathbf{w}} := (w_0, w_1, \ldots, w_{L-1})^T$ with $w_0 := -1$ and $w_i := -(1/k\pi(\beta-\alpha))(\sin(2\pi k\beta) - \sin(2\pi k\alpha))$, for $1 \le i \le L-1$.

It is evident from (13) that all the mask constraints involved are linear with respect to $\vec{\mathbf{r}}$, and from (14) that the objective is also linear. The remaining constraint is to ensure that $\vec{\mathbf{r}}$ is a valid autocorrelation; i.e., $\exists \vec{\mathbf{g}} \in R^L$ such that $r_k = \sum_{i=0}^{L-1} g_i g_{i+k}$. Fortunately, the linear constraint in (15c) below is necessary and sufficient for the existence of such a $\vec{\mathbf{g}}$. Our design problem can now be written as the following convex optimization problem:

$$\begin{array}{lllr} & \min\limits_{\vec{\mathbf{r}}} & -\psi & \text{(15a)} \\ s.t. & S_p(f) \ge 0, & \forall f & \text{(15b)} \\ & S_p(f) \le 0\text{dB}, & \forall f & \text{(15c)} \\ & S_p(f) \le -15\text{dB}, & f > 10.6\text{GHz} & \text{(15d)} \\ & S_p(f) \le -40\text{dB}, & 0 \le f < 3.1\text{GHz}. & \text{(15e)} \end{array}$$

Such a problem formulation is a semi-infinite linear programme, in the sense that it has a linear objective and an infinite number of linear constraints. (Each of (15b)-(15e) generates one linear constraint for each real value of f.) We can obtain approximately the feasible set generated by this infinite number of constraints by discretizing them—something that leads to a finite linear program. Alternatively, the infinite number of constraints can be precisely transformed into a finite number of linear constraints on some (symmetric) positive semidefinite matrices, which leads to an (exact) SDP formulation of (7). In the cases of (15b) and (15c) the transformation corresponds to instances of the Positive-Real and Bounded-Real Lemmas from systems theory, respectively, but the transformations of (15d) and (15e) require more recent results [6]. To describe those transformations succinctly, we will borrow two adjoint operators $L^*(\cdot)$ and $\Lambda^*(\cdot;\cdot;\cdot)$ that are defined by Eqs. (35) and (36) in [6], respectively.

- The constraint in (15b) holds if and only if there exists a symmetric positive semidefinite (S-PSD) matrix $\mathbf{X} \in R^{L\times L}$, such that

$$\sum_{k=0}^{L-1-n} [\mathbf{X}]_{k,k+n} = r_n, n = 0, 1, \ldots, L-1. \tag{16}$$

- The constraint in (15c) holds if and only if there exists a S-PSD matrix $\mathbf{X}_1 \in R^{L\times L}$ such that

$$\sum_{k=0}^{L-1-n} [\mathbf{X}_1]_{k,k+n} = (0\text{dB})\delta[n] - r_n, n = 0, 1, \ldots, L-1. \tag{17}$$

- The constraint in (15d) holds if and only if there exist S-PSD matrices $\mathbf{X}_2 \in R^{L\times L}$ and $\mathbf{Z}_1 \in R^{(L-1)\times(L-1)}$ such that

$$L^*(\mathbf{X}_2) + \Lambda^*(\mathbf{Z}_1; \beta; 2\pi-\beta) = (-15\text{dB} + r_0)\delta[n] - 2r_n, \tag{18}$$

for $n = 0, 1, \ldots, L-1$, where $\beta = T_q \cdot 10.6$GHz.
- The constraint (15e) holds if and only if there exist S-PSD matrices $\mathbf{X}_3 \in R^{L\times L}$ and $\mathbf{Z}_2 \in R^{(L-1)\times(L-1)}$ such that

$$L^*(\mathbf{X}_3) - \Lambda^*(\mathbf{Z}_2; \alpha; 2\pi-\alpha) = (-40\text{dB} + r_0)\delta[n] - 2r_n, \tag{19}$$

for $n = 0, 1, \ldots, L-1$, where $\alpha = T_q \cdot 3.1$GHz.

Using the above expressions, pulse design problems can be precisely formulated as a SDP. We now provide two instances of these formulations.

I. Spectral Utilization Problem: The maximum spectral utilization for a fixed filter length L, is achieved by a filter whose autocorrelation achieves:

$$\begin{array}{llr} & \min\limits_{\vec{\mathbf{r}}} \;\; \vec{\mathbf{w}}^T\vec{\mathbf{r}} & \text{(20)} \\ s.t. & (16), (17), (18), (19), & \\ & \mathbf{X}, \mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{Z}_1, \mathbf{Z}_2 \succeq 0. & \end{array}$$

This problem can be efficiently solved with an existing SDP package [5] to produce the optimal autocorrelation sequence $\vec{\mathbf{r}}$. Spectral factorization [7] is then applied to extract the optimal pulse coefficients $\vec{\mathbf{g}}$.

II. Pulse Duration Problem: The minimum pulse duration $T_p = LT_q$ for a given threshold γ on the spectra utilization ratio ψ, is achieved by a filter whose autocorrelation achieves:

$$\begin{array}{llr} & \min \;\; L & \text{(21)} \\ s.t. & \vec{\mathbf{w}}^T\vec{\mathbf{r}} \le -\gamma, & \\ & (16), (17), (18), (19), & \\ & \mathbf{X}, \mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{Z}_1, \mathbf{Z}_2 \succeq 0. & \end{array}$$

For a fixed L, the constraints in (21) generate a convex feasible set, and hence infeasibility of (21) with a fixed L can be reliably determined. Therefore, (21) can be solved by a bisection search on L for the feasibility/infeasibility boundary of (21). At each stage of this SDP-BS method we solve the semidefinite feasibility problem described by (21) for the current value of L.

5. DESIGN EXAMPLES

To lessen the hardware implementation difficulty, we set the sampling frequency F_q to be a relatively low value of 25GHz and accordingly the sampling interval T_q is 40ps. We first solve a Spectra Utilization Problem (with $L = 33$) using the SeDuMi SDP package for MATLAB [5]. The generated pulse spectrum is depicted in Fig. 1. Compliant to the FCC mask, the synthesized pulse achieves a maximum NESP of $\psi = 83.77\%$. In contrast, the method in [2] uses a sampling rate of 64GHz to yield a NESP value of approximately 39%.

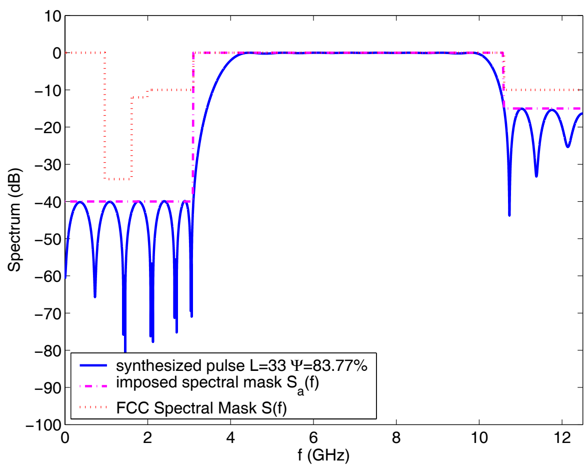

Fig. 1. Results for a Spectra Utilization Problem (L=33): Spectrum vs. f (GHz).

Considering a Pulse Duration Problem with a constraint $\gamma = 85\%$, we use the SDP-BS method to obtain the minimum L satisfying $\psi \geq \gamma$. It is shown from Fig. 2 that when $L = 36$ and $L = 37$, one has $\psi = 84.97\%$ and $\psi = 85.47\%$ respectively, bordering on the threshold. Choosing $L = 37$, we obtain our optimally synthesized pulse $p(t)$ with duration $T_p = LT_q = 1.48$ns. Fig. 2 is instrumental in striking a balance between performance (spectrum utilization ratio ψ) and complexity (filter length L). Since ψ increases dramatically for small to medium L, but increases fairly slowly for large L, the constraint γ should be chosen carefully in order to achieve an appropriate design tradeoff.

6. CONCLUSIONS

We have proposed a new pulse design method for UWB radios which achieve the maximum effective power, while complying with the FCC spectral mask constraint. By formulating the pulse design problem as a semidefinite programming problem (SDP), our method is able to efficiently obtain optimal designs, as our examples have illustrated. The advantages of this convex optimization based pulse design framework will become more evident as other system-level constraints such as robustness to timing jitter is incorporated into the waveform design. We will explore these extensions along with comparisons to [3] in our future work.

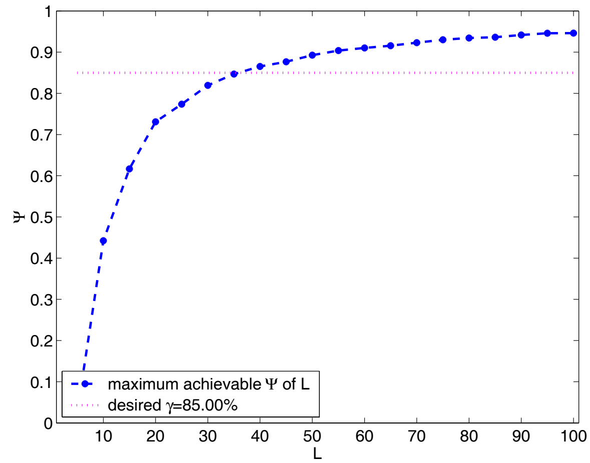

Fig. 2. Results for a Pulse Duration Problem ($\gamma = 85\%$).

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