# ANALYSIS AND COMPARISON OF AUTOCORRELATION RECEIVERS FOR IR-UWB SIGNALS BASED ON DIFFERENTIAL DETECTION

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**ABSTRACT**

In this paper we propose two innovative detection schemes for IR-UWB communication systems based on the autocorrelation receiver and differential detection. The proposed schemes avoid the need for a complex analog multiplier by employing: i) a limiter in the reference branch only, or ii) employing also a limiter in the signal branch. Analytical models for the statistics of the decision variables are presented for the receivers, and the analytical and simulated BER exhibit an excellent agreement. We point out the dependence of the performance on some important system parameters, that shows the system design trade-off between receiver complexity and performance.

## 1. INTRODUCTION

Before the ultra wideband (UWB) communication system promises of high data rate, low power consumption at low cost become a reality, signal processing research on topics as the design of practical algorithms for synchronization and channel estimation, as well as the design of a feasible receiver, needs to be further addressed.

The optimal detector for the AWGN channel requires a locally generated template waveform, composed of a low duty-cycle sequence of sub-nanosecond pulses, perfectly synchronized to the received sequence. Autocorrelation receivers (ACR) simplify and partially solve this issue by correlating the received signal in the current transmission interval with a waveform reference transmitted in a previous interval [1]. The basic difference from the optimum matched filter is that this reference is corrupted by noise. Transmitted reference (TR) ACRs that operate by transmitting a pair of unmodulated and modulated signals and employing the former to demodulate the latter, are studied in [2], [3].

Here we consider ACRs with differential detection schemes that employ as reference template the previous symbol waveform [4], thus avoiding the transmission of unmodulated pulses and therefore doubling the effective bit rate. The promise of a cheap receiver might be overcome by the requirement of an accurate and precise analog multiplier. Therefore, we propose as first alternative an ACR that passes the template signal through a limiter, so that the output waveform can assume only two discrete values, i.e. $+1, -1$, and a second one that employs limiters in both the receiver's branches. In both the cases, the analog multiplier can be removed and substituted by cheaper components. We consider an AWGN channel and we develop an accurate analytical model of the decision variables statistics. We also derive simple BER expressions for the case of rectangular monocycles, that provide a good understanding of the system parameters influence on the receiver performance.

## 2. SIGNAL DEFINITION

We consider a binary set of signal waveforms $S = \{s_0(t) = s(t), s_1(t) = -s(t)\}$, where $s(t)$ is

$$s(t) = \sum_{j=0}^{N_s-1} w(t - jT_f - c_jT_c), \quad 0 \le t \le N_sT_f, \qquad (1)$$

and $w(t)$ is a very short pulse typically of duration $T_w < 1$ ns, that provides the "ultra wide" bandwidth $W$. The time frame $T_f$ is the average pulse repetition time, $N_s$ is the number of pulses transmitted per symbol and $c_jT_c$ determines the pulse position within a frame. Without loss of generality, we assume $c_j = 0$, $j = 0, \ldots, N_s - 1$. We call the transmission of "1" $H_1$ and the transmission of "0" $H_0$. When $H_0$ is true, the transmitter generates the same signal waveform as transmitted in the previous symbol time $T_s = N_sT_f$. When $H_1$ is true, the transmitter switches to the other signal waveform. In the following, we assume $H_0$ to be true and therefore the received waveform $r(t)$, after a rectangular filter of bandwidth $W$, is

$$H_0 : r(t) = s_m(t) + s_m(t - T_s) + n(t), \quad 0 \le t \le 2T_s, \qquad (2)$$

where $m = 0, 1$ and the noise $n(t)$ is a zero mean, bandlimited white Gaussian random process, with autocorrelation function

$$R_n(\tau) \triangleq E[n(t)n(t + \tau)] = N_0W \mathrm{sinc}(2\pi W\tau). \qquad (3)$$

## 3. AUTOCORRELATION RECEIVERS

In this section we describe three different kinds of ACRs, as shown in Fig. 1, and we perform the statistical analysis of the observed variables $r^{(I)}, r^{(II)}, r^{(III)}$, where the notation $(\cdot)^{(I)}, (\cdot)^{(II)}, (\cdot)^{(III)}$ refers respectively to the first, second and third receiver.

### 3.1. Differential Autocorrelation Receiver

The receiver correlates the received signal with a symbol-time delayed version, as shown in Fig. 1. The signal template is corrupted by noise, but is synchronized to the used pulses sequence without any acquisition algorithm. At the output of the integrator the observed variable $r^{(I)}$, assuming that $H_0$ is true, is given by

$$r^{(I)} = \int_{T_s}^{2T_s} r(t)r(t - T_s)dt = r_s^{(I)} + r_n^{(I)} + n_e, \qquad (4)$$

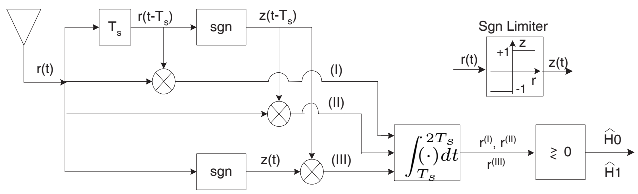


**Fig. 1**. Differential ACRs. ACR$^{(I)}$: No limiters; ACR$^{(II)}$: One limiter; ACR$^{(III)}$: Two limiters.

where

$$r_s^{(I)} = \int_0^{T_s} s(t)^2 dt = \sum_{j=0}^{N_s-1} \int_0^{T_f} w(t)^2 dt = N_s E_w = E_s, \quad (5)$$

$$r_n^{(I)} = \int_0^{T_s} s(t)[n(t) + n(t - T_s)]dt, \quad (6)$$

$$n_e = \int_0^{T_s} n(t)n(t + T_s)dt. \quad (7)$$

It is easy to see that $r_n^{(I)}$ is a Gaussian random variable (RV) with expectation $E[r_n^{(I)}] = 0$ and variance $Var[r_n^{(I)}] = N_0 E_s$, $N_0/2$ being the power spectral density of the noise within the bandwidth $[0, W]$. We show that also $n_e$ is a Gaussian RV. By using the Shannon sampling theorem, we have

$$n(t)n'(t) = \sum_{i=0}^{N} n_{(i)} \text{sinc}(t - \frac{i}{2W}) \sum_{j=0}^{N} n_{(j+N)} \text{sinc}(t - \frac{j}{2W}) \quad (8)$$

where $N = 2WT_s$ is the number of independent samples, $n'(t) \triangleq n(t+T_s)$ and $n_{(i)} = n(t)|_{t=(i/2W)}$, $n_{(j+N)}$ are independent samples of a Gaussian random process, with zero mean and variance $\sigma_n^2 = N_0 W$. Since usually $T_s >> 1/2W$, by substituting the following approximation

$$\int_0^{T_s} \text{sinc}(t - \frac{i}{2W}) \text{sinc}(t - \frac{j}{2W}) dt \simeq \begin{cases} \frac{1}{2W} & i = j \\ 0 & i \neq j \end{cases} \quad (9)$$

in (8), we can rewrite (7) as

$$n_e = \frac{1}{2W} \sum_{i=0}^{N} n_{(i)} n_{(i+N)} \triangleq \sum_{i=0}^{N} N_i, \quad (10)$$

where $N_i = (n_{(i)} n_{(i+N)})/2W$ are i.i.d RVs with $E[N_i] = 0$ and $Var[N_i] = (1/2W)^2 (N_0 W)^2 = (N_0/2)^2$. From the central limit theorem, it follows that $n_e$ is a Gaussian RV with

$$E[n_e] = 0, \qquad Var[n_e] = 2WT_s(N_0/2)^2. \quad (11)$$

The observed variable $r^{(I)}$ is therefore a Gaussian RV, with

$$E[r^{(I)}] = E_s, \quad Var[r^{(I)}] = N_0 \left( E_s + WT_s \frac{N_0}{2} \right). \quad (12)$$

### 3.2. Differential Autocorrelation Receiver with One Limiter

The delayed version of the received signal is passed through a non-linear circuit. The output $z(t) = \text{sgn}[r(t)]$ is equal to +1 for positive values of the input $r(t)$ and equal to -1 for negative values, as shown in Fig. 1. The implementation of this receiver is simpler and therefore the cost will be lower. The output $r^{(II)}$, given that $H_0$ is true, is

$$r^{(II)} = \int_{T_s}^{2T_s} r(t)z(t - T_s)dt = \int_0^{T_s} [s(t) + n'(t)]z(t)dt \quad (13)$$

$$= \sum_{j=0}^{N_s-1} r_{wj} + \sum_{j=0}^{N_s-1} r_{nwj} + \sum_{j=0}^{N_s-1} r_{nj}, \quad (14)$$

where $z(t)$ is given by

$$z(t) = \sum_{j=0}^{N_s-1} z_{wj}(t) + \sum_{j=0}^{N_s-1} z_{nj}(t), \quad (15)$$

$$z_{wj}(t) = \begin{cases} \text{sgn}[w(t - jT_f) + n(t)] & jT_f \leq t \leq jT_f + T_w \\ 0 & \text{otherwise,} \end{cases} \quad (16)$$

$$z_{nj}(t) = \begin{cases} \text{sgn}[n(t)] & jT_f + T_w \leq t \leq (j+1)T_f, \\ 0 & \text{otherwise,} \end{cases} \quad (17)$$

for $j = 0, \ldots, N_s - 1$, and then

$$r_{wj} = \int_{jT_f}^{jT_f+T_w} w(t - jT_f) z_{wj}(t) dt \quad (18)$$

$$r_{nwj} = \int_{jT_f}^{jT_f+T_w} n'(t) z_{wj}(t) dt \quad (19)$$

$$r_{nj} = \int_{jT_f+T_w}^{(j+1)T_f} n'(t) z_{nj}(t) dt. \quad (20)$$

We first compute the expected value of $r^{(II)}$. It is easy to see that $\{r_{nwj}, r_{nj}\}$ are zero mean RVs, and therefore $E[r^{(II)}] = N_s E[r_{wj}]$. The non-stationary random process $z_{wj}(t)$ assumes one out of two values $\{\text{sgn}[w(t - jT_f)], -\text{sgn}[w(t - jT_f)]\}$, and its expectation is

$$E[z_{wj}(t)] = [1 - 2q(t - jT_f)] \text{sgn}[w(t - jT_f)], \quad (21)$$

where $q(t) = Pr[z_{w0}(t) = \text{sgn}[w(t)]]$ equals to

$$q(t) = Pr[n(t) \geq |w(t)|] = Q(\frac{|w(t)|}{\sigma_n}). \quad (22)$$

By recalling that $x \cdot \text{sgn}(x) = |x|$, it follows that the expected value of $r^{(II)}$ conditioned to $H_0$ is

$$E[r^{(II)}] = N_s \int_0^{T_w} [1 - 2Q(\frac{|w(t)|}{\sigma_n})] |w(t)| dt. \quad (23)$$

It can be shown that $Cov[r_{nwj}, r_{nj}] \simeq 0$, and therefore it is straightforward to see that all the RVs $\{r_{wj}, r_{nwj}, r_{nj}\}_{j=0}^{N_s-1}$ are mutually uncorrelated. Hence the computation of the variance is reduced to

$$Var[r^{(II)}] = N_s(Var[r_{wj}] + Var[r_{nwj}] + Var[r_{nj}]), \quad (24)$$

$$Var[r_{wj}] = \int_0^{T_w} \int_0^{T_w} w(t)w(\lambda) R_{zw}(t, \lambda) dt d\lambda - E^2[r_{wj}], \quad (25)$$

$$Var[r_{nwj}] = \int_0^{T_w} \int_0^{T_w} R_n(\lambda - t) R_{zw}(t, \lambda) dt d\lambda, \quad (26)$$

$$Var[r_{nj}] = \int_{T_w}^{T_f} \int_{T_w}^{T_f} R_n(\lambda - t) R_{zn}(t, \lambda) dt d\lambda, \quad (27)$$

where $R_{zw}(t,\lambda)$, $R_{zn}(t,\lambda)$, are the autocorrelation functions of the signal waveforms $z_{wj}(t)$ and $z_{nj}(t)$. According to the arcsin law [5], $R_{zn}(t,\lambda)$ is given by

$$R_{zn}(\tau) = 2/\pi \arcsin \rho(\tau), \qquad (28)$$

with $\tau = \lambda - t$ and $\rho(\tau) = R_n(\tau)/R_n(0)$. We substitute (3), (28) in (27), obtaining

$$Var[r_{nj}] = \int_{-(T_f-T_w)}^{T_f-T_w} (T_f - T_w - |\tau|) R_n(\tau) R_{zn}(\tau) d\tau \qquad (29)$$

$$\simeq \frac{4}{\pi} N_0 W (T_f - T_w) \int_0^{T_f-T_w} \rho(\tau) \arcsin \rho[(\tau)] d\tau \qquad (30)$$

For brevity of notation we refer to the integrand function in (30) with the symbolic expression $\Xi(\tau)$. By considering the series expansion of the function $\arcsin[\rho(\tau)]$, we have

$$\Xi(\tau) \simeq \rho(\tau)^2 + \frac{\rho(\tau)^4}{6}, \quad |\tau| \geq 1/5W \qquad (31)$$

$$\Xi(\tau) \simeq 5W[\Xi(\frac{1}{5W}) - \frac{\pi}{2}]\tau + \frac{\pi}{2} \triangleq m\tau + q \quad |\tau| \leq \frac{1}{5W} \qquad (32)$$

where the proof of (32) is omitted for lack of space. We note that

$$\int_0^{T_f-T_w} \Xi(\tau) d\tau = \int_0^{T_f-T_w} \left[\rho^2(\tau) + \frac{\rho^4(\tau)}{6}\right] d\tau$$

$$+ \int_0^{\frac{1}{5W}} \{(m\tau + q) - [\rho(\tau)^2 + \frac{\rho(\tau)^4}{6}]\} d\tau \qquad (33)$$

$$\simeq \frac{1}{W}\left[\frac{1}{2\pi}\frac{5\pi}{9} + \frac{\pi - 0.92}{10} - \frac{1.215}{2\pi}\right], \qquad (34)$$

and therefore it follows

$$Var[r_{nj}] \simeq 0.39(T_f - T_w)N_0. \qquad (35)$$

For monocycles assuming only positive values, the autocorrelation function $R_{zw}(t,\lambda)$ is upper bounded by the constant '1'. It follows that

$$Var[r_{nwj}] \leq 2\int_0^{T_w} (T_w - |\tau|) R_n(\tau) d\tau \qquad (36)$$

$$\simeq 2T_w N_0 W \int_0^{T_w} \rho(\tau) d\tau \qquad (37)$$

$$\simeq \frac{T_w N_0}{\pi}\left(\frac{\pi}{2} - \frac{\cos(2\pi c)}{2\pi c} - \frac{\sin(2\pi c)}{(2\pi c)^2}\right), \qquad (38)$$

where the constant $c = T_w W$ is the time-bandwidth product of the monocycle waveform. For instance, in case of $c = 1$, we have $Var[r_{nwj}] \simeq 0.45 T_w N_0$. Simulation results show that for $E_w/N_0$ values larger than 3dB, $Var[r_{nwj}]$ is well approximated by (38). Furthermore, for a rectangular monocycle it can be shown that

$$Var[r_{wj}] \leq \frac{2}{\pi}\sqrt{2\pi\frac{E_w}{N_0}} e^{-\frac{E_w}{2N_0}} T_w N_0, \qquad (39)$$

$$\simeq 0.18 T_w N_0, \qquad (40)$$

where (40) holds for the $E_w/N_0$ values of interest. Therefore we have

$$Var[r^{(II)}] \simeq N_s[0.39T_f + 0.24T_w]N_0, \qquad (41)$$

The observed variable $r^{(II)}$ is the sum of $N_s$ i.i.d. RVs, and therefore for large $N_s$ values is a Gaussian RV with mean value given by (23) and with variance (41).

### 3.3. Differential Autocorrelation Receiver with Two Limiters

In the third detector, as illustrated in Fig. 1, the received signal in both the branches is passed through a limiter circuit. The decision variable $r^{(III)}$, conditioned to $H_0$, is

$$r^{(III)} = \int_{T_s}^{2T_s} z(t) z(t - T_s) dt = \sum_{j=0}^{N_s-1} r_{jw} + \sum_{j=0}^{N_s-1} r_{jn}, \qquad (42)$$

where

$$r_{jw} = \int_{jT_f}^{jT_f+T_w} z'_{wj}(t) z_{wj}(t) dt \qquad (43)$$

$$r_{jn} = \int_{jT_f+T_w}^{(j+1)T_f} z'_{nj}(t) z_{nj}(t) dt, \qquad (44)$$

and $z'_{wj}(t)$, $z'_{nj}(t)$ are defined as in (16), (17), with $n'(t)$ instead of $n(t)$. The expected value of $r^{(III)}$ can be easily computed as

$$E[r^{(III)}] = N_s \int_0^{T_w} \left[1 - 2Q\left(\frac{|w(t)|}{\sigma}\right)\right]^2 dt. \qquad (45)$$

For the variance evaluation we can follow an analog approach as for the differential autocorrelation receiver with one limiter. Since $E[r_{jw} r_{jn}] - E[r_{jw}]E[r_{jn}] \simeq 0$, we obtain

$$Var[r^{(III)}] \simeq N_s(Var[r_{jn}] + Var[r_{jw}]) \qquad (46)$$

$$Var[r_{jw}]) \simeq 0.2T_w^2 \qquad (47)$$

$$Var[r_{jn}] = \int_{T_w}^{T_f}\int_{T_w}^{T_f} \left(\frac{2}{\pi}\arcsin[\rho(\lambda - t)]\right)^2 dt d\lambda \qquad (48)$$

$$\simeq \frac{8}{\pi^2}(T_f - T_w)\int_0^{T_f-T_w} (\arcsin[\rho(\tau)])^2 d\tau, \qquad (49)$$

where (47) holds for the cases of interest and goes to zero for large $E_w/N_0$ values. By expanding $\arcsin(\tau)$ in series and in analogy with (31) - (33), we have

$$Var[r^{(III)}] \simeq N_s[0.31\frac{T_f - T_w}{W} + 0.2T_w^2], \qquad (50)$$

with $r^{(III)}$ described by the statistic of a Gaussian RV.

## 4. PERFORMANCE COMPARISON

All the decision variables $r^{(I)}, r^{(II)}, r^{(III)}$ are Gaussian RVs and therefore the bit error probability conditioned to $H_0$ of the $i-th$ receiver is given by

$$P(e|H_0)^{(i)} = Q\left(\sqrt{\gamma_o^{(i)}}\right), \qquad (51)$$

with $\gamma_o^{(i)} = E[r^{(i)}]^2/Var[r^{(i)}]$. Observing that $P(e|H_0)^{(i)} = P(e|H_1)^{(i)}$, it is clear that the probability of error $P(e)$ is equal to $P(e|H_0)^{(i)}$. The key parameter that allows UWB systems to operate under the noise level is the processing gain $G_p$, defined as

$$G_p = \frac{W}{R_b} = W N_s T_f = c N_s \frac{T_f}{T_w} = N_s G'_p, \qquad (52)$$

where $R_b$ is the bit rate, and $G'_p \triangleq cT_f/T_w$. The plot of (51), $i = II$, compared to the simulated BER results, proves the outstanding accuracy of the statistical analysis, as shown in Fig. 2.

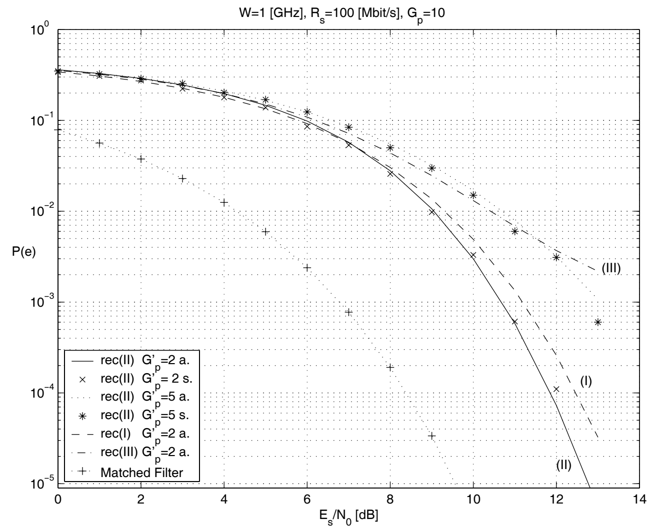


**Fig. 2**. Receivers performance, comparison of simulated (s.) and analytical (a.) results, and influence of the parameter $G'_p$.

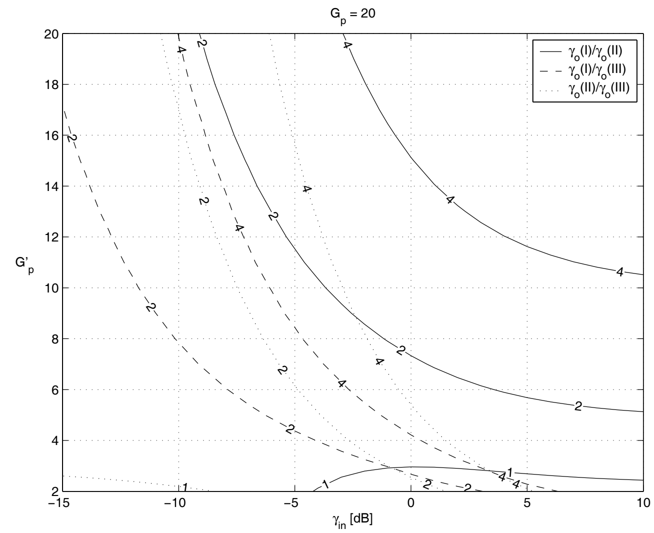


**Fig. 3**. Contour lines of the ratios $\gamma_o^{(i)}/\gamma_o^{(j)}$ showing the influence of the parameters $\gamma_{in}$ and $G'_p$ on the receivers performance.

The plot of the probability of error of receiver II, for a fixed $G_p$ but for different $G'_p$ values, shows the different role of the processing gain terms, and the BER curves of receiver (I) and (III) provide a first comparison of the receivers performance. If we denote the received power as $P_r$, we have the following relationship

$$\frac{E_s}{N_0} = \frac{P_r}{N_0 W} G_p = G_p \gamma_{in}, \tag{53}$$

where $\gamma_{in} \simeq P_r/N_O W$, and we can derive a closed form expression of (51) as a function of $\gamma_{in}, G'_p, N_s$. However the analytical expressions for $\gamma_o^{(i)}$ are quite complex, apart from the case of the rectangular monocycle waveform, where we obtain

$$\gamma_o^{(I)} \simeq \frac{G_p \gamma_{in}}{1 + (2\gamma_{in})^{-1}}, \tag{54}$$

$$\gamma_o^{(II)} \simeq 2.5 N_s \gamma_{in} \left[1 - 2Q\left(\sqrt{\gamma_{in} G'_p}\right)\right]^2, \tag{55}$$

$$\gamma_o^{(III)} \simeq 3.22 \frac{N_s}{G'_p} \left[1 - 2Q\left(\sqrt{\gamma_{in} G'_p}\right)\right]^4. \tag{56}$$

For the same $\gamma_{in}$, we see that the performance of receiver I is proportional to the processing gain $G_p$, while beyond a certain $G'_p$ level, that depends on $\gamma_{in}$, the term in brackets in (55) stays close to '1' and there is no further improvement in the receiver II bit error rate. In the third receiver, an increment of the processing gain due to $G'_p$ could lead to either higher or even lower $\gamma_o$ values, depending on $\gamma_{in}$. In Fig. 3, the contour lines of the ratios $\gamma_o^{(i)}/\gamma_o^{(j)}$ are plotted. This figure shows the influence of the parameters $\gamma_{in}$, $G'_p(= G_p/N_s)$ on the receiver performance.

## 5. CONCLUSIONS

In order to obtain a simple and cheap receiver implementation, two modified versions of the autocorrelation receiver using a linear multiplier and based on differential detection, are examined. The statistical analysis derived for the AWGN channel provides for the case of rectangular monocycles simple and insightful BER equations, that highlight the importance of an accurate choice of the system parameters involved in the processing gain. It is noticeable that the autocorrelation receiver with the non-linear limiter device provides a lower BER than the receiver with no limiters, when the pulse repetition frequency is close to the signal bandwidth. This limitation could be restrictive for a time-hopping multiple access system. However, due to low transmit power requirements, this receiver might be an interesting candidate for very short-range network, as the personal area network, where the multi-user capability is not so urgent. Moreover, we show that for pulse repetition frequencies low enough to allow consistent multiple users processing gain, the output signal-to-noise ratios of the receivers with one and two limiters are only 3 dB worse than the one of the autocorrelation receiver with a linear multiplier, and therefore the receiver choice will be a trade-off between complexity and performance.

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