Minimum MSE Transmitter and Receiver FIR MIMO Filters for Multi-User Uplink Communications

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***Abstract*— The problem of uplink communications is considered where multiple mobile terminals equipped with multiple antennas are communicating with one base station also having multiple antennas. A theory for jointly optimizing the transmitter and receiver finite impulse response (FIR) multiple-input multiple-output (MIMO) filters is developed. The signals from the individual users are assumed to be uncorrelated with each other and the additive channel noise is uncorrelated with the original signals. The FIR MIMO channel filters are assumed to be known for both at the mobile terminals and the base station. All the input signals to the FIR MIMO channels are assumed to be power-constrained. For given FIR MIMO channels with maximum allowable average input powers, the transmitter and receiver FIR MIMO filters are jointly optimized such that the mean square error (MSE) between the desired and reconstructed signals is minimized. An iterative numerical optimization algorithm is proposed. Numerical simulation results show that the proposed method has substantially better performance than a code division multiple access (CDMA) based system using minimum MSE receiver filters.**

I. Introduction

The problem of multi-user uplink FIR MIMO communications is studied. The discrete-time equivalent low-pass representation of the signals is used. K vector time-series with known second-order statistics are transmitted from K mobile stations to one base station over FIR MIMO channels when the transmitters and the receiver are equipped with multiple antennas. It is assumed that the channel is corrupted by additive signal-independent noise with zero mean and known second-order statistics, but the probability density function of the noise is arbitrary. Furthermore, it is assumed that the channel input vectors have constrained average power, and that the FIR MIMO channel filters are known at both the base station and the mobile terminals. The assumption about knowledge of the channels at both the transmitters and the receiver might be realistic in time division duplex (TDD) systems, where the channel can be estimated, and then the reciprocity of the channel is utilized.

It is assumed that K users are communicating with a base station and that the base station is decoding all the K transmitted vector time-series. As time index n is used in this article. Transmitter number $i \in \{0, 1, \ldots, K-1\}$ is communicating the original vector time-series, named $\boldsymbol{x}_i(n)$, of dimension $N_i \times 1$, and it is equipped with Q_i transmitter antennas. It is assumed that the original vector time-series of two different users are uncorrelated. However, the zero mean vector time-series $\boldsymbol{x}_i(n)$ can be correlated with itself and it is assumed that its second order statistics is known. Transmitter number i is employing a causal FIR MIMO filter of order m_i to transform the original vector input time-series $\boldsymbol{x}_i(n)$ to the channel input vector time-series $\boldsymbol{y}_i(n)$. The coefficients of transmitter filter number i are denoted $\{\boldsymbol{E}_i(k)\}_{k=0}^{m_i}$, where $\boldsymbol{E}_i(k)$ is FIR MIMO filter coefficient number k for transmitter filter number i, and $\boldsymbol{E}_i(k)$ has dimension $Q_i \times N_i$. The average power used by the transmitter output vector time-series $\boldsymbol{y}_i(n)$, of mobile station number i, is P_i.

It is assumed that the receiver is equipped with M antennas and the received vector time-series is denoted $\boldsymbol{r}(n)$. FIR MIMO channel filter from user number $i \in \{0, 1, \ldots, K-1\}$ to the base station is specified by the FIR MIMO filter $\{\boldsymbol{C}_i(k)\}_{k=0}^{q_i}$, where $\boldsymbol{C}_i(k)$ is FIR MIMO filter coefficient number k for channel filter number i, and $\boldsymbol{C}_i(k)$ has dimension $M \times Q_i$. It is assumed that the receiver knows exactly the value of all the FIR MIMO channel filters and transmitter number i knows the value of FIR MIMO channel filter number i only. The channel is corrupted with additive noise $\boldsymbol{v}(n)$, which is assumed to have a known second order statistics. Let the average variance of the components of $\boldsymbol{v}(n)$ be denoted σ_v^2. The channel noise $\boldsymbol{v}(n)$ is uncorrelated with all the original vector time-series $\boldsymbol{x}_i(n)$.

The receiver is using K causal FIR MIMO filters in order to reconstruct the K original vector time-series. In order to reconstruct the vector time-series $\boldsymbol{x}_i(n)$, a causal FIR MIMO filter of order l_i is employed and the coefficients of receiver filter number i are denoted $\{\boldsymbol{R}_i(k)\}_{k=0}^{l_i}$, where $\boldsymbol{R}_i(k)$ has dimension $N_i \times M$ and is FIR MIMO filter coefficient number k for receiver filter number i.

The system considered is shown in Figure 1, which indicates the dimensions of the vector time-series used. There are no constraints on the values of N_i, Q_i and M, except that they need to be positive integers. It is assumed that all vector time-series in Figure 1 are jointly wide sense stationary. For given values of K, N_i, P_i, Q_i, $\boldsymbol{C}_i$, and M, the transmitter and receiver FIR MIMO filters are jointly optimized with respect to minimum average block MSE between the desired output and the actual output time-series vectors of the receiver filters, subject to the average power constraints of the channel input vector time-series. The transmitter and receiver FIR MIMO filters are jointly optimized, and an iterative numerical algorithm is proposed based on formulas for finding the optimal transmitter FIR MIMO filters for given receiver FIR MIMO filters, and vice versa. The orders of all FIR MIMO filters in the system are assumed to be finite and known.

Previous literature related to the treated problem includes [1], which treats the single-user problem with zero order matrices everywhere using minimum MSE as the optimization criterion. In [2], the single-user problem having a scalar channel, with an equivalent vector channel of order 1 and no interblock interference, is treated for memoryless transforms. The single-user FIR case is studied in [3] and [4], where minimum MSE and minimum bit error rate (BER) are used as optimization criteria, respectively. The problem of multi-user downlink MIMO communications is studied in [5], using energy efficiency as the system performance criterion in the a priori designed receivers and a zero forcing (ZF) based criterion is used in the a posteriori designed transmitter. A system that can be used in uplink multi-user communications is a CDMA system employing Gold codes [6] in the transmitters and employing minimum MSE receiver filters with one antenna per mobile terminal and base station.

The rest of this article is organized as follows: Special notation is introduced in Section II, and the problem is formulated in Section III. In Section IV, the proposed iterative numerical optimization solution is given. Section V contains results using the proposed solution, and comparisons are made with a CDMA based system employing Gold codes and minimum MSE receiver filters. In Section VI, some conclusions are drawn.

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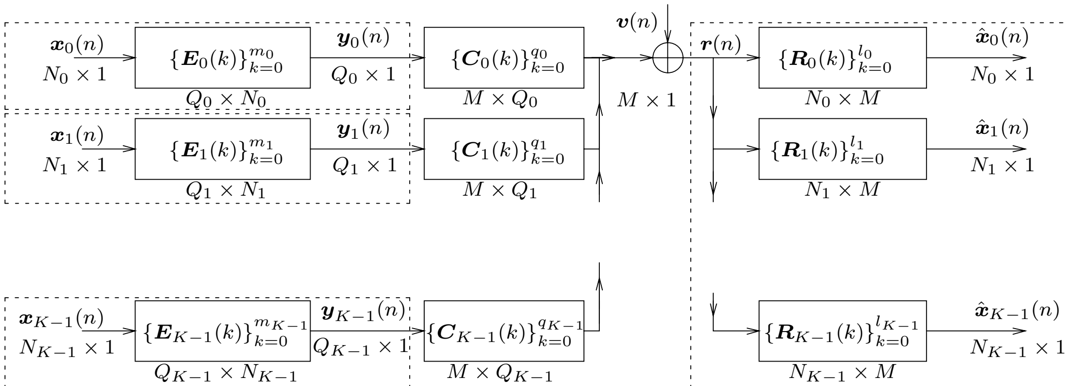

Fig. 1. System diagram of the uplink multi-user communications problem model where K mobile terminals communicate with one base station. The dashed box on the right side indicates the base station, and the dashed boxes to the left show the K mobile stations.

II. Special Notation

A. FIR MIMO Filter Expansion Operators

Three expansion operators for FIR MIMO filters will be required in this presentation, which are introduced next. Let $\{\boldsymbol{A}(i)\}_{i=0}^{k}$ be an FIR MIMO filter of order k and dimension $M_0 \times M_1$. As a short form of the filter $\{\boldsymbol{A}(i)\}_{i=0}^{k}$ only the short notation $\boldsymbol{A}$ will be used. Matrix $\boldsymbol{A}(i)$ is the ith coefficient of the FIR MIMO filter and it has dimension $M_0 \times M_1$. The *row-expanded* matrix $\boldsymbol{A}_{-}$ of the FIR MIMO filter $\boldsymbol{A}$ is an $M_0 \times (k+1)M_1$ matrix given by:

$$\boldsymbol{A}_{-} = [\boldsymbol{A}(0) \;\; \boldsymbol{A}(1) \;\; \cdots \;\; \boldsymbol{A}(k)]. \tag{1}$$

The *column-expanded* matrix $\boldsymbol{A}_{|}$ of the FIR MIMO filter $\boldsymbol{A}$ is a $(k+1)M_0 \times M_1$ matrix defined as

$$\boldsymbol{A}_{|} = \left[\boldsymbol{A}^T(k) \;\; \boldsymbol{A}^T(k-1) \;\; \cdots \;\; \boldsymbol{A}^T(1) \;\; \boldsymbol{A}^T(0)\right]^T, \tag{2}$$

where the operator $(\cdot)^T$ represents matrix transposition. Let q be a non-negative integer. The *row-diagonal-expanded* matrix $\boldsymbol{A}_{\urcorner}^{(q)}$ of order q of the FIR MIMO filter $\boldsymbol{A}$ is a $(q+1)M_0 \times (k+q+1)M_1$ block Toeplitz matrix given by:

$$\boldsymbol{A}_{\urcorner}^{(q)} = \begin{bmatrix} \boldsymbol{A}(0) & \cdots & \boldsymbol{A}(k) & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{A}(0) & \cdots & \cdots & \boldsymbol{A}(k) \end{bmatrix}. \tag{3}$$

B. Vector Time-Series Expansion

Let ν be a non-negative integer. The column-expansion of order ν of the vector time-series $\boldsymbol{x}_i(n)$ has dimension $(\nu+1)N_i \times 1$ and is defined as:

$$\boldsymbol{x}_i(n)_{|}^{(\nu)} = \left[\boldsymbol{x}_i^T(n) \;\; \boldsymbol{x}_i^T(n-1) \;\; \cdots \;\; \boldsymbol{x}_i^T(n-\nu)\right]^T. \tag{4}$$

The column-expansions of other vector time-series are defined in the same manner as shown in Equation (4). The column-expansion operator of vector time-series, while related to the column-expansion operator for FIR MIMO filters, has dimension which depends on the situation it is used. In each case, the correct dimension is given indirectly by the notation. The dimension of the column-expansion of an FIR MIMO filter is given by the dimension and order of the FIR MIMO filter.

C. Rearranging Operator

Let k be a non-negative integer. A rearranging operator denoted by $\mathcal{T}_{m_i}^{(k)} : \mathbb{C}^{N_i \times (m_i+k+1)N_i} \to \mathbb{C}^{(k+1)N_i \times (m_i+1)N_i}$ produces a $(k+1)N_i \times (m_i+1)N_i$ block Toeplitz matrix from an $N_i \times (m_i+k+1)N_i$ matrix. Let $\boldsymbol{W}_{-}$ be an $N_i \times (m_i+k+1)N_i$ matrix, where the pth $N_i \times N_i$ block is given by $\boldsymbol{w}(p)$, $p \in \{0, 1, \ldots, m_i+k\}$. Then, the operator $\mathcal{T}_{m_i}^{(k)}$ acting on the matrix $\boldsymbol{W}_{-}$ yields:

$$\mathcal{T}_{m_i}^{(k)}\{\boldsymbol{W}_{-}\} = \begin{bmatrix} \boldsymbol{w}(k) & \boldsymbol{w}(k+1) & \cdots & \boldsymbol{w}(m_i+k) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{w}(1) & \boldsymbol{w}(2) & \cdots & \boldsymbol{w}(m_i+1) \\ \boldsymbol{w}(0) & \boldsymbol{w}(1) & \cdots & \boldsymbol{w}(m_i) \end{bmatrix}. \tag{5}$$

III. Problem Formulation

In this section, the quantities involved in formulating the problem are defined, and in the end of this section the treated problem is stated mathematically. The objective function to be minimized is the average block MSE, and it is minimized with respect to the FIR MIMO transmitter and receiver filter coefficients subject to the constraint on the average power used by the input vector time-series to the channels.

The autocorrelation matrix of dimension $(\nu+1)N_i \times (\nu+1)N_i$ of the $(\nu+1)N_i \times 1$ vector $\boldsymbol{x}_i(n)_{|}^{(\nu)}$ is defined as

$$\boldsymbol{\Phi}_{\boldsymbol{x}_i}^{(\nu,N_i)} = E\left[\boldsymbol{x}_i(n)_{|}^{(\nu)} \left(\boldsymbol{x}_i(n)_{|}^{(\nu)}\right)^H\right], \tag{6}$$

where the operator $(\cdot)^H$ denotes complex conjugate transposed. For other column-expanded vectors, the autocorrelation matrices are defined in a similar way.

Let the $(m_i+1)N_i \times (m_i+1)N_i$ matrix $\boldsymbol{\Psi}_{\boldsymbol{x}_i}^{(m_i,N_i)}(k)$ be defined as

$$\boldsymbol{\Psi}_{\boldsymbol{x}_i}^{(m_i,N_i)}(k) = E\left[\left(\boldsymbol{x}_i(n)_{|}^{(m_i)}\right)^* \left(\boldsymbol{x}_i(n+k)_{|}^{(m_i)}\right)^T\right], \tag{7}$$

where k is an integer, and the operator $(\cdot)^*$ means complex conjugation of the components. From Equations (6) and (7) it is seen that the following relationship is valid:

$$\boldsymbol{\Psi}_{\boldsymbol{x}_i}^{(m_i,N_i)}(0) = \left(\boldsymbol{\Phi}_{\boldsymbol{x}_i}^{(m_i,N_i)}\right)^* = \left(\boldsymbol{\Phi}_{\boldsymbol{x}_i}^{(m_i,N_i)}\right)^T. \tag{8}$$

The desired output vector time-series $\boldsymbol{d}_i(n)$ of receiver filter number i is given by $\boldsymbol{d}_i(n) = \boldsymbol{x}_i(n-\delta_i)$, where $\delta_i \in \{0, 1, \ldots, m_i + q_i + l_i\}$ is the vector delay experienced by the input signal $\boldsymbol{x}_i(n)$ of transmitter number i. The delay δ_i should be chosen carefully, depending on the corresponding FIR MIMO channel filter $\boldsymbol{C}_i$ and the orders of the corresponding transmitter and receiver filters. The cross-covariance matrix $\boldsymbol{\phi}_{\boldsymbol{x}_i,\boldsymbol{d}_i}^{(\nu,N_i)}$ of dimension $(\nu+1)N_i \times N_i$ is defined as:

$$\boldsymbol{\phi}_{\boldsymbol{x}_i,\boldsymbol{d}_i}^{(\nu,N_i)} = E\left[\boldsymbol{x}_i(n)_{|}^{(\nu)} \boldsymbol{d}_i^H(n)\right]. \tag{9}$$

The output of the transmitter FIR MIMO filter number p in Figure 1 is represented by the $Q_p \times 1$ vector $\boldsymbol{y}_p(n)$ given by

$$\boldsymbol{y}_p(n) = \sum_{k=0}^{m_p} \boldsymbol{E}_p(k)\boldsymbol{x}_p(n-k) = \boldsymbol{E}_{p-}\boldsymbol{x}_p(n)_{|}^{(m_p)}, \tag{10}$$

where the notation introduced in Section II is used.

Let the convolution between receiver filter $\boldsymbol{R}_i$ and channel filter $\boldsymbol{C}_p$ be denoted by $\boldsymbol{H}_{(i,p)}$. The FIR MIMO filter $\boldsymbol{H}_{(i,p)}$ has order $q_p + l_i$ and dimension $N_i \times Q_p$. The row-expansion of $\boldsymbol{H}_{(i,p)}$ can be expressed as $\boldsymbol{H}_{(i,p)\_} = \boldsymbol{R}_{i\_} \boldsymbol{C}_{p}{}_{\urcorner}^{(l_i)}$, which is an $N_i \times (q_p + l_i + 1)Q_p$ matrix. Let the convolution between channel filter $\boldsymbol{C}_k$ and transmitter filter $\boldsymbol{E}_k$ be denoted by $\boldsymbol{T}_k$, and this filter has order $m_k + q_k$ and dimension $M \times N_k$. The FIR MIMO filter from the original signal $\boldsymbol{x}_i(n)$ to the output signal $\hat{\boldsymbol{x}}_k(n)$ is denoted by $\boldsymbol{W}_{(k,i)}$, and $\boldsymbol{W}_{(k,i)}$ has order $m_i + q_i + l_k$ and dimension $N_i \times N_i$. From the notation introduced in Section II, it can be verified that $\boldsymbol{W}_{(k,i)\_} = \boldsymbol{R}_{k\_} \boldsymbol{C}_{i}{}_{\urcorner}^{(l_k)} \boldsymbol{E}_{i}{}_{\urcorner}^{(q_i+l_k)}$, which has dimension $N_k \times (m_i + q_i + l_k + 1)N_i$. The matrix $\boldsymbol{C}_{i}{}_{\urcorner}^{(l_k)}$ has dimension $(l_k+1)M \times (q_i + l_k + 1)Q_i$ and the matrix $\boldsymbol{E}_{i}{}_{\urcorner}^{(q_i+l_k)}$ has dimension $(q_i+l_k+1)Q_i \times (m_i+q_i+l_k+1)N_i$. By rewriting the convolution sum with the notation introduced in Section II, it is possible to express the output vector $\hat{\boldsymbol{x}}_k(n)$ of receiver filter number k as

$$\hat{\boldsymbol{x}}_k(n) = \sum_{i=0}^{K-1} \boldsymbol{W}_{(k,i)\_} \boldsymbol{x}_i(n)_{\lfloor}^{(m_i+q_i+l_k)} + \boldsymbol{R}_{k\_} \boldsymbol{v}(n)_{\lfloor}^{(l)}, \qquad (11)$$

where the dimensions of the vectors and the matrices follow from Section II.

The average block MSE, denoted $\mathcal{E}$, is defined by:

$$\mathcal{E} = \frac{1}{K} \sum_{i=0}^{K-1} E\left[\|\hat{\boldsymbol{x}}_i(n) - \boldsymbol{d}_i(n)\|^2\right]. \qquad (12)$$

It can be shown that the block MSE $\mathcal{E}$ in Equation (12) can be expressed as

$$\begin{aligned}\mathcal{E} =& \frac{1}{K} \sum_{k=0}^{K-1} \operatorname{Tr}\Big\{ \boldsymbol{R}_{k\_} \boldsymbol{\Phi}_{\boldsymbol{v}}^{(l_k,M)} \boldsymbol{R}_{k\_}^{H} + \\ & \left(\boldsymbol{W}_{(k,k)\_} - \boldsymbol{W}_{(k,k)\_}^{(\mathrm{ZF})}\right) \boldsymbol{\Phi}_{\boldsymbol{x}_k}^{(m_k+q_k+l_k,N_k)} \left(\boldsymbol{W}_{(k,k)\_} - \boldsymbol{W}_{(k,k)\_}^{(\mathrm{ZF})}\right)^H \\ & + \sum_{\substack{i=0\\ i\neq k}}^{K-1} \boldsymbol{W}_{(k,i)\_} \boldsymbol{\Phi}_{\boldsymbol{x}_i}^{(m_i+q_i+l_k,N_i)} \boldsymbol{W}_{(k,i)\_}^{H} \Big\}, \end{aligned} \qquad (13)$$

where the ZF matrix $\boldsymbol{W}_{(k,k)\_}^{(\mathrm{ZF})}$ is defined as $\boldsymbol{W}_{(k,k)\_}^{(\mathrm{ZF})} = \left[\boldsymbol{0}_{N_k \times \delta_k N_k} \; \boldsymbol{I}_{N_k} \; \boldsymbol{0}_{N_k \times (m_k+q_k+l_k-\delta_k)N_k}\right]$. Here, the matrix $\boldsymbol{I}_{N_K}$ is the identity matrix of dimension $N_k \times N_k$, and $\boldsymbol{W}_{(k,k)\_}^{(\mathrm{ZF})}$ has dimension $N_k \times (m_k+q_k+l_k+1)N_k$. The first term inside the trace operator in Equation (13), represents the undesired additive channel noise at the output of receiver filter number k, the second term is the signal MSE for user number k, and the third term is the unwanted signals from other users at the output of receiver filter number k.

The average power constraint for transmitter number i is a constraint on the the channel input vector time-series $\boldsymbol{y}_i(n)$ and it can be expressed

$$E\left[\|\boldsymbol{y}_i(n)\|^2\right] = \operatorname{Tr}\left\{\boldsymbol{E}_{i\_} \boldsymbol{\Phi}_{\boldsymbol{x}_i}^{(m_i,N_i)} \boldsymbol{E}_{i\_}^{H}\right\} = P_i, \qquad (14)$$

where P_i is the power used by mobile station number i. Equation (14) follows from Equation (10).

From Equations (13) and (14), the constrained optimization problem to be solved can be stated as follows:

Problem 1:

$$\min_{\left\{\boldsymbol{E}_0,\ldots,\boldsymbol{E}_{K-1},\boldsymbol{R}_0,\ldots,\boldsymbol{R}_{K-1}\right\}} \mathcal{E},$$

subject to (15)

$$E\left[\|\boldsymbol{y}_i(n)\|^2\right] = P_i \;\; \forall \;\; i \in \{0,1,\ldots,K-1\}.$$

IV. Proposed Solution

Problem 1 can be converted into an unconstrained optimization problem by the use of Lagrange multipliers. The unconstrained objective function ζ can be expressed as

$$\zeta = \mathcal{E} + \frac{1}{K}\sum_{i=0}^{K-1} \mu_i \operatorname{Tr}\left\{\boldsymbol{E}_{i\_} \boldsymbol{\Phi}_{\boldsymbol{x}_i}^{(m_i,N_i)} \boldsymbol{E}_{i\_}^{H}\right\}, \qquad (16)$$

where μ_i is the positive Lagrange multiplier corresponding to the power constraint of transmitter number i. The Lagrangian multipliers μ_i will later be eliminated in the fixed point iteration proposed for the transmitter optimization. Necessary conditions for optimality are found through matrix differentiation of the positive unconstrained objective function ζ with respect to the conjugate of the complex unknown matrices.

It can be shown, that for given FIR MIMO receiver filters, the optimal transmitter FIR MIMO filter number p is obtained from

$$\left(\boldsymbol{A}_p + \mu_p \boldsymbol{F}_p\right) \cdot \operatorname{vec}(\boldsymbol{E}_{p\_}) = \boldsymbol{b}_p, \qquad (17)$$

where the operator $\operatorname{vec}(\cdot)$ stacks the columns of the matrix into a long column vector [7], and where matrix $\boldsymbol{A}$ is an $(m_p+1)N_pQ_p \times (m_p+1)N_pQ_p$ matrix given by

$$\boldsymbol{A}_p = \sum_{i=0}^{K-1} \sum_{k_0=0}^{q_p+l_i} \sum_{k_1=0}^{q_p+l_i} \boldsymbol{\Psi}_{\boldsymbol{x}_p}^{(m_p,N_p)}(k_0-k_1) \otimes \left(\boldsymbol{H}_{(i,p)}^{H}(k_0) \boldsymbol{H}_{(i,p)}(k_1)\right), \qquad (18)$$

where the operator $\otimes$ is the Kronecker product [7], the matrix $\boldsymbol{F}_p$ is an $(m_p+1)N_pQ_p \times (m_p+1)N_pQ_p$ matrix given by $\boldsymbol{F}_p = \boldsymbol{\Psi}_{\boldsymbol{x}_p}^{(m_p,N_p)}(0) \otimes \boldsymbol{I}_{Q_p}$, and the vector $\boldsymbol{b}_p$ has dimension $(m_p+1)Q_pN_p \times 1$ and is given by:

$$\boldsymbol{b}_p = \operatorname{vec}\left(\left(\boldsymbol{H}_{(p,p)_{\lfloor}}\right)^H \mathcal{I}_{m_p}^{(q_p+l_p)}\left\{\left(\boldsymbol{\phi}_{\boldsymbol{x}_p}^{(m_p+q_p+l_p,N_p)}\right)^H\right\}\right). \qquad (19)$$

From Equations (7) and (18), it is seen that $\boldsymbol{A}^H = \boldsymbol{A}$ and that the matrix $\boldsymbol{A}$ is positive semidefinite.

If the average power P_p is specified, the Lagrange multiplier μ_p can be eliminated. It can be shown from Equations (14) and (17) that the following fixed point iteration can be used to find transmitter number p:

$$\operatorname{vec}\left(\boldsymbol{E}_{p\_}\right) = \left[\boldsymbol{A}_p + \frac{\operatorname{vec}^H\left(\boldsymbol{E}_{p\_}\right)\left(\boldsymbol{b}_p - \boldsymbol{A}_p \operatorname{vec}\left(\boldsymbol{E}_{p\_}\right)\right)}{P_p} \boldsymbol{F}_p\right]^{-1} \boldsymbol{b}_p. \qquad (20)$$

When using Equation (20) in a fixed point iteration, the Lagrange multiplier μ_p is eliminated.

It can be shown that the optimized FIR MIMO receiver filter number p for given FIR MIMO transmitter filters is found by the following equation

$$\begin{aligned}\boldsymbol{R}_{p\_} =& \left(\boldsymbol{\phi}_{\boldsymbol{x}_p}^{(m_p+q_p+l_p,\delta_p,N_p)}\right)^H \left(\boldsymbol{T}_{p}{}_{\urcorner}^{(l_p)}\right)^H \\ & \cdot \left[\boldsymbol{\Phi}_{\boldsymbol{v}}^{(l_p,M)} + \sum_{k=0}^{K-1} \boldsymbol{T}_{k}{}_{\urcorner}^{(l_p)} \boldsymbol{\Phi}_{\boldsymbol{x}_k}^{(m_k+q_k+l_p,N_k)} \left(\boldsymbol{T}_{k}{}_{\urcorner}^{(l_p)}\right)^H\right]^{-1}, \end{aligned} \qquad (21)$$

where the dimension of the matrix $\boldsymbol{T}_{k}{}_{\urcorner}^{(l_p)}$ is $(l_p+1)M \times (m_k + q_k + l_p + 1)N_k$. This result can also be derived by means of the orthogonality principle [8], [9]. These filters are called FIR MIMO Wiener filters.

The problem of jointly optimizing the overall system performance is performed by the following iterative approach: For fixed transmitter FIR MIMO filters, the receiver FIR MIMO filters are optimized by solving Equation (21), then the transmitter FIR MIMO filters are optimized by using the previously optimized value of the receiver filter in Equation (20), and this procedure is repeated until convergence is reached. The algorithm is guaranteed to converge at least to a local

TABLE I
CHANNEL COEFFICIENTS $C_k(i)$ FOR THE $K = 3$ SCALAR CHANNELS USED IN SECTION V. THE ORDER OF THE CHANNELS WAS $L = 5$.

i	$C_0(i)$	$C_1(i)$	$C_2(i)$
0	$0.824 + 0.042j$	$0.881 - 0.898j$	$-0.040 + 0.265j$
1	$0.443 + 1.271j$	$-0.452 + 0.696j$	$0.363 + 0.796j$
2	$0.053 + 0.187j$	$0.408 - 0.032j$	$0.281 + 0.515j$
3	$0.249 + 0.616j$	$-0.255 - 0.565j$	$0.535 - 1.681j$
4	$-0.492 - 1.023j$	$-0.096 - 0.541j$	$0.283 - 0.194j$
5	$1.199 - 0.496j$	$-0.954 + 0.609j$	$-0.949 - 0.228j$

minimum since at each step the objective function is decreased and the objective function is lower bounded by zero.

If $K = 1$ the derived equations for the optimal transmitter and receiver filter reduce to the equations proposed in [3], which treats the corresponding single user MIMO communications problem.

For the case where some of the filter coefficients of the FIR MIMO filters $\boldsymbol{R}_i$ and $\boldsymbol{E}_i$ are equal to zero, a optimization procedure similar to the one presented in [10], can be used.

V. RESULTS AND COMPARISONS

A comparison is made against a system employing CDMA with spreading codes of length $M = 31$ in all the single-antenna mobile stations. Normalized Gold codes [6] are used as spreading codes. $K = 3$ users are assumed. In order for the studied model to agree with the single antenna CDMA model, $m_i = 0$, $N_i = 1$ and $Q_i = M$ for $i \in \{0, 1, 2\}$. The CDMA system employs minimum MSE receiver filters, i.e., Wiener filters, see Equation (21). The order of all the scalar causal FIR channel impulse responses is $L = 5$, and they are taken as samples from a white complex circularly symmetric Gaussian process with variance 1. The scalar channel coefficients $C_k(i)$ are shown in Table I, where $j = \sqrt{-1}$. Since $L \leq M$, it is shown in [2] that the *equivalent FIR MIMO channel* filter $\boldsymbol{C}_i$ of dimension $M \times M$ has order $q_i = 1$, and the two terms of the FIR MIMO channel filter $\{\boldsymbol{C}_i(k)\}_{k=0}^{q_i=1}$ can be found from Equation (19) in [2]. The order of the receiver filters is $l_i = 2$ for all receiver filters in both the CDMA system and the proposed system. The delays through the system are chosen as $\delta_i = 1$ for both systems. The additive channel noise is assumed to be white complex Gaussian with variance σ_v^2.

Assume that BPSK signals $\{-1, +1\}$ are transmitted, and assume that the original BPSK signals are uncorrelated with equally likely symbols. Let the average energy per bit be denoted $E_b = \frac{1}{K}\sum_{i=0}^{K-1} E\left[\boldsymbol{y}_i^H(n)\boldsymbol{y}_i(n)\right]$. Using the theory developed in [4], it is possible to develop an exact theoretical expression for the BER for the system described in this section. When finding the the BER versus channel quality, measured in E_b/σ_v^2, performance of the two systems, these theoretical expressions are used to find the performances. These performances are verified by Monte Carlo simulations for relatively high values of BER, since the Monte Carlo estimates depend on the number of bits used in the simulations. Figure 2 compares the theoretical BER versus E_b/σ_v^2 performances of the CDMA based system that uses minimum MSE receiver filters and Gold codes in the transmitters and the proposed system. From Figure 2, it is seen that for example for $\mathrm{BER} = 10^{-10}$, about 5.5 dB can be gained by the proposed system over the CDMA based system. The proposed system and the CDMA based system have the same number of filter coefficients in all the filters. The receiver filters, in both systems, have the same complexity. The transmitter filters in the CDMA based system are easier to implement since the transmitter filter coefficients just have two real values, and the proposed system, in general, has M different complex coefficients in each transmitter filter for the parameter values chosen in this section. The proposed

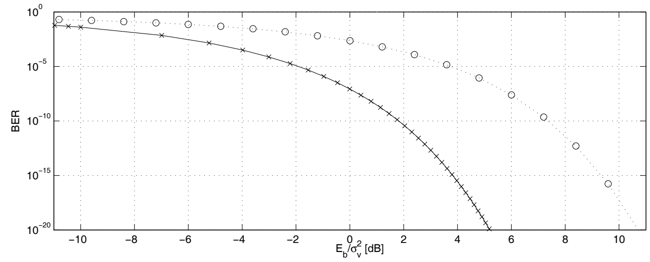

Fig. 2. BER versus E_b/σ_v^2 performance of the CDMA system using minimum MSE receivers and Gold spreading codes ($\cdots\circ\cdots$) and the proposed system ($-\times-$).

system is more complicated to optimize than the CDMA system. However, the significant gain of the proposed system might justified the increase in implementation and design complexity. This depends on the application.

VI. CONCLUSIONS

Equations were derived for finding jointly optimized transmitter and receiver FIR MIMO filters for multi-user uplink communications over power-constrained vector channels. Based on these equations an iterative solution was proposed which is able to converge to a locally optimal solution. The receiver filters are FIR MIMO Wiener filters. A numerical example was given showing the potential of the proposed method compared to a CDMA based method using Gold codes and Wiener receiver filters. It was shown that the proposed method had significantly better BER versus channel quality performance than the CDMA based method. The reason for the improvement is that the proposed system jointly optimizes the transmitter and receiver FIR MIMO filters.

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