Iterative Joint Optimization of Minimal Transmit Redundancy FIR Zero-Forcing Precoder-Equalizer System for MIMO-ISI Channel

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Abstract-An iterative joint FIR precoder-equalizer optimization algorithm for MIMO ISI channel is proposed. This algorithm provides a suboptimal solution of the FIR zero-forcing precoder (ZFP) and equalizer (ZFE) for the Space-Time Modulated Code (STMC) system. Although serval joint precoder-equalizer design methods for MIMO ISI were proposed, existing design methods require a long guard period, which is larger than or equal to the channel order, to be inserted in the transmitted signal for avoiding the inter-block interference (IBI). This reduces the spectral efficiency of those systems. In this paper, we adopted the STMC structure, which does not require a long guard period. Using the FIR LS ZFE designing techniques previously proposed by us, we proposed a FIR LS ZFP design method. By combining the proposed ZFP design method with the ZFE design method, we propose an iterative joint FIR precoder-equalizer optimization algorithm. Although the iterative algorithm can only provide the suboptimal precoder-equalizer design, simulation results showed that substantial performance gain can be obtained when comparing with those cases without the joint precoderequalizer design.

I. INTRODUCTION

Block based high speed communication can be obtained using the spatial diversity provided by the multiple transmit and receive antennas [5], and reducing the redundancy introduced in the transmitted signals for error correction purpose. Examples of such systems include V-BLAST [6], [7] and space-time codes [8], [9]. However, those systems only support flat fading channel communications. If selective fading channel is considered, the system will need to eliminate both the inter-symbol interference (ISI) and the inter-block interference (IBI). Inserting a guard period of length ρ in between each block of K transmitted signal is a common method to avoid IBI [10], [11]. However, ρ has to be at least as long as the channel length to achieve IBI free communication. This results in a reduction in the spectral efficiency by a factor of $K/(K + \rho)$, which will be very severe with the increase in channel bandwidth. Although increasing K will reduce the loss, it also increase the system delay and complexity.

Recently, a promising system, *Space-Time Modulated Codes* (*STMC*), is proposed by Xia [1]. STMC is a special type of the space time block codes for selective fading MIMO channel. Systems using STMC can release the requirement of guard period duration. Hence, it avoided the degradation of system throughput suffered by most of the block base transmission system. The STMC based system is basically constructed by a pair of precoder and equalizer. Different equalizer design methods for a given precoder were proposed [1], [3]. However, since the precoder was predefined, it can not be optimized towards the channel property. Several joint precoder-equalizer design methods for MIMO or SISO channel were proposed [11]–[15], but none of them allow the guard period to be shorter than the channel order. This means that they cannot take the advantage of minimal transmit redundancy provided by STMC.

In this paper, we propose a joint precoder-equalizer design method for MIMO ISI channel based on the STMC precoder structure. This means that our design method will not require the grand period to be great than or equal to the channel order. We will make use of the iterative optimization idea used in [15] as a framework. The method in [15] derived a pair of equations that describe the optimal equalizer with a given precoder, as well as the optimal precoder with a given equalizer. With an initial precoder equalizer pair, these two equations are used alternatively to jointly optimize both of the precoder and equalizer. However, the algorithm proposed in [15] can only be used

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for fading channel environment. In order to transform this idea to support the selective fading environment with STMC, we will apply the FIR least square (LS) zero-forcing equalizer (ZFE) designing method proposed in [3] to design the LS optimal equalizer with a given STMC precoders. By modifying this equalizer design method, we then proposed a FIR LS zero-forcing precoder (ZFP) design method with a given equalizer in Section IV. By combining the FIR LS ZFE and ZFP design methods with the iterative optimization idea in [15], the iterative joint precoder-equalizer algorithm is proposed in Section VI. The sufficient conditions for the iterative method are also derived. Although this iterative joint optimization algorithm only provides suboptimal result, the simulation results in Section VII shows that substantial performance improvement can be obtained when compared to systems without using the joint optimization algorithm.

II. CHANNEL MODEL

Consider a MIMO communication system with N transmit and M receive antennas. Let $\tilde{x}_n(z)$ be the transmitted signal at the nth transmit antenna, $\tilde{y}_m(z)$ be the received signal in the mth receive antenna, $\eta_m(z)$ be the AWGN noise received at the mth receive antenna, and $h_{m,n}(k)$ be the length L impulse response of the ISI channel corresponding to the nth transmit antenna and the mth receive antenna. The MIMO communication system can be represented by,

$$\tilde{\mathbf{y}}(z) = \mathbf{H}(z)\tilde{\mathbf{x}}(z) + \tilde{\eta}(z), \tag{1}$$

where

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$$\dot{\Psi}(z) \stackrel{\Delta}{=} [\tilde{y}_1(z), \dots, \tilde{y}_M(z)]^T,$$
(2)

$$\tilde{\mathbf{x}}(z) \stackrel{\simeq}{=} [\tilde{x}_1(z), \dots, \tilde{x}_N(z)]^T,$$
(3)

$$\tilde{\eta}(z) \stackrel{\simeq}{=} [\tilde{\eta}_1(z), \dots, \tilde{\eta}_M(z)]^T, \qquad (4)$$
$$\begin{bmatrix} h_{1,1}(z) & \cdots & h_{1,N}(z) \end{bmatrix}$$

$$\mathbf{H}(z) \stackrel{\triangle}{=} \begin{bmatrix} \vdots & \ddots & \vdots \\ h_{M,1}(z) & \cdots & h_{M,N}(z) \end{bmatrix}.$$
(5)

Here $(\cdot)^T$ denotes the matrix transpose.

III. STMC CODED SYSTEM

The STMC proposed in [1] enhances the system performance by adding a linear precoding block $\mathbf{G}(z)$ and equalization block $\mathbf{F}(z)$ in the transmitter and receiver respectively. The input source signal, $\mathbf{x}(z)$, is a length K complex valued column vector, which can be the signal after binary to complex mapping, or the encoded signal of an outer code. The signal is precoded using the linear STMC precoder $\mathbf{G}(z)$. $\mathbf{G}(z)$ is a $NP \times K$ polynomial matrix with STMC block size P. The NP outputs of the precoder $\mathbf{G}(z)$ represent the P polyphase components of the signals in each transmit antenna. These signal will go through a P-to-1 parallel to serial block and then launched to the MIMO channel. Hence, $\mathbf{G}(z)$ is precoding block with rate K/(NP). When NP > K, the precoder induces redundancy to the transmitted signal that is exploited in the receiver to compensate for the spectral nulling effect of the ISI channel.

The signal received by the M receiver antennas is converted by a 1-to-P serial-to-parallel converter. The length NP output vector is processed by a linear equalizer $\mathbf{F}(z)$. The linear equalizer makes use of the redundant information induced by the precoder to recover the source signal. Readers should refer to [1], [2] for further details.

The design of the LS optimal FIR ZFE with a given channel and STMC precoder was proposed in [3]. However, there does not existing

an algorithm that jointly optimized both the STMC ZFE and ZFP. By modifying the ZFE design method proposed in [3], an optimal FIR ZFP algorithm with a given equalizer is constructed. After that, an iterative joint optimization method for the suboptimal solution of joint ZF precoder-equalizer is proposed.

A. Formulation of FIR ZF Precoder-Equalizer

The STMC precoder-equalizer system can be formulated using polyphase representation. Let $\mathbf{H}_p(z)$ and $\eta_p(z)$ be the *p*th polyphase component of $\mathbf{H}(z)$ and $\tilde{\eta}(z)$ in eq.(1), such that $\mathbf{H}(z) = \sum_{p=0}^{P-1} \mathbf{H}_p(z^P) z^{-p}$ and $\tilde{\eta}(z) = \sum_{p=0}^{P-1} \eta_p(z^P) z^{-p}$. The equalized output, $\mathbf{y}(z)$, can be written as,

$$\mathbf{y}(z) = \mathbf{F}(z)[\mathcal{H}(z)\mathbf{G}(z)\mathbf{x}(z) + \eta(z)], \tag{6}$$

where $\mathcal{H}(z)$ is the blocked version of $\mathbf{H}(z)$ that has a pseudo-circulant form

$$\mathcal{H}(z) = \begin{bmatrix} \mathbf{H}_{0}(z) & z^{-1}\mathbf{H}_{P-1}(z) & \cdots & z^{-1}\mathbf{H}_{1}(z) \\ \mathbf{H}_{1}(z) & \mathbf{H}_{0}(z) & \cdots & z^{-1}\mathbf{H}_{2}(z) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{P-2}(z) & \mathbf{H}_{P-3}(z) & \ddots & z^{-1}\mathbf{H}_{P-1}(z) \\ \mathbf{H}_{P-1}(z) & \mathbf{H}_{P-2}(z) & \cdots & \mathbf{H}_{0}(z) \end{bmatrix}, (7)$$

$$\mathbf{y}(z) \stackrel{\triangle}{=} [y_0(z) \ y_1(z) \ \cdots \ y_{K-1}(z)]^T, \tag{8}$$

$$\mathbf{x}(z) \stackrel{\triangle}{=} [x_0(z) \ x_1(z) \ \cdots \ x_{K-1}(z)]^T, \tag{9}$$

$$\mathbf{r}(\mathbf{x}) \stackrel{\Delta}{\longrightarrow} [\mathbf{r}^T(\mathbf{x}) \quad \mathbf{r}^T(\mathbf{x}) \quad \mathbf{r}^T \quad (\mathbf{x})]^T \tag{10}$$

$$\eta(z) = [\eta_0^-(z) \ \eta_1^-(z) \ \cdots \ \eta_{P-1}^-(z)]^-.$$
(10)

The STMC code rate is K/(NP). Since M, N and P are fixed, the code rate is maximized when the source signal block size K is maximized. Denotes K_{max} as the maximal K that can achieve ZF precoding and equalization. [3] showed that

$$K_{max} = \min(NP, MP) - \rho_{\mathcal{H}},\tag{11}$$

where $\rho_{\mathcal{H}}$ is the number of nonunity terms in the diagonal of the Smith form of $\mathcal{H}(z)$. The FIR ZF precoder-equalizer system achieve *minimal transmit redundancy* when $K = K_{max}$. Interested reader please refers to [3] for the proof.

IV. FIR ZFP DESIGN

Denotes the virtual channel $\mathcal{F}(z)$ as the combined $\mathbf{F}(z)$ and $\mathcal{H}(z)$ in eq.(6), $\mathcal{F}(z) \stackrel{\triangle}{=} \mathbf{F}(z)\mathcal{H}(z)$. ISI-free communication is achieved when y(z) in eq.(6) equals to the scaled delayed input signal x(z), i.e., $y_k(z) = c_k z^{-r_k} x_k(z)$ with non-zero constants c_k and integers r_k for all $k = 0, 1, \ldots, K - 1$. Therefore, ISI-free communication is achieved when

$$\mathcal{F}(z)\mathbf{G}(z) = \operatorname{diag}(c_0 z^{-r_0}, c_1 z^{-r_1}, \dots, c_{K-1} z^{-r_{K-1}}).$$
(12)

Without lost of generality, we assume $c_k = 1$ and $r_k \ge 0$ for $k = 0, 1, \ldots, K - 1$, in the following discussion. Let $\gamma = \{r_0, r_1, \ldots, r_{K-1}\}$ be the set of delay of the sub-channels. As a result, the ZFP is given by $\mathbf{G}(z) = \mathcal{F}^{\dagger}(z)\mathbf{D}_{\gamma}(z)$, where $\mathbf{D}_{\gamma}(z) = \text{diag}(z^{-r_0}, z^{-r_1}, \ldots, z^{-r_{K-1}})$, and $(\cdot)^{\dagger}$ denotes the matrix pseudoinverse.

A. Minimal Order Requirement for FIR ZFP

The ZFP $\mathbf{G}(z)$ equals to the pseudo-inverse of the polynomial matrix $\mathcal{F}(z)$. As a result, the solution of $\mathbf{G}(z)$ may not be unique. One of the free parameter in designing $\mathbf{G}(z)$ is the order of the polynomial matrix. Let L_D , $L_{\mathcal{F}}$ and L_G be the order of $\mathbf{D}(z)$, $\mathcal{F}(z)$ and $\mathbf{G}(z)$, such that $\mathbf{D}_{\gamma}(z) \stackrel{\Delta}{=} \sum_{\ell=0}^{L_D} \mathbf{D}_{\ell} z^{-\ell}$, $\mathcal{F}_{\gamma}(z) \stackrel{\Delta}{=} \sum_{\ell=0}^{L_C} \mathcal{F}_{\ell} z^{-\ell}$, and $\mathbf{G}_{\gamma}(z) \stackrel{\Delta}{=} \sum_{\ell=0}^{L_G} \mathbf{G}_{\ell} z^{-\ell}$. The polynomial matrix multiplication in eq.(12) can be rewritten in terms of a multiplication of a set of scalar block matrices,

$$\tilde{\mathbf{D}}_{\gamma} = \tilde{\mathcal{F}}\tilde{\mathbf{G}},\tag{13}$$

where,

$$\tilde{\mathbf{D}}_{\gamma} \stackrel{\Delta}{=} \begin{bmatrix} \mathbf{D}_{L_D}^T & \mathbf{D}_{L_D-1}^T & \cdots & \mathbf{D}_0^T \end{bmatrix}^T, \quad (14)$$

$$\tilde{\mathbf{G}} \stackrel{\Delta}{=} \begin{bmatrix} \mathbf{G}_{L_G}^T & \mathbf{G}_{L_G-1}^T & \cdots & \mathbf{G}_0^T \end{bmatrix}^T, \quad (15)$$

$$\tilde{\mathcal{F}} \stackrel{\triangle}{=} \begin{bmatrix} \mathcal{F}_{L_{\mathcal{F}}} & \cdots & \mathcal{F}_{0} & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & \mathcal{F}_{L_{\mathcal{F}}}^{T} & \cdots & \mathcal{F}_{0}^{T} \end{bmatrix} .$$
(16)

The ZF condition in eq.(12) can be represented using eq.(13) with,

$$[\tilde{\mathbf{D}}_{\gamma}]_{\{i,j\}} = \begin{cases} 1 & \text{, for } i = (L_D - r_j - 1)K + j + 1, \\ 0 & \text{, otherwise,} \end{cases}$$
(17)

for all j = 0, 1, ..., K - 1, where $[\tilde{\mathbf{D}}_{\gamma}]_{\{i,j\}}$ denotes the element in the *i*th row and *j*th column of $\tilde{\mathbf{D}}_{\gamma}$. This formulated a *scalar matrix representation* of the polynomial matrix equation in eq.(12).

Observed from eq.(13), the existence of ZFP required the set of system delay γ to be chosen with

$$[\tilde{\mathcal{F}}]_{\{[(L_D - r_j - 1)K + j + 1] \text{-th row}\}} \neq \mathbf{0}, \text{ for } j = 0, 1, \dots, K - 1.$$
 (18)

Assume the set of system delays is chosen appropriately, the ZFP $\mathbf{G}(z)$ can be obtained from $\tilde{\mathbf{G}}$ in eq.(13).

$$\tilde{\mathbf{G}} = \tilde{\mathcal{F}}^{\dagger} \tilde{\mathbf{D}}.$$
(19)

Assume $\tilde{\mathcal{F}}$ has ξ non-zero columns, and the non-zero rows are indexed by $\ell_1, \ell_2, \ldots, \ell_{\xi}$, with $1 \leq \ell_1 < \ell_2 < \cdots < \ell_{\xi} \leq K(L_{\mathcal{F}} + L_G + 1)$. Form a matrix $\hat{\mathcal{F}}$ with the non-zero rows of $\hat{\mathcal{F}}$,

$$\hat{\mathcal{F}} \stackrel{\triangle}{=} [\tilde{\mathcal{F}}]_{\{(\ell_1,\ell_2,\ldots,\ell_{\xi})\text{-th rows}\}} = \mathbf{T}\tilde{\mathcal{F}}, \qquad (20)$$
with
$$[\mathbf{T}]_{\{i,j\}} = \begin{cases} 1 & \text{, for } j = \ell_i \text{ and } i = 1, 2, \ldots, \xi \\ 0 & \text{, otherwise.} \end{cases}$$

To simplify our discussion, we assumed $\hat{\mathcal{F}}$ has full rank. Because, the coefficients of the channel transfer function, $\mathbf{H}(z)$ is randomly distributed in real world applications, therefore, the matrix $\hat{\mathcal{F}}$ is almost sure to have full rank. The pseudo-inverse of $\tilde{\mathcal{F}}$ can be obtained by

$$\hat{\mathcal{F}}^{\dagger} = \hat{\mathcal{F}}^{\dagger} \mathbf{T}.$$
 (21)

The FIR ZFP $\mathbf{G}(z)$ exists if the right pseudo-inverse of $\hat{\mathcal{F}}$ exists. Since $\hat{\mathcal{F}}$ has full rank, the existence condition of $\hat{\mathcal{F}}^{\dagger}$ is equivalent to

Number of column of
$$\hat{\mathcal{F}} \geq N$$
 Number of row of $\hat{\mathcal{F}}$,
 $\Rightarrow NP(L_G+1) \geq \xi$,

$$\Leftrightarrow \qquad L_G \qquad \geq \quad \frac{\xi}{NP} - 1. \qquad (22)$$

Hence, the minimal possible order of $\mathbf{G}(z)$, $L_{G_{min}}$, is given by eq.(22) as

$$L_{G_{min}} = \left\lceil \xi / (NP) \right\rceil - 1, \tag{23}$$

where $\lceil \cdot \rceil$ denotes the ceiling. In most case, $\tilde{\mathcal{F}}$ will not have all zero rows. As a result, $\tilde{\mathcal{F}} = \hat{\mathcal{F}}$ and ξ equals to the total number of rows in $\tilde{\mathcal{F}}$, i.e. $\xi = K(L_{\mathcal{F}} + L_G + 1)$. Hence eq.(23) can be simplified to

$$L_{G_{min}} = \left\lceil \frac{KL_{\mathcal{F}}}{NP - K} \right\rceil - 1.$$
(24)

B. Parameterization of FIR ZFP

Assume that those conditions in the previous section are satisfied. In this section, we consider the design of ZFP. Let $\mathbf{G}_{\gamma}(z)$ be a solution of ZFP with a given set of system delays γ which satisfies the condition in eq.(18), and $\tilde{\mathbf{G}}_{\gamma}$ be the corresponding scalar matrix representation of $\mathbf{G}_{\gamma}(z)$. The solution of $\tilde{\mathbf{G}}_{\gamma}$ can be obtained by eq.(19) and eq.(20). Notice that if $L_G \leq L_{G_{min}}$, $\hat{\mathcal{F}}$ may not be a square matrix. This implies that there may be infinitely many solution of $\tilde{\mathbf{G}}_{\gamma}$ (and so as $\mathbf{G}_{\gamma}(z)$). If $\tilde{\mathbf{G}}_{\gamma}$ can be parameterized, the parameterization will help to analysis the FIR ZFP solution and construct the optimal solution.

Using singular value decomposition (SVD), $\hat{\mathcal{F}}$ can be rewritten as $\hat{\mathcal{F}} = \mathbf{U}\Sigma\mathbf{V}^{H}$, where U and V are unitary matrices with $(\cdot)^{H}$ denotes the conjugate transpose, and Σ is a $\xi \times NP(L_G+1)$ matrix, with

$$[\Sigma]_{\{i,j\}} \stackrel{\triangle}{=} \begin{cases} \sigma_i &, \text{ for } i=j=1,2,\dots,\xi \\ 0 &, \text{ otherwise.} \end{cases}$$
(25)

where σ_i , for $i = 1, 2, ..., \xi$, is the singular value of $\hat{\mathcal{F}}$. Define $\mu \stackrel{\triangle}{=} NP(L_G + 1) - \xi$. The inverse of $\hat{\mathcal{F}}_{\gamma}$ can be parameterized by an arbitrary matrix $\hat{\mathbf{A}} \in \mathbb{R}_{\mu \times \xi}$,

$$\hat{\mathcal{F}}^{\dagger} = [\hat{\mathcal{F}}^{\ddagger} \ \mathcal{V}] \begin{bmatrix} I_{\xi} \\ \hat{\mathbf{A}} \end{bmatrix}, \qquad (26)$$

where $\mathcal{V} \stackrel{\triangle}{=} [\mathbf{V}]_{\{\mu+1,\mu+2,\dots,\mu+\xi \text{ columns}\}}$, and $\hat{\mathcal{F}}^{\ddagger}$ is the Moore-Penrose pseudo-inverse of $\hat{\mathcal{F}}$,

$$\hat{\mathcal{F}}^{\ddagger} \stackrel{\triangle}{=} \mathbf{V} \Sigma^{\ddagger} \mathbf{U}^{H}, \text{ with } [\Sigma^{\ddagger}]_{\{i,j\}} \stackrel{\triangle}{=} \left\{ \begin{array}{c} \sigma_{i}^{-1} & , \text{ for } i=j=1,\ldots,\xi \\ 0 & , \text{ otherwise.} \end{array} \right.$$
(27)

The parameterized solution of $\tilde{\mathbf{G}}$ is obtained by substituting eq.(26) into eq.(19) and (21), such that

$$\tilde{\mathbf{G}}_{\gamma} = [\hat{\mathcal{F}}^{\ddagger} \ \mathcal{V}] \begin{bmatrix} \mathbf{T} \tilde{\mathbf{D}}_{\gamma} \\ \mathbf{A} \end{bmatrix}, \qquad (28)$$

where $\mathbf{A} \stackrel{\triangle}{=} \hat{\mathbf{A}} \mathbf{T} \tilde{\mathbf{D}}_{\gamma}$ is a free matrix that parameterize $\tilde{\mathbf{G}}_{\gamma}$, which indirectly parameterizes $\mathbf{G}_{\gamma}(z)$.

C. LS Optimal Solution of ZFP

With the parameterization of matrix $\tilde{\mathbf{G}}_{\gamma}$ in eq.(28), we can obtain the LS optimized $\tilde{\mathbf{G}}_{\gamma}$ by adjusting the free parameter **A**. The equalized signal vector $\mathbf{y}(z)$ in eq.(6) with a given ZFP is,

$$\mathbf{y}(z) = \mathbf{x}(z) + \mathbf{F}(z)\eta(z). \tag{29}$$

Since the ZFE $\mathbf{F}(z)$ is given in designing the ZFP $\mathbf{G}(z)$, the signal-to-noise ratio (SNR) of the equalized signal $\mathbf{y}(z)$ will not be alternated by choosing a different $\mathbf{G}(z)$ from the solution set given by eq.(28). However, different ZFP given by eq.(28) will provide different transmission power requirement. As a result, the $\mathbf{G}(z)$ which requires the minimal transmission will provide the LS optimal performance under the total transmit power constraint.

Definition 1 (Frobenius Norm of Polynomial Matrix): The

Frobenius norm of any polynomial matrix $\mathbf{X}(z) \stackrel{\triangle}{=} \sum_{n} \mathbf{X}_{n} z^{-n}$ is defined as $\|X(z)\|_{F} \stackrel{\triangle}{=} \sqrt{\sum_{i} \operatorname{trace}(X_{i}^{T} X_{i})}$.

With eq.(29), the power gain of the transmitted signal is proportional to $||\mathbf{G}(z)||_F^2$. Hence, we can obtain an optimal precoding by choosing a $\mathbf{G}_{\gamma}(z)$ from eq.(28) which has minimal $||\mathbf{G}(z)||_F$. Since $\tilde{\mathbf{G}}_{\gamma}$ is formed by the coefficient of $\mathbf{G}_{\gamma}(z)$, eq.(16) and the Frobenius norm of polynomial matrix provides the following relationship,

$$\|\tilde{\mathbf{G}}_{\gamma}\|_{F} = \|\mathbf{G}_{\gamma}(z)\|_{F}.$$
(30)

Eq.(30) simplifies the optimization because $\tilde{\mathbf{G}}_{\gamma}$ is just a scalar matrix. The optimal precoder can be obtained by finding an optimal matrix **A** in eq.(28) that minimizes $\|\tilde{\mathbf{G}}_{\gamma}\|_{F}$. By eq.(28), $\|\tilde{\mathbf{G}}_{\gamma}\|_{F}^{2}$ can be written as,

$$\|\tilde{\mathbf{G}}_{\gamma}\|_{F}^{2} = \|\hat{\mathcal{F}}^{\dagger}\mathbf{T}\tilde{\mathbf{D}}_{\gamma}\|_{F}^{2} + \|\mathcal{V}\mathbf{A}\|_{F}^{2}.$$
(31)

Hence, $\|\tilde{\mathbf{G}}_{\gamma}\|_{F}$ is minimized when $\|\mathcal{V}\mathbf{A}\|_{F} = 0$. Since the columns of \mathcal{V} are orthogonal, having $\|\mathcal{V}\mathbf{A}\|_{F} = 0$ implies $\mathbf{A} = 0$. As a result, the scalar matrix representation of LS optimal ZFP with the set of system delays γ (denoted as $\tilde{\mathbf{G}}_{\gamma,opt}$) can be obtained by,

$$\tilde{\mathbf{G}}_{\gamma,opt} = \hat{\mathcal{F}}^{\ddagger} \mathbf{T} \tilde{\mathbf{D}}_{\gamma}. \tag{32}$$

$$\begin{array}{|c|c|c|} & \text{Set } \mathbf{G}_0(z) = \mathbf{G}_{init}(z) \text{ and } n = 0. \\ & \text{Do} \\ & \text{Design LS optimal } \mathbf{F}_n(z) \text{ using } \mathcal{H}(z) \text{ and } \mathbf{G}_n(z) \\ & \text{Normalize } \mathbf{F}_n(z), \text{ i.e. } \mathbf{F}_n(z) = \frac{\mathbf{F}_n(z)}{||\mathbf{F}_n(z)||_F} \\ & \text{Design LS optimal } \mathbf{G}_{n+1}(z) \text{ using } \mathcal{H}(z) \text{ and } \mathbf{F}_n(z) \\ & n = n + 1 \\ & \text{Until } 1 - \frac{||\mathbf{G}_n(z)||_F}{||\mathbf{G}_{n-1}(z)||_F} < \alpha \end{array}$$

Fig. 1. Iterative Joint ZFP and ZFE Optimization Algorithm.

Denotes the set of optimal system delay as $\gamma_{opt} = \{r_{1,opt}, r_{2,opt}, \dots, r_{K,opt}\}$, which is obtained by selecting $\mathbf{r}_{i,opt}$ as

$$r_{i,opt} = \left\{ r: \begin{array}{l} \arg\min_{r} \left(\left\| \left[\hat{\mathcal{F}}^{\ddagger} \mathbf{T} \right]_{\{(rK+i)}\text{-th column} \right\} \right\|_{F} \right), \\ \left[\mathbf{T} \right]_{\{(rK+i)}\text{-th column} \right] \text{ is not all zero valued.} \end{array} \right\},$$
(33) for $i = 1, 2, \dots, K$.

V. FIR ZFE DESIGN

By taking the transpose in the derivation of ZFP, the parameterization of LS optimal solution for ZFE can be obtained. The method for designing ZFE was proposed in [3]. For the sake of easy reference, we quoted the key results in [3] below. Considers

$$\mathcal{G}(z) \stackrel{\triangle}{=} \mathcal{H}(z)\mathbf{G}(z). \tag{34}$$

For any particular set of system delay γ , FIR ZFE $\mathbf{F}_{\gamma}(z)$ exists if

$$\mathbf{F}_{\gamma}(z)\mathcal{G}(z) = \mathbf{D}_{\gamma}(z). \tag{35}$$

Define $\mathbf{F}_{\gamma}(z) \stackrel{\Delta}{=} \sum_{\ell=0}^{L_F} \mathbf{F}_{\ell} z^{-\ell}$ and $\mathcal{G}_{\gamma}(z) \stackrel{\Delta}{=} \sum_{\ell=0}^{L_{\mathcal{G}}} \mathcal{G}_{\ell} z^{-\ell}$. Express eq.(34) using the scalar matrix representation,

$$\tilde{\mathbf{E}}_{\gamma} = \tilde{\mathbf{F}}\tilde{\mathcal{G}},$$
(36)

$$\tilde{\mathbf{E}}_{\gamma} \stackrel{\triangle}{=} \begin{bmatrix} \mathbf{D}_{L_D} & \mathbf{D}_{L_D-1} & \cdots & \mathbf{D}_0 \end{bmatrix}, \quad (37)$$

$$\tilde{\mathbf{F}} \stackrel{\triangle}{=} \begin{bmatrix} \mathbf{F}_{L_F} & \mathbf{F}_{L_F-1} & \cdots & \mathbf{F}_0 \end{bmatrix}, \quad (38)$$

$$\tilde{\mathcal{G}} \stackrel{\triangle}{=} \begin{bmatrix} \mathcal{G}_{L_{\mathcal{G}}} & \cdots & \mathcal{G}_{0} & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & \mathcal{G}_{L_{\mathcal{G}}} & \cdots & \mathcal{G}_{0} \end{bmatrix}.$$
(39)

Let $\rho_1, \rho_2, \ldots, \rho_{\zeta}$ be the index of the non-zero column of $\tilde{\mathcal{G}}$. Define,

$$\hat{\mathcal{G}} \stackrel{\Delta}{=} \tilde{\mathcal{G}} \mathbf{T}, \text{ with } [\mathbf{T}]_{\{i,j\}} = \begin{cases} 1 & , \text{ for } i = \rho_j \text{ and } j = 1, 2, \dots, \zeta \\ 0 & , \text{ otherwise.} \end{cases}$$
(40)

The LS optimal solution of FIR ZFE can be obtained by

$$\tilde{\mathbf{F}}_{r,opt} = \tilde{\mathbf{E}}_{\gamma} \mathbf{T} \hat{\mathcal{G}}^{\ddagger}.$$
(41)

Let the required minimal order of $\mathbf{F}(z)$ for zero-forcing be $L_{F_{min}}.$ Then,

$$L_{F_{min}} = \lceil \zeta/(MP) \rceil - 1. \tag{42}$$

If $\tilde{\mathcal{G}}$ does not has all zeros column, such that $\zeta = K(L_{\mathcal{G}} + L_F + 1)$, then eq.(42) can be written as,

$$L_{F_{min}} = \left\lceil \frac{KL\mathcal{G}}{MP - K} \right\rceil - 1. \tag{43}$$

VI. ITERATIVE JOINT OPTIMIZATION ALGORITHM

A. Algorithm

Section IV and V have shown the methods for designing the LS optimal FIR ZFP and ZFE with a given precoder or the equalizer respectively. By combining these two algorithms, we can jointly optimize the precoder and equalizer of a zero-forcing system iteratively as shown in Fig.1. Once an initial precoder or equalizer is given, the iterative algorithm will alternatively design the FIR LS optimal ZFP and ZFE under the filter order constraints of precoder and equalizer. Since the ZFP and ZFE design algorithm in the iteration can provide the LS optimal result, the system performance is guaranteed to be improving during the iteration process until the optimal system

 TABLE I

 Iterative joint optimization sufficient conditions.

K	$L_{\mathcal{H}}$	Required L_F and L_G
$< \frac{NMP}{M+N}$	Any	$L_G \geq \frac{K(L_{\mathcal{H}}+L_F)}{NP-K} - 1 \geq \frac{(L_{\mathcal{H}}-1)KM}{NMP-K(N+M)} - 1,$ $L_F \geq \frac{K(L_{\mathcal{H}}+L_G)}{MP-K} - 1 \geq \frac{(L_{\mathcal{H}}-1)KN}{NMP-K(N+M)} - 1.$
$= \frac{NMP}{M+N}$	= 0	$M \le ML_F - NL_G \le N$
	= 1	$ML_F - NL_G = N - M$
$> \frac{NMP}{M+N}$	= 0	$\frac{KL_F}{NP-K} - 1 \le L_G \le \frac{KM}{K(N+M)-NMP} - 1,$ $\frac{KL_G}{MP-K} - 1 \le L_F \le \frac{KN}{K(N+M)-NMP} - 1.$

is obtained. Noted that, the iterative algorithm will compare the percentage improvement of the gain of $\mathbf{G}(z)$ designed in each loop. If the percentage improvement is less than a given threshold value, α , the iteration will stop.

B. Sufficient Conditions

The proposed iterative design must satisfy the order requirements of the ZFP and ZFE designs. As discussed, under random channel transfer function environment, $\tilde{\mathcal{F}}$ and $\tilde{\mathcal{G}}$ can be assumed not to contain all zero rows and all zeros columns respectively. Therefore, the order of the ZFP and ZFE should be larger then or equal to the minimum order requirement in eq.(24) and (43). Let $L_{\mathcal{H}}$ be the order of $\mathcal{H}(z)$, and $L_{\mathcal{G}} = L_{\mathcal{H}} + L_{G}$ and $L_{\mathcal{F}} = L_{\mathcal{H}} + L_{F}$. The sufficient conditions for the iterative algorithm to has a solution are

$$L_G \geq \frac{K(L_{\mathcal{H}} + L_F)}{NP - K} - 1, \qquad (44)$$

$$L_F \geq \frac{K(L_{\mathcal{H}} + L_G)}{MP - K} - 1. \tag{45}$$

As a consequence, the sufficient conditions in terms of N, K and $L_{\mathcal{H}}$ are listed in table I. The detail derivation can be found in [16].

VII. SIMULATION RESULTS

Simulation results of a MIMO system with 3 transmit and 3 receive antennas are presented. The orders of channel $\mathbf{H}(z)$, precoder $\mathbf{G}(z)$ and equalizer $\mathbf{F}(z)$ are set to be 5 (i.e. $L_H = L_G = L_F = 5$). The coefficients of the channel transfer function $\mathbf{H}(z)$ are Rayleigh distributed with uniform delay profile [4]. Moreover, both the source vector size and the oversampling rate of the system are set to be 3 (i.e. K = P = 3). The source data are modulated using QPSK. Fig.2 showed the BER performance of the system with and without using the iterative joint optimization algorithm. For the case without using the joint optimization algorithm, a trailing zero precoder (i.e. $\mathbf{G}(z) = \mathbf{I}_3$) and a corresponding FIR LS optimal equalizer were used. In the case of joint optimization algorithm, the performance of the system using different values of stopping threshold, α , are considered.

Compare with the case without using the joint optimization algorithm, the simulation results showed that the system using joint optimization algorithm can obtained about 3dB gain. This performance gain increases with the increase of channel SNR. In the cases of joint optimization algorithm, the average number of iteration required for α equal 1%, 10% and 20% are 4.933, 2.136 and 2 respectively. The simulation results in Fig.2 showed that the system BER performance is improved with the reduction in the value of α .

VIII. CONCLUSION

An iterative joint FIR precoder-equalizer design algorithm for STMC based MIMO system is proposed. This is the first algorithm available in literature that jointly designs both the FIR ZFE and ZFP without the requirement of having a guard period larger than or equal to the channel order. Although the proposed algorithm can only guarantee suboptimal precoder-equalizer design, simulation results showed that a substantial improvement in the error performance can be obtained using the proposed algorithm without decreasing the spectral efficiency of the system. In addition, the LS FIR ZFP design is parameterized which found applications in communication systems with fixed equalizer structure.

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Fig. 2. BER of system with and without iterative joint optimization algorithm. $(K = N = M = P = 3, L_H = L_G = L_F = 5).$