# OPTIMAL SWITCHING THRESHOLDS FOR SPACE–TIME BLOCK CODED RATE–ADAPTIVE M–QAM

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## ABSTRACT

Channel state information (CSI) at the transmitter can be used to adapt transmission rate or antenna gains to time–varying fading channels in multi–antenna systems. However, the CSI that is fed back will get outdated if the channel is changing rapidly. We have developed a rate–adaptive M–QAM technique equipped with space–time block coding for multi–input multi–output (MIMO) systems and imperfect signal to noise ratio (SNR) feedback as opposed to the more complex full channel feedback in [6]. Based on the closed–form expressions for the average BER and average throughput derived in [6], we derive herein an optimal switching thresholds which maximize the average throughput under average BER constraint for MIMO fading channels. Our numerical results illustrate the immunity of our optimal thresholds to error due to time–delayed feedback.

## 1. INTRODUCTION

Future wireless data transmission systems will require novel spectrally efficient techniques as the demand for services claiming the limited radio spectrum grows. Multiantenna arrays are well known to offer improvements in spectral efficiency along with diversity and coding benefits over fading channels. When the channel state information (CSI) can be estimated at the receiver and sent back to the transmitter, adaptive transmission techniques compensate for the time–varying fading channels by taking advantage of favorable channel conditions.

Optimal and practical adaptive modulation techniques are well established for single input single output (SISO) systems [1]–[2]. In [1], several adaptation policies are introduced, and the spectral efficiency for each policy is compared with the capacity of fading channels with transmitter knowledge of CSI. Reference [2] investigates the rate–adaptive multilevel M–QAM scheme over Nakagami fading channels, and describes the effect of time–delay on the average BER for a SISO system.

Adaptive modulation techniques for multiple antenna systems have also been pursued in [3]–[6]. Power control and beamforming for multiple transmit and receive antennas are provided in [3]. Adapting optimal transmitter eigen–beamforming equipped with space–time block coding (STBC) is introduced over channels where the transmitter has imperfect CSI [4]. In [5], transmit antenna selection to maximize the resultant capacity for a low–rank channel is considered. Reference [6] proposed space–time block coded rate–adaptive M–QAM based on uncertain SNR feedback. The closed–form expressions for average BER and average throughput are derived, and the impact of delayed CSI on the achievable performance gains are investigated in [6].

In this paper, our emphasis is on enhancing the average throughput by adapting rate while reference [4] minimizes a bound on the symbol error rate (SER) for a fixed rate. Adaptive modulation techniques for MIMO systems can feedback either SNR or a matrix of channel coefficients. We focus on SNR feedback coupled with orthogonal STBC, which is less complex than full channel matrix feedback used in [4]. We propose to feed back to the transmitter the appropriate constellation size based on the SNR at the receiver for a system with rate–adaptive modulation coupled with STBC and linear ML decoding. In contrast with our work in [6], we derive the *optimal* thresholds that maximizes the average data rate subject to the average BER constraint by using Lagrange multipliers. Using the optimal thresholds, we then analyze the impact of delayed CSI on the average BER and throughput.

## 2. CHANNEL AND SYSTEM MODELS

We now outline channel and system models for our novel MIMO adaptive modulation scheme. Suppose that the $N_r \times N_t$ wireless channel matrix $\mathbf{H}$ has complex Gaussian random entries for a system with $N_t$ transmit and $N_r$ receive antenna elements. The $(i,j)$th entry $[\mathbf{H}]_{i,j} = h_{i,j}$ has zero mean with variance 1/2 per dimension. Suppose that $h_{i,j}$ is correlated with $\hat{h}_{i,j}$ by the correlation factor $\rho := J_o(2\pi f_d \tau)$, where $\hat{h}_{i,j}$ is the $\tau$ time–delayed version of $h_{i,j}$, $J_o(.)$ is the zero–order Bessel function of the first kind, and $f_d$ is the maximum Doppler spread. The matrix input/output relationship is given by

$$\mathbf{X} = \sqrt{\bar{\gamma}}\mathbf{HC} + \mathbf{V}, \tag{1}$$

where $\mathbf{X}$ is the $N_r \times P$ received matrix, $\mathbf{C}$ is the $N_t \times P$ transmitted STBC codeword matrix, $\bar{\gamma}$ is the expected SNR per channel and $\mathbf{V}$ is the $N_r \times P$ additive white Gaussian noise (AWGN) matrix with the $(i,j)$th entry $[\mathbf{V}]_{i,j} = v_{i,j} \sim \mathrm{CN}(0,1)$. We have dropped the time index for convenience.

A block diagram of our system is given in Fig. 1. We assume perfect channel knowledge available at the receiver, which is where the SNR is computed. Based on this partial CSI, a constellation size selector at the receiver decides the constellation to be transmitted, reconfigures the demodulator accordingly, and ac-

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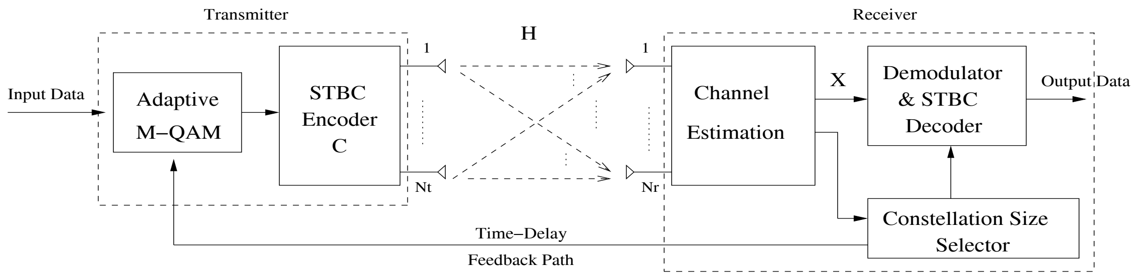



**Fig. 1**. Block Diagram of Space–Time Block Coded Adaptive M–QAM scheme

quaints the transmitter with the information regarding the next constellation size via the feedback path. The transmitter adapts the modulator and transmits space–time block coded symbols over flat MIMO fading channels.

## 3. ADAPTIVE M–QAM COUPLED WITH STBC

Assume that a finite set of candidate constellation sizes $\mathcal{M} := \{M_0, \cdots, M_{J-1}\}$ is used for this adaptive M–QAM with Gray bit mapping, where $M_0$ means no–transmission and $M_j > M_{j-1}, \forall j$. Given a vector of thresholds $\mathbf{t} := [t_0, \cdots, t_J]$, the constellation size $M_j$ is selected and $R_j := \log_2 M_j$[bits/s] for $j \geq 1$ are transmitted when $t_j \leq \gamma_t < t_{j+1}$, where $\gamma_t = \bar{\gamma}||\mathbf{H}||^2$ is the total received SNR.

In [6], the closed–form expressions for the average throughput and average BER are derived with a time–delay $\tau$:

$$\bar{R}(\mathbf{t}) = \sum_{j=0}^{J-1} \frac{R_j}{\Gamma(N_t N_r)} \times \left[\Gamma\left(N_t N_r, \frac{t_j}{\bar{\gamma}}\right) - \Gamma\left(N_t N_r, \frac{t_{j+1}}{\bar{\gamma}}\right)\right], \tag{2}$$

$$\overline{\mathrm{BER}}^{\tau}(\mathbf{t}) \leq \bar{R}(\mathbf{t})^{-1} \sum_{j=0}^{J-1} R_j C_j \left(\frac{1}{\bar{\gamma}}\right)^{N_t N_r} \frac{1}{\Gamma(N_t N_r)} \times \left[\Gamma\left(N_t N_r, \frac{t_j}{A_j^{\tau}}\right) - \Gamma\left(N_t N_r, \frac{t_{j+1}}{A_j^{\tau}}\right)\right], \tag{3}$$

where $\Gamma(\cdot)$ is the complete Gamma function, $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function,

$$A_j^{\tau} := \frac{[3\bar{\gamma}(1-\rho^2) + 2(M_j - 1)]\bar{\gamma}}{3\bar{\gamma} + 2(M_j - 1)}, \quad j = 0, \cdots, J-1. \tag{4}$$

and $C_j := 0.2(A_j^{\tau})^{N_t N_r}|_{\rho=1}$. Clearly, (2) and (3) depend on the switching thresholds $\mathbf{t}$.

## 4. OPTIMAL VECTOR OF THRESHOLDS

The thresholds can be chosen so that for any realization of $\mathbf{H}$ the instantaneous BER $\leq$ BER$_o$. We will refer to this choice as "generic thresholds". Note that this condition on the instantaneous BER is a more strict condition than requiring the *average* $\overline{\mathrm{BER}} \leq$ BER$_o$. Assuming generic thresholds, our novel adaptive M–QAM scheme coupled with STBC obtains an average BER much less than BER$_o$ at high SNR ranges in [6]. Intuitively, since there is a trade–off between BER and the rate for adaptive modulation systems, there is potential for improving the average throughput by adjusting the thresholds, as long as $\overline{\mathrm{BER}} \leq \mathrm{BER}_o$. Both $\bar{R}(\mathbf{t})$ and $\overline{\mathrm{BER}}^{\tau}(\mathbf{t})$ depend on a given set of thresholds as shown in (2) and (3). We will search the optimal set of thresholds $\mathbf{t}^{opt}$ maximizing the average data rate $\bar{R}(\mathbf{t})$ subject to an average BER constraint. Hence, our main goal is to optimize the set of thresholds $\mathbf{t}$ that maximizes the average throughput $\bar{R}(\mathbf{t})$ with the constraint $\overline{\mathrm{BER}}^{\tau}(\mathbf{t}) \leq \mathrm{BER}_o$ by using Lagrage multipliers. Assuming $J$ constellation sizes from the set $\mathcal{M}$ for an adaptive M–QAM scheme and a time–delay $\tau$ from the receiver to the transmitter, we can express the average BER constraint by defining $\overline{\mathrm{BER}}^{\tau}(\mathbf{t}) := \bar{R}(\mathbf{t})^{-1} \ \overline{\mathrm{BER}}'(\mathbf{t})$:

$$\overline{\mathrm{BER}}^{\tau}(\mathbf{t}) \leq \mathrm{BER}_o \Rightarrow \overline{\mathrm{BER}}'(\mathbf{t}) \leq \bar{R}(\mathbf{t})\mathrm{BER}_o. \tag{5}$$

Since our goal is to maximize $\bar{R}(\mathbf{t})$ with respect to $\mathbf{t}$ under the constraint (5), we have a $J-1$ dimensional optimisation problem because we assume $t_0 = 0$ and $t_J = \infty$ as in the adaptive modulation literature. We will convert the $J-1$ dimensional optimisation into a set of one dimensional optimisation problems by using Lagrange multipliers. The modified objective function $\mathcal{F}(\mathbf{t})$ is given in terms of a Lagrange multiplier $\lambda$ as

$$\mathcal{F}(\mathbf{t}) = \bar{R}(\mathbf{t}) + \lambda(\overline{\mathrm{BER}}'(\mathbf{t}) - \bar{R}(\mathbf{t})\mathrm{BER}_o). \tag{6}$$

The optimal set of thresholds $\mathbf{t}^{opt}$ should satisfy the following two conditions:

$$\frac{\partial \mathcal{F}(\mathbf{t})}{\partial \mathbf{t}} = 0, \tag{7}$$

$$\overline{\mathrm{BER}}'(\mathbf{t}) - \bar{R}(\mathbf{t})\mathrm{BER}_o = 0. \tag{8}$$

This modified objective function is to be made stationary with respect to all variations so that

$$\frac{\partial \mathcal{F}(\mathbf{t})}{\partial t_j} = \frac{\partial \bar{R}(\mathbf{t})}{\partial t_j}(1 - \lambda \mathrm{BER}_o) + \lambda \frac{\partial \overline{\mathrm{BER}}'(\mathbf{t})}{\partial t_j} = 0, \tag{9}$$

and the $\lambda$ is to be selected so as to satisfy the constraint (8).

The derivatives in (9) can be expressed using (2) and (3) as

$$\frac{\partial \bar{R}(\mathbf{t})}{\partial t_j} = \frac{t_j^{N_t N_r - 1}}{\Gamma(N_t N_r)} \left(\frac{1}{\bar{\gamma}}\right)^{N_t N_r} \exp\left(-\frac{t_j}{\bar{\gamma}}\right) \times (R_{j-1} - R_j), \quad j = 1, \cdots, J-1. \tag{10}$$

$$\frac{\partial \overline{\mathrm{BER}}'(\mathbf{t})}{\partial t_j} = \frac{1}{\Gamma(N_t N_r)} \left(\frac{1}{\bar{\gamma}}\right)^{N_t N_r} t_j^{N_t N_r - 1} \times B_j^{\tau}, \quad j = 1, \cdots, J-1 \tag{11}$$

where $B_j^\tau$ is defined as

$$B_j^\tau := \left(\frac{1}{A_{j-1}^\tau}\right)^{N_t N_r} R_{j-1} C_{j-1} \exp\left(-\frac{t_j}{A_{j-1}^\tau}\right) - \left(\frac{1}{A_j^\tau}\right)^{N_t N_r} R_j C_j \exp\left(-\frac{t_j}{A_j^\tau}\right). \tag{12}$$

Thus, substituting (10) and (11) into (9), the following relationship must be satisfied for all $t_j$, $j = 1, \cdots, J-1$:

$$(1 - \lambda \mathrm{BER}_o) = \frac{\lambda}{R_{j-1} - R_j} \exp\left(\frac{t_j}{\bar{\gamma}}\right) \times \left[D_j^\tau \exp\left(-\frac{t_j}{A_j^\tau}\right) - D_{j-1}^\tau \exp\left(-\frac{t_j}{A_{j-1}^\tau}\right)\right], \tag{13}$$

where $D_j^\tau := \left(\frac{1}{A_j^\tau}\right)^{N_t N_r} R_j C_j$. As the impact of delayed feedback becomes negligible ($\rho \to 1$ or $\tau \to 0$), we can see that the relationship between thresholds becomes independent of $\bar{\gamma}$ from (13). Since it is not easy to express $t_j$ with respect to $\lambda$ as shown in (13), we will find the optimal vector of thresholds $\mathbf{t}^{opt}$ numerically. The left side of (13) is identical for all $t_j$. Thus, it is possible to relate $t_1$ to all other $t_j$, $j = 2, \cdots, J-1$, as

$$\left(\frac{1}{A_1^\tau}\right)^{N_t N_r} C_1 \exp\left[-t_1\left(\frac{1}{A_1^\tau} - \frac{1}{\bar{\gamma}}\right)\right] = -\frac{\exp(t_j/\bar{\gamma})}{R_{j-1} - R_j} \times \left[D_j^\tau \exp\left(-\frac{t_j}{A_j^\tau}\right) - D_{j-1}^\tau \exp\left(-\frac{t_j}{A_{j-1}^\tau}\right)\right]. \tag{14}$$

According to (14), $t_1$ determines the other variables $t_j$ for $j = 2, \cdots, J-1$, which provides a vector of thresholds $\mathbf{t}(t_1)$. Thus, the optimal vector of thresholds $\mathbf{t}^{opt}$ is found numerically as the vector $\mathbf{t}(t_1^{opt})$ having the maximum data rate among vectors of thresholds, $\mathbf{t}(t_1)$, satisfying the constraint in (5). As an example, Fig. 2 illustrates the average BER constraint and throughput corresponding to the vector of thresholds as a function of $t_1$ when $\bar{\gamma} = 20$dB, $\mathrm{BER}_o = 10^{-4}$, $f_d \times \tau = 0.05$, $N_t = 2$, and $N_r = 2$. In Fig. 2 (a), $t_1 \in [8.4, 27\text{dB}]$ satisfies the average BER constraint in (5). Therefore, $t_1^{opt} = 8.4$dB, because it achieves the maximum throughput (4.45 $\mathrm{bits/s/Hz}$) in Fig. 2 (b), and $\mathbf{t}^{opt}$ can be found by using (14).

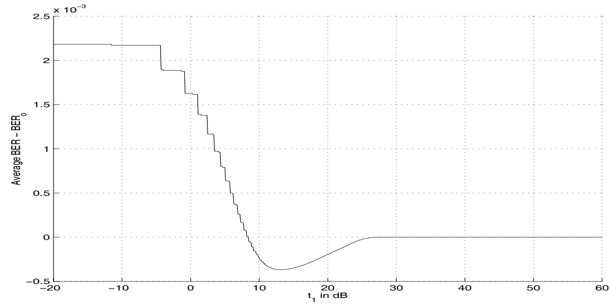
Combining the optimal vector of thresholds $\mathbf{t}^{opt}$ with our proposed adaptive scheme, the numerical performance for the average BER and throughput and its sensitivity to delayed CSI feedback are also discussed in Section 5.

## 5. NUMERICAL RESULTS

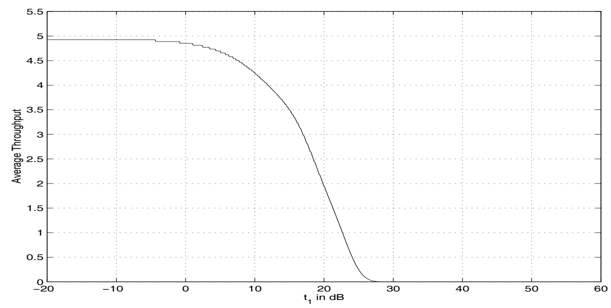
We assume a land mobile communication system using the constellation sizes $\mathcal{M} = \{0, 4, 16, 64, 256\}$, a carrier frequency $f_c = 900$MHz, and a mobile moving at a speed of $90\mathrm{km/hr}$ over a scattering environment, which produces a Doppler spread of $f_d = 75$Hz.

### 5.1. Optimal Thresholds with Average BER Constraint

Assuming a normalized time–delay, $f_d \times \tau = 0.01$, Fig. 3 illustrates the average BER and throughput with the optimal thresholds subject to $\overline{\mathrm{BER}}^\tau(\mathbf{t}) \leq \mathrm{BER}_o = 10^{-4}$. The optimal thresholds




(a)




(b)

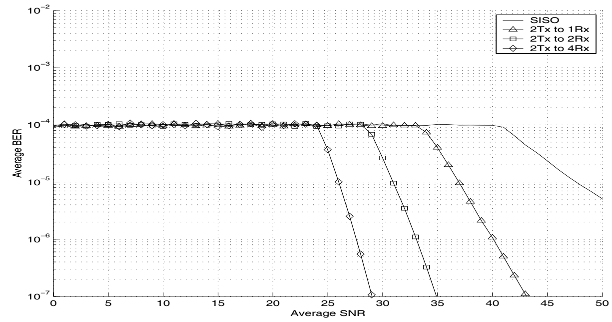
**Fig. 2**. (a)Average BER constraint, (b) Average throughput with respect to $t_1$ in dB ($\mathrm{BER}_o = 10^{-4}$, $\bar{\gamma} = 20$dB, $f_d \times \tau = 0.05$, $\mathcal{M} = \{0, 4, 16, 64, 256\}$, $N_t = 2$ and $N_r = 2$).

under the average BER constraint, the adaptive M–QAM technique in a SISO system achieves $\overline{\mathrm{BER}}^\tau(\mathbf{t}) = \mathrm{BER}_o = 10^{-4}$ and $\bar{R}(\mathbf{t}) = 5.6$ bits/s/Hz at $\bar{\gamma} = 28$dB, and the average BER and throughput in a $2 \times 2$ system obtains $\bar{R}(\mathbf{t}) = 7.9$ bits/s/Hz at $\bar{\gamma} = 28$dB for the same target BER. Moreover, our technique with the optimal thresholds under the average BER constraint achieves an additional 3.5 bits/s/Hz over a space–time block coded *non–adaptive* 4–QAM at $\bar{\gamma} = 22$ dB in a $2 \times 1$ system for $\overline{\mathrm{BER}} = \mathrm{BER}_o$. Due to the discrete set of constellations, increasing the number of transmit/receive antenna elements results in the ripples on the average BER and throughput in Fig. 3 (b).

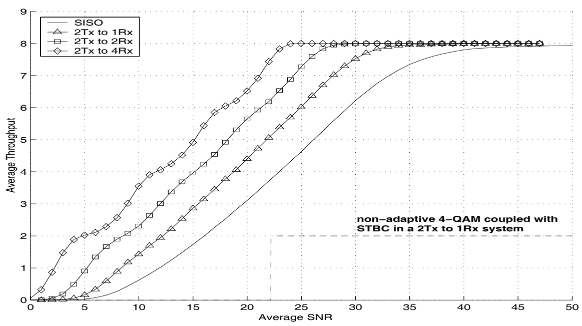
### 5.2. Impact of Delayed Feedback

The effect of time–delay on the average BER and throughput at $\bar{\gamma} = 20$dB is shown in Fig. 4 when the optimal vector of switching thresholds is used. Fig. 4 (a) shows that our scheme using the optimal vector of thresholds with the average BER constraint maintains $\overline{\mathrm{BER}}^\tau(\mathbf{t}) \leq \mathrm{BER}_o = 10^{-4}$ as the delay increases while with generic thresholds the average BER becomes worse than the desired BER as shown in [6]. In Fig. 4 (b), as the delay $\tau$ gets greater, the proposed adaptive M–QAM scheme using the optimal thresholds converges to a non–adaptive modulation scheme. This is intuitive because with increasing $\tau$, the feedback becomes less reliable yielding a non–adaptive system that disregards the feedback.

We note that the optimal vector of switching thresholds are less sensitive to delayed feedback compared with generic thresholds given in [6].

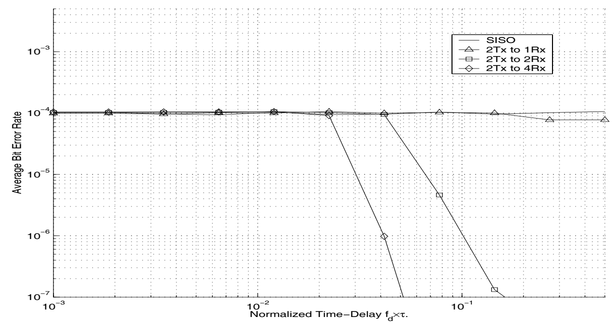



(a)

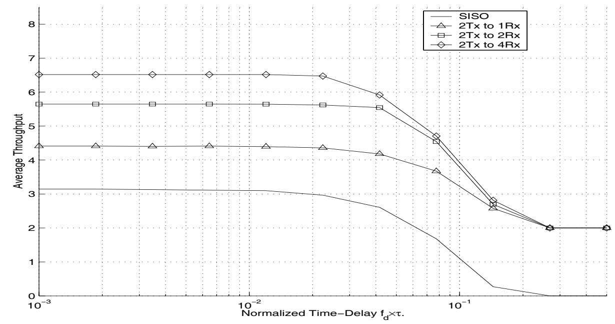



(b)

**Fig. 3**. Space–time block coded rate–adaptive M–QAM coupled with $\mathbf{t}^{opt}$ subject to the average BER constraint ($\mathrm{BER}_o = 10^{-4}$ and $\mathcal{M} = \{0, 4, 16, 64, 256\}$).




(a)




(b)

**Fig. 4**. Space–time block coded rate–adaptive M–QAM scheme coupled with $\mathbf{t}^{opt}$ subject to the average BER constraint at $\bar{\gamma}_c =$ 20dB ($\mathrm{BER}_o = 10^{-4}$, $\mathcal{M} = \{0, 4, 16, 64, 256\}$).

## 6. CONCLUSION AND FUTURE WORK

We have developed the *optimal* vector of thresholds for a robust space–time block coded rate–adaptive M–QAM technique with delayed SNR feedback in order to maximize the average data rate $\bar{R}(\mathbf{t})$ subject to the average BER constraint $\overline{\mathrm{BER}}^{\tau}(\mathbf{t}) \leq \mathrm{BER}_o$, which reduces the impact of delayed CSI on the achievable performance gains. We observed that our optimal thresholds gives rise to a system that is less sensitive to time delay.

As our future work, we will perform the numerical results for the optimal thresholds subject to the outage probability, which is expected to be related with the case under the average BER constraint. Systems adapting both the power and rate, using the SNR information for MIMO systems will also be considered.

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