ORTHOGONAL SPACE-TIME BLOCK CODE FROM AMICABLE COMPLEX ORTHOGONAL DESIGN

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Abstract—**Amicable Complex Orthogonal Design (ACOD), which is the complex version of Amicable Orthogonal Design first reported in [1], is proposed and its existence is proven. Using ACOD, new orthogonal space-time block codes (O-STBC) for four and eight transmit antennas are constructed. Their maximum achievable code rates are proven to be as high as the existing O-STBC designs. In addition, the new O-STBCs are shown to have better practical implementation features than the existing O-STBCs, as they do not require any transmit antenna to be turned off intermittently (which is beneficial for the power amplifier design), and they do not have irrational-number coefficients inside the codeword (which simplifies the hardware implementation).**

I. INTRODUCTION

Orthogonal Space-Time Block Code (O-STBC) has been first proposed in [2] and generalized in [3]. It can provide full transmit diversity with simply linear decoding complexity. Due to these advantages, O-STBC has drawn a lot of research attraction, for example in [4], [5], [6] and etc. However, some of these O-STBCs require some transmit antennas to be turned off at regular intervals [4][5], while others have irrational-numbers in the code coefficients [3]. Regular switching-off of transmit antennas may lead to undesirable low-frequency interference, while irrational-number coefficient may require floating-point multiplication which lead to higher hardware cost. In addition, the authors of [4] also highlighted an open issue in [7] regarding the design of O-STBC based on Amicable Orthogonal Design (AOD), i.e. whether the complex version of AOD exists and what is the maximum O-STBC code rate achievable from this design. In this paper, we address this unsolved issue by defining a new orthogonal design, called Amicable Complex Orthogonal Design (ACOD). The maximum code rate of O-STBC achievable by this design is derived. New O-STBCs for four and eight transmit antennas are constructed. Their advantages over existing O-STBC designs are discussed.

Section II of this paper will give an overview on O-STBC as well as the STBC signal model. ACOD will be defined and solved in Section III, followed by new designs of O-STBC for four and eight transmit antennas in Section IV. The power distribution properties of the proposed and existing codes are discussed in detail in Section V. Finally, Section VI will end this paper with conclusion.

II. ORTHOGONAL STBC

It has been shown in [8] that a linear STBC, $\mathbf{G}$, can be represented as:

$$\mathbf{G} = \sum_{i=1}^{k} (x_i^{\mathrm{R}} \mathbf{A}_i + j x_i^{\mathrm{I}} \mathbf{B}_i) \tag{1}$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ are called the dispersion matrices (both of dimension $p\times n$), x^{R} and x^{I} represent the real and imaginary part of a transmitted symbol, p represents the code length, n represents the number of transmit antennas, and k represents the number of complex symbols being transmitted across p period of time (hence the code rate of a STBC is k/p).

An O-STBC is one that can achieve full transmit diversity with linear decoding complexity, maximum SNR, and minimum Union PEP. To design such a code, it has been proven in [3], [4], and [9] that its dispersion matrices $\mathbf{A}$ and $\mathbf{B}$ must satisfy the following constraints:

$$\begin{array}{llll} \text{(i)} & \mathbf{A}_i^{\mathrm{H}}\mathbf{A}_i = \mathbf{I}_n \quad \& \quad \mathbf{B}_i^{\mathrm{H}}\mathbf{B}_i = \mathbf{I}_n & \forall i & \\ \text{(ii)} & \mathbf{A}_i^{\mathrm{H}}\mathbf{A}_q = -\mathbf{A}_q^{\mathrm{H}}\mathbf{A}_i \quad \& \quad \mathbf{B}_i^{\mathrm{H}}\mathbf{B}_q = -\mathbf{B}_q^{\mathrm{H}}\mathbf{B}_i & i \neq q & (2) \\ \text{(iii)} & \mathbf{A}_i^{\mathrm{H}}\mathbf{B}_q = \mathbf{B}_q^{\mathrm{H}}\mathbf{A}_i & \forall i,q & \end{array}$$

In addition, it is also desirable that the O-STBC has high code rate and minimum code length, i.e. k as large as possible and p as small as possible respectively. For full diversity, it is also required that $p \geq n$. Hence a square design, i.e. $p=n$, is the minimum possible code length.

In the next section, we propose a new orthogonal design, called the Amicable Complex Orthogonal Design (ACOD), to construct O-STBCs that satisfy the above-stated design objectives.

III. AMICABLE COMPLEX ORTHOGONAL DESIGN

Before discussing ACOD, a related orthogonal design called Complex Orthogonal Design (COD) is first reviewed.

Definition 1 [10]: A COD of order n and of type $(h_1,\ldots h_r)$ (h_i positive integers) on the real commuting variables $z_1, \ldots, z_r$ is an $n\times n$ matrix $\mathbf{Z}$, with entries from $\{0, \varepsilon_1 z_1,\ldots, \varepsilon_r z_r$ where ε_i is a fourth root of 1$\}$ satisfying

$$\mathbf{Z}^{\mathrm{H}}\mathbf{Z} = \left(\sum_{i=1}^{r} h_i z_i^2\right)\mathbf{I}_n \tag{3}$$

$\mathbf{Z}$ can be expressed as

$$\mathbf{Z} = \mathbf{D}_1 z_1 + \cdots + \mathbf{D}_r z_r \tag{4}$$

where the $\mathbf{D}_i$ are matrices of size $n\times n$ with elements $\{0,\pm1, \pm j\}$, satisfying:

$$\begin{aligned} &\text{(i) } \mathbf{D}_i^{\mathrm{H}}\mathbf{D}_i = h_i\mathbf{I}_n && 1\le i\le r \\ &\text{(ii) } \mathbf{D}_i^{\mathrm{H}}\mathbf{D}_q + \mathbf{D}_q^{\mathrm{H}}\mathbf{D}_i = 0 && 1\le i\ne q\le r \end{aligned} \tag{5}$$

Theorem 1 [10, Theorem 4]: Let $\tau(n)$ denote the maximum number of variables in a COD of order n (i.e. $\tau(n)=\max(r)$), it has been shown that $\tau(n) \le H(n)$, where $H(n)=2a+2$ if $n=2^a b$, b odd.

Next, we proceed to define ACOD by following the approach adopted in [1] to define Amicable Orthogonal Design from Orthogonal Design.

Definition 2: Let the matrices $\mathbf{X}=\mathbf{A}_1x_1+\ldots+\mathbf{A}_s x_s$ and $\mathbf{Y}=\mathbf{B}_1y_1+\ldots+\mathbf{B}_t y_t$ be COD of the same order n where $\mathbf{X}$ is of type $(f_1,\ldots f_s)$ on the variables $\{x_1,\ldots,x_s\}$ and $\mathbf{Y}$ is of type $(g_1,\ldots,g_t)$ on the variables $\{y_1,\ldots,y_t\}$. $\mathbf{X}$ and $\mathbf{Y}$ are said to be ACOD if

$$\mathbf{X}^{\mathrm{H}}\mathbf{Y} = \mathbf{Y}^{\mathrm{H}}\mathbf{X} \tag{6}$$

A necessary and sufficient condition for ACOD, as defined in *Definition 2*, to exist is that there exist a family of matrices $\{\mathbf{A}_1,\ldots,\mathbf{A}_s; \mathbf{B}_1,\ldots,\mathbf{B}_t\}$ satisfying:

$$\begin{aligned} &\text{(i) } \mathbf{A}_i^{\mathrm{H}}\mathbf{A}_i = f_i\mathbf{I}_n && 1\le i\le s \\ &\quad\ \mathbf{B}_q^{\mathrm{H}}\mathbf{B}_q = g_q\mathbf{I}_n && 1\le q\le t \\ &\text{(ii) } \mathbf{A}_i^{\mathrm{H}}\mathbf{A}_l + \mathbf{A}_l^{\mathrm{H}}\mathbf{A}_i = 0 && 1\le i\ne l\le s \\ &\quad\ \mathbf{B}_q^{\mathrm{H}}\mathbf{B}_m + \mathbf{B}_m^{\mathrm{H}}\mathbf{B}_q = 0 && 1\le q\ne m\le t \\ &\text{(iii) } \mathbf{A}_i^{\mathrm{H}}\mathbf{B}_q = \mathbf{B}_q^{\mathrm{H}}\mathbf{A}_i && 1\le i\le s, 1\le q\le t \end{aligned} \tag{7}$$

where $\mathbf{A}_i$ and $\mathbf{B}_q$ are all $\{0, \pm1, \pm j\}$ matrices of order n.

By comparing (7) and (2), it can be observed that the $\mathbf{A}_i$ and $\mathbf{B}_q$ matrices defined above in (7) can be used as the dispersion matrices of an O-STBC since they satisfy the design constraints in (2). On the other hand, the total number of variables $(s+t)/2$ of the ACOD represents the number of complex symbols k carried by the O-STBC. Next, the existence of ACOD and the upper bound on the maximum code rate k/p of O-STBC derived from ACOD are established.

Proposition 1: Assume that the CODs $\mathbf{X}=\mathbf{A}_1x_1+\ldots+\mathbf{A}_s x_s$ and $\mathbf{Y}=\mathbf{B}_1y_1+\ldots+\mathbf{B}_t y_t$ as defined in *Definition 2* exist. By letting:

$$\begin{aligned} \mathbf{D}_i &= \mathbf{A}_i && 1\le i\le s \\ \mathbf{D}_{s+q} &= j\mathbf{B}_q && 1\le q\le t \end{aligned} \tag{8}$$

a COD $\mathbf{Z}=\mathbf{D}_1z_1+\ldots+\mathbf{D}_r z_r$ with $r=s+t$ variables of type $(f_1,\ldots f_s, g_1,\ldots,g_t)$ will be formed by $\mathbf{D}_i$ in (8), $1\le i\le s+t$.

Proof of *Proposition 1*: It can be shown that (8) satisfies all the constraints of (5). For example, 5(ii) can be verified as follows, by considering the case $1\le i\le s$ and $1\le q\le t$.

$$\begin{aligned} &\mathbf{D}_i^{\mathrm{H}}\mathbf{D}_{s+q} + \mathbf{D}_{s+q}^{\mathrm{H}}\mathbf{D}_i \qquad 1\le i\le s, 1\le q\le t \\ &= \mathbf{A}_i^{\mathrm{H}}(j\mathbf{B}_q) + (j\mathbf{B}_q)^H\mathbf{A}_i \\ &= j\mathbf{A}_i^{\mathrm{H}}\mathbf{B}_q - j\mathbf{B}_q^{\mathrm{H}}\mathbf{A}_i \\ &= 0 \end{aligned}$$

Proposition 2: The upper bound on the maximum total number of variables of an ACOD, i.e. $s+t$, is bounded by $H(n)$.

Proof of *Proposition 2*: From *Proposition 1*, it is clear that whenever an ACOD design with $s+t$ variables exist, a COD with $r=s+t$ variables will exist. Furthermore it has been stated in *Theorem 1* that the maximum number of variables of a COD is bounded by $H(n)$, i.e. $\max(r) \le H(n)$. Hence the maximum total number of variables of an ACOD is also bounded by $H(n)$, i.e. $\max(s+t) \le H(n)$. As a result *Proposition 2* is proved.

Proposition 3: The maximum total number of variables of an ACOD, i.e. $\max(s+t)$, is equal to the maximum total number of variables of an AOD.

Proof of *Proposition 3*: The upper bound of maximum total number of variables of an AOD has been shown in [11, Section 5.32] to be $2a+2$, if $n=2^a b$, b odd. This is the same value as $H(n)$, which is the upper bound for $s+t$. As a result *Proposition 3* is proved.

With the proof of *Proposition 3*, we have solved the open issue raised in [7], i.e. ACOD exists and it can achieve the same maximum code rate as AOD, which has been shown in [4] to have maximum code rate of ¾ for four transmit antennas and ½ for eight transmit antennas for a square complex O-STBC design.

In the next section, two new O-STBCs are constructed based on our proposed ACOD design, and shown to be more suitable for practical implementation than the existing O-STBCs.

IV. O-STBC FROM ACOD

Two complex O-STBCs for four transmit antennas have been proposed in the literature. In this section, O-STBC proposed in [3] by Tarokh, Jafarkhani and Calderbank is denoted as "**TJC**", while O-STBC proposed in [4] by Ganesan and Stoica is denoted as "**GS**".

$$\mathbf{TJC} = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1 - x_1^* + x_2 - x_2^*}{2} & \frac{-x_2 - x_2^* + x_1 - x_1^*}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{x_2 + x_2^* + x_1 - x_1^*}{2} & -\frac{(x_1 + x_1^* + x_2 - x_2^*)}{2} \end{bmatrix} \quad (9)$$

$$\mathbf{GS} = \begin{bmatrix} x_1 & 0 & x_2 & -x_3 \\ 0 & x_1 & x_3^* & x_2^* \\ -x_2^* & -x_3 & x_1^* & 0 \\ x_3^* & -x_2 & 0 & x_1^* \end{bmatrix} \quad (10)$$

By using complex weighting matrices of order 4 and weight 2 [11], we can construct ACOD matrices that lead to the following new O-STBC designs, denoted as **G4**, where $\mathbf{A}_1$ to $\mathbf{A}_3$, $\mathbf{B}_1$ to $\mathbf{B}_3$ are its dispersion matrices.

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ j & -j & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & j & -j \end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix} -1 & -1 & 0 & 0 \\ j & -j & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & j & -j \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} -1 & 1 & 0 & 0 \\ j & j & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & j & j \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -j & -j & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & j & j \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & j & j \\ -1 & 1 & 0 & 0 \\ -j & -j & 0 & 0 \end{bmatrix} \quad \mathbf{B}_3 = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -j & -j \\ -1 & 1 & 0 & 0 \\ -j & -j & 0 & 0 \end{bmatrix}$$

$$\mathbf{G4} = \begin{bmatrix} x_1^* - x_2 & x_1^* + x_2 & x_3^* & -x_3^* \\ jx_1 + jx_2^* & -jx_1 + jx_2^* & jx_3^* & jx_3^* \\ -x_3 & x_3 & x_1^* - x_2^* & x_1^* + x_2^* \\ -jx_3 & -jx_3 & jx_1 + jx_2 & -jx_1 + jx_2 \end{bmatrix} \quad (11)$$

It can be observed from (9) that the **TJC** code contains the irrational-number $1/\sqrt{2}$ in some of the codeword entries. This makes hardware implementation more difficult, as floating-point multiplication is needed. For the **GS** code in (10), on the other hand, one of the transmit antennas has to be turned off once in every four code symbol durations. In (11), our proposed **G4** code contains no irrational-number within the codeword. Furthermore, by choosing the x_1 and x_2 symbols from different modulation constellation, e.g. x_1 from QPSK and x_2 from rotated-QPSK [12], it can be ensured that no zero will exist in the **G4** codeword. As a result, **G4** does not require any transmit antenna to be turned off. Due to these reasons, our proposed **G4** code is advantageous over the **TJC** and **GS** codes in terms of practical implementation.

Next, we consider the O-STBC designs for 8 transmit antennas. Two ½-rate O-STBCs for eight transmit antennas have been proposed in the literature: one is a non-square design from [3], the other is a square design from [5]. Due to the numerous advantages of square design (e.g. minimum decoding delay, applicability to differential/unitary space-time coding [13]), we only consider the square O-STBC from Tirkkonen and Hottinen [5], herein referred as "**TH**", for comparison with our new code for 8 antennas, which is denoted as **G8**.

$$\mathbf{TH} = \begin{bmatrix} x_1 & x_2 & x_3 & 0 & x_4 & 0 & 0 & 0 \\ -x_2^* & x_1^* & 0 & -x_3 & 0 & -x_4 & 0 & 0 \\ -x_3^* & 0 & x_1^* & x_2 & 0 & 0 & -x_4 & 0 \\ 0 & x_3^* & -x_2^* & x_1 & 0 & 0 & 0 & x_4 \\ -x_4^* & 0 & 0 & 0 & x_1^* & x_2 & x_3 & 0 \\ 0 & x_4^* & 0 & 0 & -x_2^* & x_1 & 0 & -x_3 \\ 0 & 0 & x_4^* & 0 & -x_3^* & 0 & x_1 & x_2 \\ 0 & 0 & 0 & -x_4^* & 0 & x_3^* & -x_2^* & x_1^* \end{bmatrix} \quad (12)$$

$$\mathbf{G8} = \begin{bmatrix} x_1^* & x_1^* & x_2 & -x_2 & x_3 & -x_3 & x_4 & -x_4 \\ jx_1 & -jx_1 & jx_2^* & jx_2^* & jx_3^* & jx_3^* & jx_4^* & jx_4^* \\ -x_2 & x_2 & x_1^* & x_1^* & x_4^* & -x_4^* & -x_3^* & x_3^* \\ -jx_2^* & -jx_2^* & jx_1 & -jx_1 & jx_4 & jx_4 & -jx_3 & -jx_3 \\ -x_3 & x_3 & -x_4^* & x_4^* & x_1^* & x_1^* & x_2^* & -x_2^* \\ -jx_3^* & -jx_3^* & -jx_4 & -jx_4 & jx_1 & -jx_1 & jx_2 & jx_2 \\ -x_4 & x_4 & x_3^* & -x_3^* & -x_2^* & x_2^* & x_1^* & x_1^* \\ -jx_4^* & -jx_4^* & jx_3 & jx_3 & -jx_2 & -jx_2 & jx_1 & -jx_1 \end{bmatrix} \quad (13)$$

By comparing **TH** and **G8**, it can be seen that there is no zero inside the codeword of **G8**. In other words, new design **G8** does not require any transmit antenna to be turned off at any period of time. This is in sharp contrast to the **TH** code, in which four transmit antennas to be turned off at any one time.

In terms of decoding bit error rate performance, it can be showed that the new constructed O-STBCs perform the same as existing O-STBCs. A detailed discussion on the power distribution of O-STBC will next be discussed.

V. POWER UNIFORMIZATION

One of the targets of a good wireless front-end design is to minimize the linearity requirements of its power amplifier. To achieve this, the transmitted signal should have the following power-distribution characteristics [14]:

- low peak-to-average power (Peak/ave)
- low average-to-minimum power (Ave/min)
- low probability P_o that an antenna transmits zero
- the average transmit power should be stationary (power uniformity)

The power distribution characteristics of our new ¾-rate O-STBC for four transmit antennas and ½-rate O-STBC for eight transmit antennas are compared against existing O-STBCs in Table 1 and Table 2 respectively.

Table 1 Power distribution characteristics for four-antennas O-STBC with QPSK modulation

	Peak/ave	Ave/min	P_o
TJC [3]	1.33	1.5[1]	0
GS [4]	1.33	∞	25%
GS with power-uniformized 1 [14]	3	3	0
GS with power-uniformized 2 [14]	2.6	17.5	0
G4[2]	2.28	2.56	0

Table 2 Power distribution characteristics for eight-antennas O-STBC with QOSK modulation

	Peak/ave	Ave/min	P_o
TH [4]	2	∞	50%
G8	1	1	0

As shown in Table 1, in terms of the peak/ave and ave/min characteristics, our newly constructed **G4** code is better than the **GS** code in [14] (which has been optimized for power uniformity). Although the **TJC** code has the best power-distribution characteristics, it contains irrational-number coefficient inside its codeword, as discussed earlier.

Finally, Table 2 shows that our proposed **G8** code has significantly better power-distribution characteristics over the **TH** code.

VI. CONCLUSION

In this paper, Amicable Complex Orthogonal Design (ACOD) is defined and its existence is proven. The maximum code rate of an O-STBC constructed by this new ACOD design is shown to be as high as existing O-STBC.

Using the proposed ACOD design, new O-STBCs for four and eight transmit antennas that support complex constellation have been constructed. In terms of practical implementation, these new codes show certain advantages over existing O-STBCs because the new codes do not require any transmit antenna to be turned off intermittently, they do not require floating-point scaling in the transmitted codeword, and they have better power distribution characteristics that lighten the design requirements of front-end power amplifiers.

Finally, it should be noted that the examples presented in this paper are not the only possible new O-STBCs that can be constructed using our proposed ACOD design. Many more codes with similar properties can be constructed. Being square orthogonal designs, the new codes presented in this paper also enjoy the benefits of having minimum code length, and they can be used as differential/unitary space-time codes.

[1] This value has been corrected from the original value stated in [14]

[2] Assume that x_1 is QPSK, while x_2 is rotated QPSK, which is 45 degree with respect to x_1.

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