LINEAR SPACE-TIME PRECODING FOR RICIAN FADING MISO CHANNELS

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ABSTRACT

We study a space-time precoding technique for MISO wireless systems by employing a linear prefilter at each transmit antenna. The channel is Rician fading, where the mean and variance of the propagation paths are known to the transmitter. This model includes the Rayleigh fading channels as special cases. We use channel capacity as the optimizing criterion for the prefilter design. This criterion provides a unified design of the prefilters for both Rician and Rayleigh fading channels. The optimum prefilters are functions of the channel mean and variance. The solution ranges from beamforming for Rician channels with high K-factor, to unitary diversity for Rayleigh fading cases, where delay diversity is an example. MMSE equalizer is then used to detect the signal at the receiver. Analysis of bounds on error rate performance and numerical simulations for 4QAM input signal show significant diversity gains and array gains. The results also illustrate that having partial channel knowledge at the transmitter can strongly enhance the system performance.

1. INTRODUCTION

Multiple transmit antennas have long been recognized as an effective mean to improve wireless system performance [4, 6]. The use of multiple antennas in wireless communication also increases the channel capacity significantly [2]. Many space-time coding techniques have been proposed, which include coding with memory [4, 6, 5], and block coding [9, 10]. These works however, apply specifically to Rayleigh fading channels. It is also of interest to design space-time codes for Rician fading channels, which are often found in practice.

In this paper, we study a particular space-time coding scheme with memory for Rician MISO wireless channels. The scheme entitles to placing linear prefilters in front of the transmit antennas. The optimum prefilters are found based on maximizing the channel ergodic capacity. This is a new criterion for designing space-time codes, as in the previous works, the link between capacity and space-time code design has not been examined. The optimum prefilters act as a combination between beamforming on the channel mean vector and transmit diversity coding. The design resolves to only beamforming when the K factor of the channel is large enough, and to only transmit diversity coding when $K = 0$ (i.e. Rayleigh channels).

A linear MMSE receiver is used to detect the signal. We analyze a bound on the error rate performance for 4QAM uncoded input signal. The analysis however is applicable to any rectangular QAM constellation. Both analysis and simulation results show that we were able to extract diversity and array gains in the channel using the optimum prefilter structure above, eventhough the MMSE is a sub-optimum receiver. The design achieves a lower error rate with more transmit antennas or with stronger K factor in the Rician channel, at the same average total transmit power.

The result in this paper is also applicable to MISO wireless systems with partial channel knowledge at the transmitter, where the channel coefficients are known with some errors. The receiver is always assumed to know the channel perfectly.

Some notations used in this paper are: $(.)^T$ is transpose, $(.)^*$ is conjugate, $(.)^\dagger$ is conjugate transpose, $||.||$ is the Euclidean norm, $||.||_F$ is the Frobenius norm and $E[.]$ is expectation.

2. PROBLEM SETUP

2.1. Channel model

We consider frequency flat wireless MISO channels with independent Rician fading statistics. Let M be the number of transmit antennas. The channel coefficient vector $\mathbf{h} = [h_1\ h_2\ \dots\ h_M]^T$ is complex Gaussian distributed with mean $\mathbf{h}_0$ and variance $\alpha\mathbf{I}$, i.e. $\mathbf{h} \sim \mathcal{N}(\mathbf{h}_0, \alpha\mathbf{I})$. The total power gain in the channel is normalized such that

$$\frac{1}{M}||\mathbf{h}_0||^2 + \alpha = 1\ . \tag{1}$$

With this normalization, K factor of the channel, which is the ratio between power in the fixed and random components of the channel, becomes

$$K = \frac{||\mathbf{h}_0||^2}{M\alpha}\ . \tag{2}$$

When $\mathbf{h}_0 = \mathbf{0}$, the channel is Rayleigh fading with independent identically distributed components.

This channel model also encompasses the model for partial channel knowledge at the transmitter characterized by an estimated coefficient vector $\mathbf{h}_0$ and i.i.d Gaussian estimation error with zero mean and variance α.

2.2. Signal model

The space-time precoder structure is depicted in Fig. 1. The uncoded input symbols $x[k]$ is passed through the prefilters $p_i[k]$ before being sent from the transmit antennas. We assume that all the prefilters are FIR of length $L+1$. The signal transmitted from each antenna therefore is

$$z_i[k] = \sum_{n=0}^{L} p_i[n]x[k-n]\ ,\quad i = 1\dots M\ . \tag{3}$$

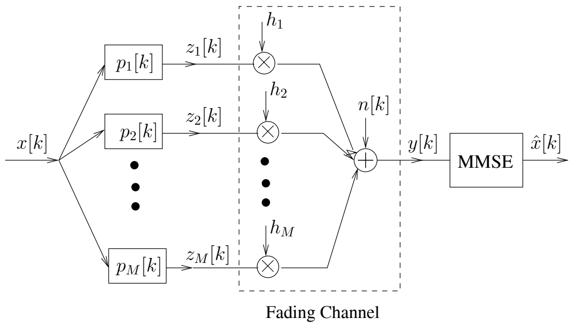

Fig. 1. Space-time precoding with prefilter structure.

Let $\mathbf{z}[k] = \left[z_1[k] \; z_2[k] \ldots z_M[k]\right]^T$ be the vector of signal sent from the transmit antenna array at time k. Form the prefilter matrix $\mathbf{P}$

$$\mathbf{P} = \begin{bmatrix} p_1[0] & p_1[1] & \ldots & p_1[L] \\ p_2[0] & p_2[1] & \ldots & p_2[L] \\ \vdots & \vdots & \ddots & \vdots \\ p_M[0] & p_M[1] & \ldots & p_M[L] \end{bmatrix},$$

then we have the relation

$$\mathbf{z}[k] = \mathbf{P}\mathbf{x}_k \; , \tag{4}$$

where $\mathbf{x}_k = \left[x[k] \; x[k-1] \; \cdots \; x[k-L]\right]^T$. To keep the average total transmit power constant, we constrain the prefilter taps to satisfy $||\mathbf{P}||_F^2 = 1$.

The signal arriving at the receive antenna is

$$y[k] = \sum_{i=1}^{M} h_i z_i[k] + n[k] = \mathbf{h}^T\mathbf{z}[k] + n[k] \; . \tag{5}$$

In the following sections, we will drop the subscript k where appropriate.

3. OPTIMUM SPACE-TIME PREFILTERS BASED ON CHANNEL CAPACITY

3.1. Capacity achieving transmit signal characteristics

We are interested in the channel ergodic capacity, which is the maximum average mutual information between the transmit and receive signals E $[\mathcal{I}(\mathbf{z}; y)]$, subject to an average transmit power constraint. The capacity achieving transmit signal $\mathbf{z}$ is Gaussian distributed with zero mean and a normalized covariance matrix $\mathbf{R}_{zz}$ that is the solution of the optimization problem

$$\begin{aligned} \mathcal{C} \quad &= \quad \max_{\mathbf{R}_{zz}} \mathrm{E}\left[\log(1 + \rho\mathbf{h}^T\mathbf{R}_{zz}\mathbf{h}^*)\right] \\ &\quad\;\; \text{s.t.} \quad \mathrm{tr}(\mathbf{R}_{zz}) = 1 \; . \end{aligned} \tag{6}$$

Here $\rho = \sigma_s^2/N_0$ is the SNR, with σ_s^2 as average symbol power and N_0 as the noise power.

This problem with the channel model $\mathbf{h} \sim \mathcal{N}(\mathbf{h}_0, \alpha\mathbf{I})$ has been solved analytically in [1]. Since we are going to make use of this result, we recite the solution here. Let $\mathbf{R}_{zz} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^\dagger$. The optimum covariance matrix will have the eigenvectors $\mathbf{U}$ such that the first column $\mathbf{u}_1 = \frac{\mathbf{h}_0^*}{||\mathbf{h}_0||}$, and all other columns are arbitrary except for the constraint that $\mathbf{U}$ is unitary. The power allocations are

$$\lambda_2 = \cdots = \lambda_M = \frac{1-\lambda_1}{M-1} \; . \tag{7}$$

Let $\mathbf{v} = \mathbf{h}^T\mathbf{U}/\sqrt{\alpha}$, then $v_1 \sim \mathcal{N}(\sqrt{MK}, 1)$ and $v_i \sim \mathcal{N}(0,1)$ with $2 \le i \le M$, where all the v_i are independent. The problem becomes finding λ_1 such that

$$\begin{aligned} &\max_{\lambda_1} \quad \mathrm{E}\left[\log\left(\tfrac{1}{\rho\alpha} + \lambda_1|v_1|^2 + \tfrac{1-\lambda_1}{M-1}\textstyle\sum_{i=2}^{M}|v_i|^2\right)\right] \\ &\text{s.t.} \qquad\qquad 0 \le \lambda_1 \le 1 \; . \end{aligned} \tag{8}$$

This is a convex optimization problem which can be solved efficiently [3]. We use Newton method with the gradient and Hessian approximated by Monte-Carlo simulations to find the optimum value $\lambda_1^\star$. This optimum value is a function of the number of transmit antennas M, SNR and K factor. The $\lambda_1^\star$ values can be stored as a table lookup for use in prefilters design.

3.2. Optimum space-time prefilters

From (4), we have the normalized covariance matrix

$$\mathbf{R}_{zz} = \mathbf{P}E[\mathbf{x}_k\mathbf{x}_k^\dagger]\mathbf{P}^\dagger/\sigma_s^2 \; .$$

Assume that the input symbols $x[k]$ are zero mean and independent, then $E[\mathbf{x}_k\mathbf{x}_k^\dagger] = \sigma_s^2\mathbf{I}$. Hence the prefilters $\mathbf{P}$ satisfies

$$\mathbf{P}\mathbf{P}^\dagger = \mathbf{R}_{zz} \; . \tag{9}$$

For the case of Rayleigh fading, where the channel mean $\mathbf{h}_0 = \mathbf{0}$, the optimum transmit covariance is $\mathbf{R}_{zz} = \frac{1}{M}\mathbf{I}$ [2], and the optimum prefilters must satisfy

$$\mathbf{P}\mathbf{P}^\dagger = \frac{1}{M}\mathbf{I} \; . \tag{10}$$

This is the same condition as derived by Wittneben in [4], and developed further by Wornell and Trott in [5], although the optimization criterion in [4] was to minimize the variance of the receive power for a single symbol. From (10), the length of each FIR has to be at least the number of transmit antennas. The optimum prefilter matrix $\mathbf{P}$ is then a scaled *unitary matrix*, hence we call this "unitary diversity". This type of optimum prefilters includes delay diversity [6] as a special case. Other matrices such as the DFT, Hadamard matrices also satisfy, although they are more restricted on dimension than delay diversity.

For Rician fading channels, since $\mathbf{h}_0 \neq \mathbf{0}$, the optimum covariance matrix will have the form

$$\mathbf{P}\mathbf{P}^\dagger = \lambda_2\mathbf{I} + (\lambda_1 - \lambda_2)\frac{\mathbf{h}_0^*\mathbf{h}_0^T}{||\mathbf{h}_0||^2} \; , \tag{11}$$

i.e., a rank-one update to a scaled identity matrix. There can be different prefilter matrices which satisfy this condition. We propose the following design for the prefilters

$$\mathbf{P} = (\sqrt{\lambda_1} - \sqrt{\lambda_2})[\mathbf{u}_1 \; \mathbf{0}_{M\times L}] + \sqrt{\lambda_2}\mathbf{U} \; , \tag{12}$$

where $\mathbf{U}$ is a unitary matrix with the first column as $\mathbf{u}_1 = \mathbf{h}_0^*/||\mathbf{h}_0||$. This scheme is a combination between beamforming, which is the first term in (12), and unitary diversity, which is the second term. When K factor is above a certain threshold (SNR dependent) then $\lambda_2 = 0$, and the scheme is beamforming, where the prefilters become just a scalar weight vector. When $K = 0$ then $\lambda_1 = \lambda_2 = 1/M$, and the scheme becomes unitary diversity. Note that in Rayleigh fading channels, the prefilter matrix $\mathbf{P}$ can be any scaled unitary matrix, but for Rician fading, the unitary portion of $\mathbf{P}$ must have the first column as $\mathbf{h}_0^*/||\mathbf{h}_0||$.

4. PERFORMANCE ANALYSIS WITH MMSE RECEIVER

4.1. MMSE receiver and the optimum MSE

The space-time prefilters and the flat fading channel together act as an effective frequency selective channel with impulse response

$$h[k] = \sum_{i=1}^{M} h_i p_i[k] \quad , \quad k = 0 \dots L \; . \tag{13}$$

The frequency response of this effective channel is

$$H(\omega) = \sum_{k=0}^{L} h[k] e^{-j\omega k} = \mathbf{h}^T \mathbf{P} \mathbf{e}(\omega) \; , \tag{14}$$

where $\mathbf{e}(\omega) = [1 \; e^{-j\omega} \; \cdots \; e^{-jL\omega}]^T$.

We choose the linear MMSE receiver for simplicity and efficiency reasons. It has to be noted however, that the MMSE is a sub-optimum receiver and hence does not operate close to the capacity. Achieving the capacity also requires optimum channel coding, which we do not study here. Since the prefilters design is based on maximizing the capacity, only part of the gain promised by the prefilters can be realized with this receiver, an effect which shows on the error rate performance.

The ideal equalizer response is given by

$$Q(\omega) = \frac{\rho H^*(\omega)}{1 + \rho |H(\omega)|^2} \; . \tag{15}$$

With this equalizer, the mean square error is

$$\epsilon = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\sigma_s^2}{1 + \rho |\mathbf{h}^T \mathbf{P} \mathbf{e}(\omega)|^2} d\omega \; . \tag{16}$$

4.2. Error rate performance

The error performance analysis in this section applies when the input symbols $x[k]$ comes from a 4QAM (or equivalently QPSK) constellation. Based on the results in [7], which is an application of a more general result in [8], the average symbol error probability can be upper bounded based on the mean square error as

$$\bar{P}_e \le E\Big[\exp\Big(\frac{d^2}{\sigma_s^2} - \frac{d^2}{\epsilon}\Big)\Big] \; , \tag{17}$$

where $2d$ is the distance between two neighboring points along a dimension of the constellation, ϵ is given in (16) and the expectation is over the channel response $\mathbf{h}$. In general cases, this bound can be evaluated numerically. For the two boundary cases of unitary prefilters and beamforming, more detail analytical results can be obtained as follows.

For Rayleigh fading channels, since $\mathbf{P}$ is scaled unitary and the statistics of $\mathbf{h}$ is unitary invariant, the upper bound on the average symbol error probability becomes

$$\bar{P}_e \le E\Big[\exp\Big(\frac{d^2}{\sigma_s^2} - d^2 \Big\{\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\sigma_s^2}{1 + \frac{\rho}{M} |\mathbf{h}^T \mathbf{e}(\omega)|^2} d\omega\Big\}^{-1}\Big)\Big] \; . \tag{18}$$

This upper bound is independent of the prefilters used.

For strong Rician fading channels (high K factor) where the optimum prefilters is beamforming, then $\mathbf{P} = [\frac{\mathbf{h}_0^*}{||\mathbf{h}_0||} \; \mathbf{0}]$ and the MSE becomes

$$\epsilon = \frac{\sigma_s^2}{1 + \rho |\frac{\mathbf{h}^T \mathbf{h}_0^*}{||\mathbf{h}_0||}|^2} \; . \tag{19}$$

Taking the expectation over $\mathbf{h}$, the upper bound is

$$\bar{P}_e \le \Big(\frac{1}{\eta + 1}\Big) \exp\Big(-KM\big(1 - \frac{1}{\eta + 1}\big)\Big) \; , \tag{20}$$

where $\eta = d^2 \rho \alpha / \sigma_s^2$. As the number of transmit antennas M increases, this upper bound will go to zero exponentially. The bound also shows that the average error rate decreases with increasing K factor or SNR (which is proportional to η). However, due to the variation in the channel (variance $\alpha \neq 0$) which is present in both K and η, the bound only decreases sub-exponentially with SNR. As $\alpha = 0$ for fixed channel, the bound becomes $\exp(-M\rho d^2/\sigma_s)$, which decreases exponentially with both the number of transmit antennas M and the SNR ρ. This agrees with the classical beamforming result.

5. NUMERICAL RESULTS

We perform the simulation for three different values of K factor: $K = 0$ (Rayleigh fading channels), $K = 0.2$ and $K = 3$. For each SNR and M configuration, we run blocks of 1024, 128 and 32768 symbols respectively over 20000 realizations of the channel.

For the first case of Rayleigh fading, the optimum prefilters are unitary diversity. Using delay diversity where $\mathbf{P} = \mathbf{I}/\sqrt{M}$, we obtain the plots in Fig. 2. The upper bound (18) is superimposed on the simulation curves. The bound tracks closely the diversity gain in the channel but has a constant gap of around 2.4dB compared to the actual performance. The diversity gain by multiple transmit antennas only starts to show up at SNR larger than 5dB. This gain tends to diminish with higher number of transmit antennas, which is also observed in [5].

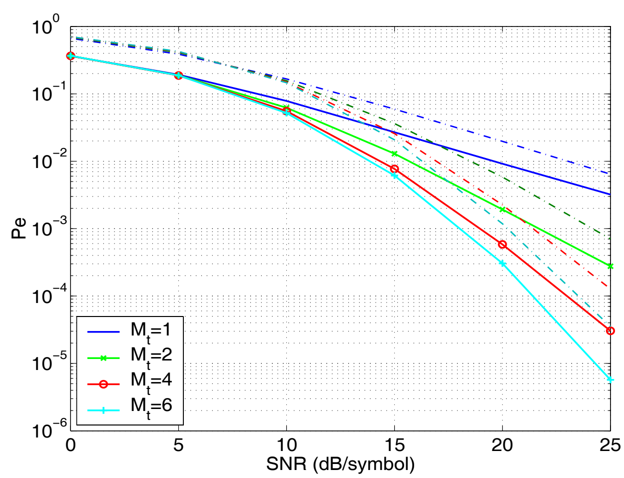

Fig. 2. Average symbol error probability for Rayleigh flat fading channels ($K = 0$) using delay diversity. The dotted lines are the corresponding upper bounds in (18).

For the case of strong Rician fading with $K = 3$, the optimum $\lambda_1^\star = 1$ for the range of SNRs that we are interested in. Thus beamforming on the channel mean vector is optimum. The symbol error rate is plotted in Fig. 3. The result shows strong array gain. The error rate is significantly lower than the corresponding Rayleigh channel. This error rate is also a strong function of the number of transmit antennas. The upper bound (20) tracks the actual performance well at low to medium SNRs (up to 15dB) at a constant gap of around 2dB, but tends to diverge from the actual performance at higher SNRs.

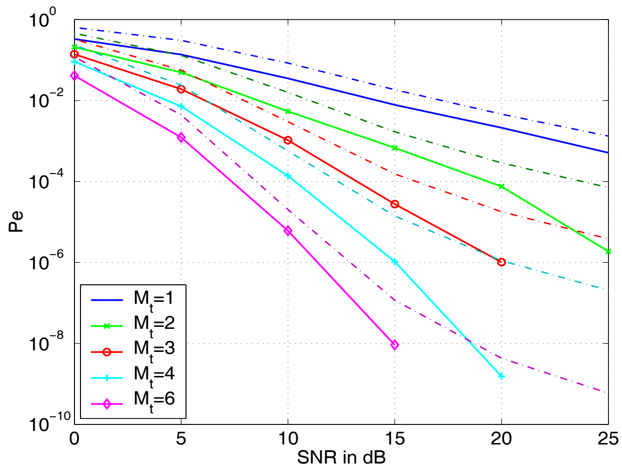

Fig. 3. Average symbol error probability for Rician flat fading channels with $K = 3$ using beamforming. The dotted lines are the corresponding upper bound in (20).

SNR	0dB	5dB	10dB	15dB	20dB	25dB
M = 2	0.921	0.775	0.715	0.693	0.681	0.679
M = 4	1.000	0.931	0.802	0.754	0.738	0.732

Table 1. Optimum $\lambda_1^\star$ for $K = 0.2$ Rician channels.

With $K = 0.2$, the optimum $\lambda_1^\star$ varies depending on the SNR and number of antennas M, and is shown in Table 1. Using these values to design the prefilters according to (12), we obtain the performance curves in Fig. 4. The plot shows an improved performance with more antennas at low SNRs (up to 10dB) compared to delay diversity. This is due to array gain on the Rician factor of the channel. At higher SNRs however, the error rates are similar to that of delay diversity. This is due to MMSE being a suboptimum receiver, hence it does not achieve all the gain promised by the capacity of the channel here. It must also be stressed that these curves are for uncoded QAM symbols. With coding, which is mandatory for operating close to capacity region, the error performance is expected to improve with more transmit antennas over delay diversity at all SNRs.

6. CONCLUSION

We have derived an optimal space-time precoding scheme for MISO wireless channels with Rician fading statistics by using linear prefilters at the transmitter. The optimum prefilter design is based on maximizing the channel ergodic capacity. Performance analysis by error rate bounds and numerical simulations have been carried out using MMSE receiver for 4QAM uncoded input signal. Results show that performance gain can be obtained by exploiting the channel knowledge at the transmitter, and also by having more transmit antennas, especially in high K factor channels.

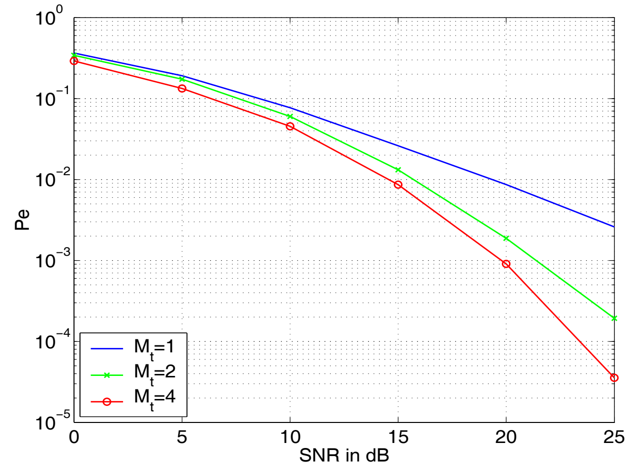

Fig. 4. Average symbol error probability for Rician flat fading channels with $K = 0.2$ using prefilters design (12) and MMSE receiver.

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