A RECURSIVE QR APPROACH TO SEMI-BLIND EQUALIZATION OF TIME-VARYING MIMO CHANNELS

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ABSTRACT

This paper presents a novel adaptive equalization algorithm for time-varying, frequency-selective MIMO systems that harnesses the finite alphabet property inherent in digital communication. The algorithm leads to a direct, cost-efficient QR-based recursive updating procedure for the equalizer coefficients that forces adaptation to changing channel characteristics. The proposed method does not require precise channel estimation and uses significantly less pilot symbols than other traditional equalizers, implying a drastic reduction in bandwidth overhead. Simulation results confirm that this approach outperforms the traditional recursive least squares (RLS) adaptive equalizer for this application and rivals the MMSE equalizers with perfect channel knowledge.

1. INTRODUCTION

The major challenge in signal recovery for MIMO (multiple-input-multiple-output) systems is mitigating the ISI (inter-symbol- interference) due to multi-path propagation and channel distortion, as well as ICI (inter-channel-interference), a result of multiple transmitters sharing the communication medium. In general, signal recovery and tracking schemes can be classified into two categories: equalizers that attempt to estimate the channel before equalization, or those which directly equalize the received signal with no attention to channel estimation [1]. We focus on the second type, because precise channel state information may not be readily or reliably available at the receiver, especially when the channel is changing very fast. Furthermore, imprecision in CSI (channel state information) may cause significant performance degradation for equalizers relying on channel estimation. Such an approach bears some resemblance to the well-known class of blind equalizers for MIMO systems [2], but we refer to it as "semi-blind" because it requires a minimal amount of training symbols for initialization. In addition, we exploit the finite alphabet property inherent in digital communication signals, which enables the identification of equalization of unknown MIMO channels [3][4].

1.1. MIMO Channel Model

For simplicity, we start the analysis with the stationary channel model. In Section 4, the proposed algorithm will be extended to the time-varying scenario, and the time-varying channel model will be introduced in Section 5. We consider a frequency-selective fading MIMO model with t inputs and r outputs:

$$\underline{x}(k) = \sum_{l=0}^{L} H(l)\underline{s}(k-l) + \underline{n}(k), \qquad (1)$$

where L is the maximal degree among all the channels, $\underline{s}(k)$, $\underline{x}(k)$ are $t \times 1$, $r \times 1$ sample stacks of the transmitted, received data sequences. The expression of (1) in D-transform domain is:

$$\underline{\mathbf{x}}(D) = \mathbf{H}(D)\underline{\mathbf{s}}(D) + \underline{\mathbf{n}}(D), \qquad (2)$$

where $\mathbf{H}(D)$ is the $r \times t$ transfer function of the MIMO system.

1.2. Zero-Forcing Equalizer for ISI MIMO

For ISI MIMO Channels, an equalizer can be viewed as a bank of FIR space-time filters at the receiver side, all with finite taps. Let the $t \times r$ polynomial matrix $\mathbf{G}(D)$ denote the FIR coefficient matrix of the equalizer, where each row $\underline{\mathbf{g}}_i^T(D)$ represents the FIR array for the estimate of a particular input. Then the zero-forcing constraint [5] requires that

$$\begin{aligned} \mathbf{V}(D) = \mathbf{G}(D)\mathbf{H}(D) &= diag\{D^{\tau_i}\}_{i=1}^{t} \\ \text{or,} \quad \underline{\mathbf{g}}_i^T(D)\mathbf{H}(D) &= D^{\tau_i}\underline{\mathbf{e}}_i^T, \end{aligned} \qquad (3)$$

where τ_i are nonnegative system delays that are allowed upon recovery. Here $\underline{\mathbf{e}}_i^T$ is a $1 \times t$ vector with all entries zero except 1 at position i, and $(\cdot)^T$ denotes transpose. In the absence of noise, perfect symbol recovery can be accomplished by applying $\underline{\mathbf{g}}_i^T(D)$ on the receiving data. For noisy channels, we have

$$\begin{aligned} \hat{\mathbf{s}}_i(D) &= \underline{\mathbf{g}}_i^T(D)\mathbf{H}(D)\underline{\mathbf{s}}(D) + \underline{\mathbf{g}}_i^T(D)\underline{\mathbf{n}}(D) \\ &= D^{\tau_i}\mathbf{s}_i(D) + \underline{\mathbf{g}}_i^T(D)\underline{\mathbf{n}}(D). \end{aligned} \qquad (4)$$

2. ITERATIVE SIGNAL RECOVERY ALGORITHM

The lack of CSI makes it impossible to find a closed-form expression for the equalizer coefficients in $\mathbf{G}(D)$. We adopt a joint estimation approach that iterates between the two spaces (equalizer taps and source symbols) for signal recovery. WLOG, assume the objective is the recovery of the first stream $s_1(k)$. The key steps of our approach are listed below:

1. Predefine equalizer order and system delay from the feasible region (cf. Section 3);
2. Obtain a proper initial estimate $\hat{s}_1^{(0)}(k)$ with a block of symbols drawn from the digital constellation;
3. Repeat the following two steps until convergence:

   (a) Fix the estimate of $\hat{s}_1^{(i)}(k)$ as if it is correct and find the equalizer $\underline{\hat{g}}(k)$ that minimizes the detection error:

$$\underline{\hat{g}}^{(i+1)}(k) = \arg\min_{\underline{\hat{g}}(k)} \|\hat{s}_1^{(i)}(k) - \underline{\hat{g}}^T(k) * \underline{x}(k)\|; \qquad (5)$$

(b) To drive $\hat{s}_1(k)$ towards a valid finite alphabet sequence, the new estimate $\hat{s}_1^{(i+1)}(k)$ is updated to the nearest symbols in the constellation:

$$\hat{s}_1^{(i+1)}(k) = \arg \min_{\hat{s}_1(k) \in \mathcal{FA}} \|\hat{s}_1(k) - [\underline{\hat{g}}^{(i+1)}(k)]^T * \underline{x}(k)\|, \tag{6}$$

where $\mathcal{FA}$ is the set of all finite alphabet sequences.

To introduce the matrix-vector expression of (5) and (6), for any two positive integers ρ and N, we define a $r(\rho+1) \times N$ block Toeplitz matrix $\Gamma_N^\rho[\underline{x}]$ of output signals: $\Gamma_N^\rho[\underline{x}] \equiv$

$$\begin{bmatrix} \underline{x}(N-1) & \underline{x}(N-2) & \dots & \underline{x}(0) \\ \underline{x}(N) & \underline{x}(N-1) & \dots & \underline{x}(1) \\ \vdots & \vdots & \ddots & \vdots \\ \underline{x}(N+\rho-1) & \underline{x}(N+\rho-2) & \dots & \underline{x}(\rho) \end{bmatrix}. \tag{7}$$

The equalizer filters $\underline{g}(k)$ with order ρ (or, $\underline{\mathbf{g}}(D)$ in D-transform domain) can also be represented in an extended vector form:

$$\Gamma^\rho[\underline{\hat{g}}]^T \equiv \left[\begin{array}{cccc} \underline{\hat{g}}^T(\rho) & \underline{\hat{g}}^T(\rho-1) & \dots & \underline{\hat{g}}^T(0) \end{array}\right]. \tag{8}$$

Similarly, we stack N consecutive symbols of the signal estimate:

$$\Gamma_N^0[\hat{s}_1] \equiv \left[\begin{array}{ccc} \hat{s}_1(N+\rho-1) & \dots & \hat{s}_1(\rho) \end{array}\right]. \tag{9}$$

The convolution of $\underline{\hat{g}}^T(k)$ with $\underline{x}(k)$ is equivalent to the multiplication of the corresponding Toeplitz-structured matrices. The matrix-vector formulation admits a simple least square solution to (5) and non-linear mapping solution to (6):

$$\Gamma^\rho[\underline{\hat{g}}^{(i+1)}]^T = \Gamma_N^0[\hat{s}_1^{(i)}] \cdot \Gamma_N^\rho[\underline{x}]^\dagger; \tag{10}$$

$$\Gamma_N^0[\hat{s}_1^{(i+1)}] = T(\Gamma^\rho[\underline{\hat{g}}^{(i+1)}]^T \cdot \Gamma_N^\rho[\underline{x}]), \tag{11}$$

where $T(\cdot)$ denotes the mapping to the nearest valid symbol in the digital constellation and $(\cdot)^\dagger$ denotes the Moore-Penrose inverse of a matrix. Furthermore, the two parts in each loop can be combined to yield an update formula for the signal estimate $\hat{s}_1(k)$:

$$\Gamma_N^0[\hat{s}_1^{(i+1)}] = T(\Gamma_N^0[\hat{s}_1^{(i)}] \cdot \Gamma_N^\rho[\underline{x}]^\dagger \cdot \Gamma_N^\rho[\underline{x}]). \tag{12}$$

In short, each update involves the following operations:

1. Least-square approximation (linear subspace projection): The multiplication of $\Gamma_N^0[\hat{s}_1^{(i)}]$ with a weighting matrix $W = \Gamma_N^\rho[\underline{x}]^\dagger \Gamma_N^\rho[\underline{x}]$ is a linear projection of the vector $\Gamma_N^0[\hat{s}_1^{(i)}]$ into the subspace spanned by the row vectors in $\Gamma_N^\rho[\underline{x}]$.
2. Nonlinear decision making (finite alphabet mapping): The projected vector is mapped into the finite alphabet constellation to obtain the new estimate vector $\Gamma_N^0[\hat{s}_1^{(i+1)}]$.

3. THEORETICAL FOUNDATION

The most critical concern of an iterative approach lies in its convergence behavior. For this, the following issues must be considered:

1. *Existence:* Will there exist correct fixed points for the signal recovery iterations using an FIR combiner?
2. *Exclusiveness:* In the noise free case, how do we assure that the system will not converge to a wrong solution?
3. *Robustness:* How do we make the algorithm's convergence behavior robust against noise disturbance?

We elaborate on these three issues in the following sections.

3.1. Existence of Fixed Points in Noise-free Channels

For simplicity, let us temprorarily ignore the effect of noise. The condition for the existence of fixed points is inherently tied to three properties of a MIMO system: channel characteristics, system delay, and equalizer order.

The effect of channel characteristics on the existence of fixed points is established through the (Generalized) Bézout Identity [6]. More precisely, given an FIR MIMO channel $\mathbf{H}(D)$, there exists a polynomial matrix $\mathbf{G}(D)$ satisfying the zero-forcing constraint in (3) if and only if $\mathbf{H}(D)$ is delay-permissive right coprime[1]. For such a MIMO channel, there is in general a necessary minimum delay elapsed before an input signal can be reconstructed by any FIR equalizer. The minimum system delays for different input signals can be different and are denoted by $\{\tau_j^\star\}_{j=1}^t$. In addition, the existence of fixed points is also related to the equalizer order. In other words, there is a minimal degree requirement on the matrix $\mathbf{G}(D)$ in (3). As to the sufficient degrees of zero-forcing equalizers given $\mathbf{H}(D)$, a very comprehensive treatment on the degree bound is derived in [7].

Theorem 1 (Sufficient Order of Zero-Forcing Equalizer) *Suppose that the MIMO transfer function $\mathbf{H}(D)$ is delay-permissive right coprime and that $\tau \geq \tau_i^\star$. There exists a zero-forcing equalizer for the ith input with order ρ and recovery delay τ if*

$$\rho \geq \max\{\nu - 1, \tau + L - 1\}, \tag{13}$$

where ν is the degree of the null-space minimum basis for $\mathbf{H}(D)$.

Proof: For the proof, see [7]. The notion of the null-space minimal basis is proposed by G. D. Forney, see [6][7]. ■

Two useful corollaries immediately follow the theorem above:

Corollary 1 (Sufficient Equalizer Orders)
Given a t-in-r-out MIMO system with a transfer function $\mathbf{H}(D)$:

1. *if $\mathbf{H}(D)$ is right coprime, a sufficient degree to reconstruct any signal is $\rho \geq max(\nu - 1, L - 1)$.*
2. *if $\mathbf{H}(D)$ is column-reduced, a sufficient degree to reconstruct signal i with delay τ (assuming that $\tau \geq \tau_i^\star$) is $\rho \geq \max(\nu - 1, \tau)$.*

In practice, we generally do not have direct knowledge of ν, while we do have a reasonable estimate on L. Thus, the following approximation for ν may prove useful: $\nu \approx \frac{Lt}{r-t}$.

3.2. Exclusiveness of Fixed Points

In digital transmission, each symbol in an information stream is an element drawn from a finite set. A sequence is called a (valid) finite alphabet sequence if and only if each of its symbols is a valid point in a digital constellation. To establish the exclusiveness of convergent points, we must exploit the pivotal finite alphabet property inherent in digital communication systems:

[1] A polynomial matrix $\mathbf{C}(D)$ is said to be a right common divisor (rcd) of the rows in $\mathbf{H}(D)$ if a finite-order polynomial matrix $\mathbf{H}'(D)$ can be found to support the factorization $\mathbf{H}(D) = \mathbf{H}'(D)\mathbf{C}(D)$. A matrix $\mathbf{R}(D)$ is the grcd of $\mathbf{H}(D)$ if and only if any rcd $\mathbf{R}'(D)$ of $\mathbf{H}(D)$ is also an rcd of $\mathbf{R}(D)$. A polynomial matrix (of D) is said to be delay-permissive (right) coprime if and only if the determinant of any its grcd (greatest right common divisor) has the form of a pure delay D^τ (if $\tau = 0$, the matrix is called coprime).

Theorem 2 (FAE Property for FIR Filters)
Let the $1 \times t$ polynomial vector $\underline{\mathbf{v}}^T(D)$ represent a linear FIR system. For the output $\underline{\mathbf{v}}^T(D)\underline{\mathbf{s}}(D)$ to always be a valid symbol sequence given any input symbol sequence $\underline{\mathbf{s}}(D)$, it is necessary that $\underline{\mathbf{v}}^T(D) = cD^\tau \underline{\mathbf{e}}_i^T$. Here c is a constant, e.g. $c = \pm 1$ in the case of BPSK constellation (or $c = e^{jn\pi/2}$ for any integer n in case of QPSK).

Proof: See [7]. ■

FAE holds only when the symbol patterns are sufficiently random. The failure rate decreases exponentially with an increase of the block size N. Therefore, the FAE property is a practical assumption as long as N is reasonably large. Without invoking the FAE property, it is well-known that a MIMO channel is identifiable only up to a transformation of a unimodular matrix. Fortunately, such ambiguity can be resolved once the FA constraint is imposed. Given t (sufficiently long and random) FA sequences, by the FAE property it is impossible to produce by linear FIR filters a FA output other than one of the t original sequences or its scaled and/or delayed version; thus the system becomes identifiable except for a scaling factor, a delay, and/or a permutation on the sources.

3.3. Robustness of Convergence

An important question to be addressed is how to ensure equalizer robustness in the presence of inevitable noise. Given an $\mathbf{H}(D)$ that is delay-permissive right coprime, the choice of $\mathbf{G}(D)$ satisfying (3) is highly non-unique. A commonly accepted solution is one which yields the minimum postprocessing noise power $\mathcal{E}[|\underline{g}^T(k) * \underline{n}(k)|^2] = \sigma_n^2 \|\Gamma^\rho[\underline{g}]\|^2$, or equivalently, maximum postprocessing SNR. Clearly, the minimum-norm equalizer is the most desirable.

The optimal SNR gain can be improved by (1) adopting optimal equalizer order and (2) purposefully imposing some system delay. Equalizer order selection is governed by the following trade-offs. Given a precisely known channel, the 2-norm of the optimal equalizer vector is a monotonically decreasing function with respect to the FIR order ρ. On the other hand, for blind channels, a higher ρ implies (a) a greater failure possibility of the FAE property, which could result in convergence to incorrect solution, and (b) an unnecessary expansion of search space, which could hamper the convergence process. Thus, the equalizer order must be chosen in such a way that a desirable SNR gain can be achieved while still maintaining a manageable search space. With regard to SNR optimization, the system delay can be more or less treated as a free parameter. Given the FIR order ρ, the possible range of the system delay τ is $[0, 1, \cdots, L + \rho - 1]$. The 2-norm, $\|\Gamma^\rho[\underline{g}]\|$, displays a $\bigcup$-shape as a function τ, i.e. they have lower values in the middle portion of the possible delay range. Hence the optimal system delay can be empirically chosen as $\tau_{opt} \approx \frac{\rho+L}{2}$.

4. RECURSIVE SIGNAL TRACKING ALGORITHMS

We extend the iterative signal recovery algorithm in Section 2 to time-varying channels by developing recursive QR-based equalization schemes capable of fast computation with minor overhead. We adopt a time-varying MIMO channel model as proposed by Komninakis in [8]. Each coefficient is the sum of a constant and time-varying part: $h_{ij}(k,l) = c_{ij}(l) + \bar{h}_{ij}(k,l)$ where $\bar{h}_{ij}$ is a zero-mean, wide-sense stationary, complex Gaussian process whose autocorrelation is related to the Doppler rate f_{ij} between the corresponding transmitter and receiver. For further details of the channel model, see [8]. Such a channel model will generically satisfy the delay-permissive coprime condition in Section 3.1 [2].

To adapt to changing channel characteristics, we introduce a recursive QR factorization on the modified receiver data matrix, which now takes the form $\Gamma_N^\rho[\underline{x}]\Phi^N$ to allow multiplication by a forgetting factor ($\Phi^N = diag(\phi^i)_{i=1}^N$ where $\phi \in [0,1]$). Assume we have a lower-triangular matrix R at time N such that

$$R\,\Gamma_N^\rho[\underline{x}]\Phi^N = Q,$$

where Q is unitary. With the arrival of Δ additional symbols (or columns of $\Gamma_N^\rho[\underline{x}]$), we wish to find $\bar{R}$ satisfying:

$$\bar{R}\,\Gamma_{N+\Delta}^\rho[\underline{x}]\Phi^{N+\Delta} = \bar{Q}. \qquad (14)$$

To expedite the computation of $\bar{R}$, we compute an updating matrix C such that $\bar{R} \leftarrow CR$. Note that

$$\begin{aligned} \bar{Q} &= CR \cdot \Gamma_{N+\Delta}^\rho[\underline{x}] \cdot \Phi^{N+\Delta} \\ &= C\left[\ Q_\Delta \quad | \quad \phi^\Delta Q\ \right], \end{aligned} \qquad (15)$$

where Q_Δ denotes the submatrix formed by the Δ newly added columns in $\Gamma_{N+\Delta}^\rho[\underline{x}]$, pre-multiplied by R. Then we have:

$$\bar{Q}\bar{Q}^* = C[Q_\Delta Q_\Delta^* + \phi^{2\Delta} I_{r(\rho+1)}]C^* = I_{r(\rho+1)}, \qquad (16)$$

where $(\cdot)^*$ is the conjugate transpose. Thus C can be computed via a Cholesky factorization. Based on the analysis above, each time a new block of data is received, the recursive QR formulation includes three procedures: whitening, signal and equalizer tracking, and interference cancellation.

1. Whitening: The first step is to "whiten" the data matrix, $\Gamma_{N+\Delta}^\mu[\underline{x}] \cdot \Phi^{N+\Delta}$, i.e. find $\bar{R}$ such that the rows of $\bar{Q}$ are orthonormal. Equalizer tracking on this whitened space has empirically demonstrated more numerical stability and is quite practical given the recursive QR factorization. The whitening transformation can be recursively updated by the matrix C according to $\bar{R} \leftarrow CR$.

2. Signal and equalizer tracking: Letting $\Gamma^\rho[\underline{\tilde{g}}_i^{(N)}]^T = \Gamma^\rho[\underline{\hat{g}}_i^{(N)}]^T \cdot R^{-1}$ denote the transformed equalizer operating on the whitened data space Q, we obtain an estimate on the newly arrived symbols: $\Gamma_\Delta^0[\hat{s}_i^{(N)}] = T(\Gamma^\rho[\underline{\tilde{g}}_i^{(N)}]^T Q_\Delta)$, where $\underline{\tilde{g}}_i^{(N)}$ is the equalizer estimate in the previous block. After obtaining the new estimate $\Gamma_\Delta^0[\hat{s}_i^{(N)}]$, we update the equalizer by:

$$\Gamma^\rho[\underline{\tilde{g}}_i^{(N+1)}]^T = \left[\ \Gamma_\Delta^0[\hat{s}_i^{(N)}] \quad | \quad \Gamma_N^0[\hat{s}_i]\ \right]\bar{Q}^*. \qquad (17)$$

At first glance, (17) seems to require keeping track of both $\hat{s}_i(k)$ and Q in the updates. However, neither is necessary, and in fact all we need to update is R. Substituting (15) into (17), we have:

$$\begin{aligned} \Gamma_\mu[\underline{\tilde{g}}_i^{(N+1)}]^T &= (\Gamma_\Delta^0[\hat{s}_i^{(N)}] \cdot Q_\Delta^* + \phi^\Delta \Gamma_N^0[\hat{s}_i] \cdot Q^*)C^* \\ &= (\Gamma_\Delta^0[\hat{s}_i^{(N)}] \cdot Q_\Delta^* + \phi^\Delta \Gamma^\rho[\underline{\tilde{g}}_i^{(N)}]^T)C^* \end{aligned} \qquad (18)$$

Thus the update takes on a relatively simple and efficient form.

3. Interference cancellation: When assuming all the source signals uncorrelated, the channel parameters corresponding to the

[2] A simple way to test the coprimeness is: $\mathbf{H}(D)$ is delay-permissive coprime if and only if $\mathbf{H}(D)$ has full column rank for any complex number D except $D = 0$.

extracted ith input can be estimated as $\underline{\hat{h}}_i(k) = \frac{\mathcal{E}[\underline{x}(k+l)\hat{s}_i^*(l)]}{\mathcal{E}[|\hat{s}_i(l)|^2]}$. Once the channel is estimated, then the interference caused by that stream can be accordingly cancelled.

The proposed recursive update resembles the traditional recursive least square (RLS) filtering (derived from Kalman filter) [9] in structure, but with different updating and feedback weights. We claim, and provide supporting simulations in the following section, that the RQR algorithm provides better tracking capability and increased robustness while incurring little extra computation.

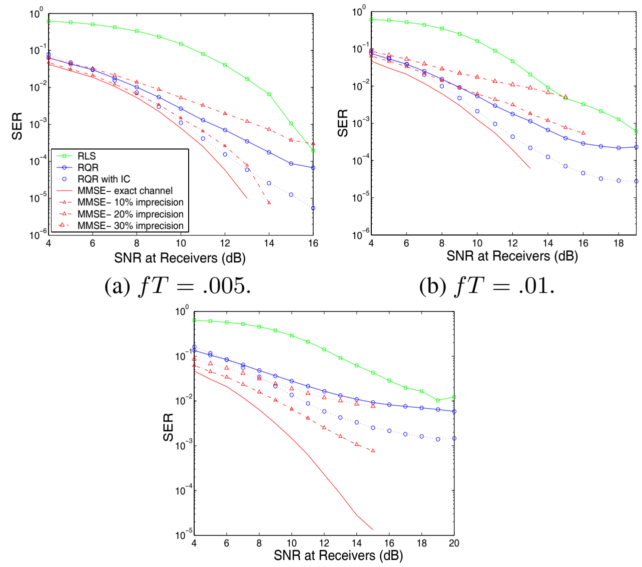
5. SIMULATION

We present simulation results to demonstrate the performance of the proposed RQR scheme. The time-varying channels are generated based on the model presented in Section 4. Figure 1 shows the symbol error rate comparison for RLS, RQR and MMSE equalizers under different channel stationarity. We specify channel stationarity by the Doppler rate, fT (Doppler frequency multiplied by the symbol period)[3]. For comparison, we plot the performance of MMSE equalizer with varying amounts of channel knowledge imprecision (channel imprecision is introduced by adding a Gaussian random variable to the exact channel coefficient). Each of the three plots corresponds to a 2-in-5-out MIMO system with $L = 4$, and an equalizer order of $\rho = 5$. Simulations are conducted for 600 times over a block of 1150 symbols, with randomly generated initial channels. Both RLS and RQR equalizers are given the first 150 symbols for training purposes and perform unsupervised equalizer tracking on the remaining 1000 symbols. The proposed RQR scheme clearly outperforms the traditional RLS method in all the simulated scenarios, particularly in the low SNR regime. For the most stationary channel ($fT = .005$) RQR performs better than the MMSE given channel knowledge with 20% imprecision. For less stationary channels, both the RLS and RQR methods are decisively inferior to the MMSE equalizer with exact channel knowledge, but the RQR scheme offers comparable performance to that of the MMSE equalizer with slightly more imprecision $(20 - 30\%)$. It is also demonstrated in the plots that the interference cancellation in these two-input cases leads to improvements of approximately $1-2$dB. For the two most stationary channels in with $fT = .005, .01$, the interference cancellation approach rivals the MMSE with exact channel knowledge.

6. CONCLUSION

This paper presents a recursive QR approach to semi-blind equalization of time-varying ISI MIMO channels. The theoretical foundations of the proposed approach are rooted in signal recovery results derived from the generalized Bézout identity and the finite alphabet property inherent in digital communication. Concerning the convergence behavior of the algorithm, three issues are addressed: existence, exclusiveness, and robustness. It is recognized that it is necessary to impose a proper equalizer order and system delay for correct and robust results. Theoretical and practical bounds for such parameters are provided. Under the theoretical framework, we develop a computationally efficient recursive QR scheme for adaptive equalization of time-varying MIMO systems and present simulation results confirming its performance.

[3]For reference, a system with a 2.4 GHz carrier frequency, 20 ksps rate, and travelling at 60 mph corresponds to a Doppler rate of around .01.



(a) $fT = .005$. (b) $fT = .01$.

(c) $fT = .03$.

Fig. 1. Comparison of RLS, RQR and MMSE schemes (SER vs. SNR). QPSK signaling with the parameters: $N = 1150$, $t = 2$, $r = 5$, $L = 4$, $\rho = 5$, $\tau = 5$, $\Delta = 1$, $K_{l,i,j} = 10$dB.

7. REFERENCES

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