MIMO BLIND DECONVOLUTION USING SUBSPACE-BASED FILTER DEFLATION

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ABSTRACT

We solve the overdetermined MIMO Blind Deconvolution problem for independent, stationary, and temporally white sources. Our approach is based on the SVD analysis of the output auto-correlation matrices at different time lags. The key idea is the identification of suitable subspace projectors which achieve an effective reduction of the filter length. We refer to this process as "filter deflation". Recursive applications of the filter deflation transform simplify the problem into an instantaneous BSS problem.

1. INTRODUCTION

Blind Source Separation (BSS) refers to the estimation of n unknown independent signals, given just a set of mixtures observed at m sensors. The fact that the underlying mixing operator is unknown explains the use of the term "*blind*". According to the mixing process we can divide models into memoryless linear mixture BSS (also known as instantaneous BSS) and convolutive mixture BSS (also referred to as MIMO Blind Deconvolution/Equalization). In the instantaneous case the mixing operator is a constant matrix and there is no time shift of the source signals. Instantaneous BSS can't tolerate source multipath dispersion caused by reflections from obstacles between the source and the observation. Such multipath phenomena often appear in many applications in mobile communications, acoustics etc. Those cases are modeled by MIMO systems and are the subjects of the convolutive BSS problem. In the past, BSS was treated using either second-order statistics (SOS) [1, 2, 3] and or higher-order statistics (HOS) [4, 5, 6]. Most Blind Deconvolution approaches can be classified according to their processing domain: time [6, 7, 8] or frequency [9, 10]. From the algorithmic point of view methods can be grouped as iterative [6, 8] or batch [7, 9].

In 1995 Moulines *et al.* [11] investigated the eigenspace of the observation covariance matrix. They showed that the eigenvectors of a SIMO system corrupted with white noise can be used in order to identify it up to a multiplicative constant. The extension of this approach to a MIMO problem was proposed in [12]. In [13] Chevreuil and Loubaton reduced the MIMO problem into multiple SIMO problems by multiplying each observation by a complex exponential at a characteristic frequency (the conjugate cyclic frequency of each observation). Ma *et al.* [14] proposed a second order algorithm using the generalized eigenvalue decomposition of a matrix pencil formed by output auto-correlation matrices at different time-lags. This approach assumes that the sources have distinct, nonstationary, colored power spectral densities.

In this paper we investigate the MIMO Blind Deconvolution problem for independent, stationary, and temporally white sources. We propose a new method based on Second-Order Statistics which uses auto-correlation matrices of the output at different time lags. The key idea is the concept of "filter deflation", i.e. the effective reduction of the filter length achieved by suitable subspace projections. The proper subspace projector is identified using the SVD analysis of the output auto-correlation matrices. Necessary condition is the existence of more observations than sources. We show that recursive filter deflation reduces the MIMO Blind Deconvolution into instantaneous BSS, provided that sufficient observations are available.

2. MATHEMATICAL APPROACH

The problem formulation is described below. Consider a MIMO system involving n independent source signals $s_1(k)$, $\cdots$, $s_n(k)$, and m observations $x_1(k), \cdots, x_m(k)$, $(m > n)$ including additive white noise $v_1(k), \cdots, v_m(k)$

$$x_i(k) = \sum_{l=0}^{L} \sum_{j=1}^{n} a_{ij}(l) s_j(k-l) + v_i(k) \qquad (1)$$

or

$$\mathbf{x}(k) = \sum_{l=0}^{L} \mathbf{A}(l) \mathbf{s}(k-l) + \mathbf{v}(k) \qquad (2)$$

where we used the obvious definitions for the source, observation, and noise vectors $\mathbf{s}(k)$, $\mathbf{x}(k)$, and $\mathbf{v}(k)$, respectively. Clearly, the taps $\mathbf{A}(0)$, ..., $\mathbf{A}(L)$, of the above MIMO FIR filter are in the form of $m \times n$ matrices. We will assume, in general, that the sources and the filter are complex as in a typical multipath situation in digital communications.

We assume that the sources are mutually independent and temporally white

$$E\{s_i(p)s_j(q)^*\} = 0 \qquad i \neq j, \text{ any } p, q \tag{3}$$
$$E\{s_i(k)s_i(k-l)^*\} = 0 \qquad \text{any } i, k, l \tag{4}$$

where * denotes the complex conjugate. Without loss of generality we may also assume that the sources are normalized to unit variance

$$E\{|s_i(k)|^2\} = 1 \qquad i = 1...n\ . \tag{5}$$

Therefore the time-lagged auto-correlation matrices for the source vector $\mathbf{s}$ have the properties

$$\mathbf{R}_s(l) = E\{\mathbf{s}(k)\mathbf{s}(k-l)^H\} = \mathbf{0}\ , \tag{6}$$
$$\mathbf{R}_s(0) = E\{\mathbf{s}(k)\mathbf{s}(k)^H\} = \mathbf{I}\ . \tag{7}$$

The noise is independent to the sources and has variance

$$E\{\mathbf{v}(k)\mathbf{v}(k)^H\} = \sigma^2\mathbf{I}\ .$$

The proposed approach is based in the analysis of the time-lagged auto-correlation matrices of the observation vector $\mathbf{x}$.

Computing the series of the output auto-correlation matrices for different time-lags we can estimate the filter length L by observing that $\mathbf{R}_x(L+1)$ has all eigenvalues equal to σ^2 while the same is not true for $\mathbf{R}_x(L)$. Indeed,

$$\mathbf{R}_x(L+1) = E\{\mathbf{x}(k)\mathbf{x}(k-(L+1))^H\} = \sigma^2\mathbf{I}\ , \tag{8}$$

$$\begin{aligned}\mathbf{R}_x(L) &= E\{\mathbf{x}(k)\mathbf{x}(k-L)^H\} \\ &= \mathbf{A}(L)\mathbf{A}(0)^H + \sigma^2\mathbf{I}\ .\end{aligned} \tag{9}$$

By the same token we get an estimate $\hat{\sigma}^2$ of the noise variance. We can then subtract $\hat{\sigma}^2\mathbf{I}$ from all auto-correlations to remove the effect of noise:

$$\tilde{\mathbf{R}}_x(l) = \mathbf{R}_x(l) - \hat{\sigma}^2\mathbf{I}\ . \tag{10}$$

Now recall that $m > n$, thus for any l, $\mathbf{A}(l)$ is a "tall" matrix. Let

$$\mathbf{A}(l) = \mathbf{U}_A(l)\Sigma_A(l)\mathbf{V}_A(l)^H$$

be the "economy-size" SVD of $\mathbf{A}(l)$, so $\Sigma_A(l)$ is a $n \times n$ diagonal matrix involving only the non-zero singular values, and the $m \times n$ and $n \times n$ matrices $\mathbf{U}_A(l)$, $\mathbf{V}_A(l)$ involve only those singular vectors associated with non-zero singular values. We therefore have

$$\tilde{\mathbf{R}}_x(L) = \mathbf{U}_A(L)\Sigma_A(L)\mathbf{V}_A(L)^H\mathbf{V}_A(0)\Sigma_A(0)\mathbf{U}_A(0)^H\ . \tag{11}$$

We notice that the left null space of $\tilde{\mathbf{R}}_x(L)$ is the same as the null space of $\mathbf{U}_A(L)$ since any vector $\mathbf{z}$ that yields $\mathbf{z}^H\mathbf{U}_A(L) = 0$ will also produce $\mathbf{z}^H\tilde{\mathbf{R}}_x(L) = 0$. The same relationship holds between the right null space of $\tilde{\mathbf{R}}_x(L)$ and the null space of $\mathbf{U}_A(0)$. Consequently, if

$$\tilde{\mathbf{R}}_x(L) = \mathbf{U}_x(L)\Sigma_x(L)\mathbf{V}_x(L)^H$$

is the economy-size SVD of $\tilde{\mathbf{R}}_x(L)$ we must have

$$\text{span}\{\mathbf{U}_x(L)\} = \text{span}\{\mathbf{U}_A(L)\}\ , \tag{12}$$
$$\text{span}\{\mathbf{V}_x(L)\} = \text{span}\{\mathbf{U}_A(0)\}\ . \tag{13}$$

2.1. Filter deflation by subspace projection

Let us define

$$\mathbf{Q}(L) = [\mathbf{U}_x(L) \mid \mathbf{V}_x(L)] \tag{14}$$

and take the orthogonal projector

$$\mathbf{P}(L) = \mathbf{I} - \mathbf{Q}(L)\mathbf{Q}(L)^+ \tag{15}$$

where the superscript $^+$ denotes the pseudo-inverse operator. According to our previous discussion $\mathbf{P}(L)$ projects data onto the subspace orthogonal to both $\mathbf{U}_A(L)$ and $\mathbf{U}_A(0)$. Therefore the projection $\mathbf{P}(L)\mathbf{x}(k)$ should "kill" the taps $\mathbf{A}(0)$, $\mathbf{A}(L)$ thus *deflating* the MIMO filter to a length $L' = (L+1) - 2$ (less 2 than the original length). In other words,

$$\begin{aligned}\mathbf{x}'(k) &= \mathbf{P}(L)\mathbf{x}(k) \\ &= \mathbf{A}'(1)\mathbf{s}(k-1) + \cdots + \mathbf{A}'(L-1)\mathbf{s}(k-L+1) \\ &\quad + \mathbf{P}(L)\mathbf{v}(k)\end{aligned} \tag{16}$$

where $\mathbf{A}'(l) = \mathbf{P}(L)\mathbf{A}(l)$. This result can be used to set a recursive mechanism into motion. If $m > 3n$ the resulting taps $\mathbf{A}'(l)$ will have rank $r > n$ since $2n$ dimensions have been removed by the projection. Similar steps as above can be repeated in order to further deflate the filter down to a length less 2 than before. In fact the projected data $\mathbf{x}'(k)$ need not be computed. One only needs to obtain the new auto-correlation

$$\tilde{\mathbf{R}}_{x'}(L-2) = \mathbf{P}(L)\tilde{\mathbf{R}}_x(L-2)\mathbf{P}(L)^H = \mathbf{A}'(L-1)\mathbf{A}'(1)^H$$

and compute the economy-size SVD

$$\tilde{\mathbf{R}}_{x'}(L-2) = \mathbf{U}_{x'}(L-2)\Sigma_{x'}(L-2)\mathbf{V}_{x'}(L-2).$$

The new projector $\mathbf{P}(L-2) = \mathbf{I} - \mathbf{Q}(L-2)\mathbf{Q}(L-2)^+$, where $\mathbf{Q}(L-2) = [\mathbf{U}_{x'}(L-2) \mid \mathbf{V}_{x'}(L-2)]$, will now "kill" the taps $\mathbf{A}'(L-1)$, $\mathbf{A}'(1)$.

In general, if

$$m > (2K+1)n$$

then the filter-deflation procedure can be repeated K times reducing the filter to a length $(L+1) - 2K$.

We consider separately two different cases

- If L is even, then we repeat the filter-deflation procedure $K = L/2$ times reducing the problem to a memoryless (instantaneous) BSS problem.
- If L is odd, then we repeat the deflation procedure $K = (L-1)/2$ times reducing the filter to length 2. Now we are left with the auto-correlation

  $$\tilde{\mathbf{R}}_{x'}(1) = \mathbf{A}'(L-K)\mathbf{A}'(K)^H \ .$$

  If we remove both $\mathbf{A}'(L-K)$ and $\mathbf{A}'(K)^H$ there will be no more taps left. In order to kill only one tap, instead of the usual 2, we use a "small" projection $\mathbf{P} = \mathbf{I} - \mathbf{U}_{x'}(1)\mathbf{U}_{x'}(1)^T$. This will remove only $\mathbf{A}(L-K)$ and it will reduce the length of the filter by 1. The problem is again transformed into an instantaneous BSS problem

In either case, the problem is reduced into an instantaneous BSS, and methods such as ICA can be applied to obtain estimates of the sources.

2.2. Discussion

The method offers a second-order, recursive, closed-form reduction of the blind MIMO deconvolution problem to an instantaneous blind signal separation problem. The latter is much easier to solve and there is a rich literature of methods for approaching the problem. A significant advantage over other MIMO methods is that the algorithm is computationally very attractive. It is basically consisting of a small number of SVD computations followed by subspace projections. There is no specific cost function to be optimized or any iterative/nonlinear optimization to be performed. In simulation, almost all the computation effort is consumed in the calculation of the auto-correlations, while the SVD steps are instantaneous in comparison.

A drawback of the method is the large number of observations required. For a filter with $(L+1)$ taps we need at least $m = (L+1)n$ observations so that, after killing L taps, the last remaing tap has rank at least n. This, of course, implies extensive oversampling (spatial or temporal). However, with todays' extremely high processing speeds and cheap memory costs temporal oversampling becomes less of a problem in many applications.

Moreover, the deflation procedure per se can be used as a preprocessing tool for effectively reducing the length of the filter. This can help other MIMO methods whose performance is heavily influenced by the filter size. In particular, if $m > (K+1)n$ then we can remove K taps by successive projections thus reducing the filter size by K.

3. SIMULATIONS

We have simulated the method in a variety of conditions. The results reported here were produced from a system with 3 independent 16-QAM sources. The complex MIMO filter has length $L = 6$ and was randomly generated. The number of observations is $m = (L+1)n + 1 = 22$. The observation noise power is $\sigma = 0.1$. Initially, the filter was deflated down to length 1 by successive subspace projections. Subsequently, the resulting instantaneous BSS problem was treated using the JADE method [15]. In this particular experiment $N = 5000$ data samples were processed. The deconvolved sources are displayed in Fig. 1. The clustering of the recovered constellation is clear. The rotation of the lattices are due to the unknown complex scale factors multiplying the reconstructed sources. An important point that must be emphasized is the execution speed of the algorithm. Our MATLAB implementation of the whole process took less than 1 second on a 2.4 GHz Pentium-4 computer.

4. CONCLUSION

In this paper we presented a novel second-order method for blindly reducing the length of a MIMO filter by successive subspace projections called "filter deflations". The proper subspace, which is orthogonal to the first and the last filter taps, can be identified by the SVD analysis of the output auto-correlation matrices for different time lags. Recursive applications of the deflation process will shrink the filter, until it is eventually composed of only one matrix. This idea has been applied to the Blind MIMO Deconvolution problem by transforming it into the easier instantaneous Blind Source Separation problem. Subsequently, well-known BSS/ICA methods (such as JADE) can be used to estimate the sources.

The method is computationally very fast. The drawback is the need for a large number of observations. However, the channel deflation idea may be still useful even if not as many observations are available. Provided that $m > (K+1)n$ we can still deflate K taps out of the MIMO filter thus reducing its length and simplifying the problem for subsequent treatment.

5. REFERENCES

[1] A. Belouchrani, K. Abed-Meraim, J.-F. Cardoso, and E. Moulines, "A Blind Source Separation Technique Using Second-Order Statistics," *IEEE Trans. Signal Processing*, vol. 45, no. 2, pp. 434–444, Feb. 1997.

[2] K.I. Diamantaras, "Asymmetric PCA Neural Networks for Adaptive Blind Source Separation," in *Proc. IEEE Workshop on Neural Networks for Signal Processing (NNSP'98)*, Cambridge, UK, 1998, pp. 103–112.

[3] K.I. Diamantaras and Th. Papadimitriou, "Oriented PCA and blind signal separation," in *Proc. 4th Int.*

Symposium on Independent Component Analyis and Blind Signal Separation, Nara, Japan, April 1-4 2003.

[4] J.-F. Cardoso, "Source separation using higher order moments," in *Proc. IEEE ICASSP,*, Glasgow, U.K., 1989, vol. 4, pp. 2109–2112.

[5] P. Comon, "Independent component analysis, a new concept?," *Signal Processing*, vol. 36, pp. 287–314, 1994.

[6] C. B. Papadias, "Globally convergent blind source separation based on a multiuser kurtosis maximization criterion," *IEEE Trans. on Signal Processing*, vol. 48, no. 12, pp. 3508–3519, Dec 2000.

[7] J.K. Tugnait, "On blind separation of convolutive mixtures of independent linear signals," in *Proc. 8th Signal Processing Workshop on Statistical Signal and Array Processing*, Corfu, Greece, 24-26 Jun 1996, pp. 312–315.

[8] Y. Inouye and T. Sato, "Iterative algorithms based on multistage criteria for multichannel blind deconvolution," *IEEE Trans. on Signal Processing*, vol. 47, no. 6, pp. 1759–1764, Jun 1999.

[9] D. Yellin and E. Weinstein, "Criteria for multichannel signal separation," *IEEE Trans. on Signal Processing*, vol. 42, no. 8, pp. 2158–2168, Aug 1994.

[10] K.I. Diamantaras, A.P. Petropulu, and B. Chen, "Blind two-input two-output fir channel identification based on second-order statistics," *IEEE trans. Signal Processing*, vol. 48, no. 2, pp. 534–542, Feb 2000.

[11] E. Moulines, P. Duhamel, J.-F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel fir filters," *IEEE Trans. Signal Processing*, vol. 43, no. 2, pp. 516–525, Feb 1995.

[12] Ph. Loubaton and E. Moulines, "On blind multiuser forward link channel estimation by the subspace method: Identifiability results," *IEEE Trans. Signal Processing*, vol. 48, no. 8, pp. 2366–2376, Aug 2000.

[13] A. Chevreuil and Ph. Loubaton, "Mimo blind second-order equalization method and conjugate cyclostationarity," *IEEE Trans. Signal Processing*, vol. 47, no. 2, pp. 572–578, Feb 1999.

[14] C. T. Ma, Z. Ding, and S. F. Yau, "A two-stage algorithm for mimo blind deconvolution of nonstationary colored signals," *IEEE Trans. Signal Processing*, vol. 48, no. 4, pp. 1187–1192, Apr 2000.

[15] J.-F. Cardoso and A. Souloumiac, "Blind beamforming for non Gaussian signals," *IEE Proceesings-F*, vol. 140, no. 6, pp. 362–370, Dec 1993.

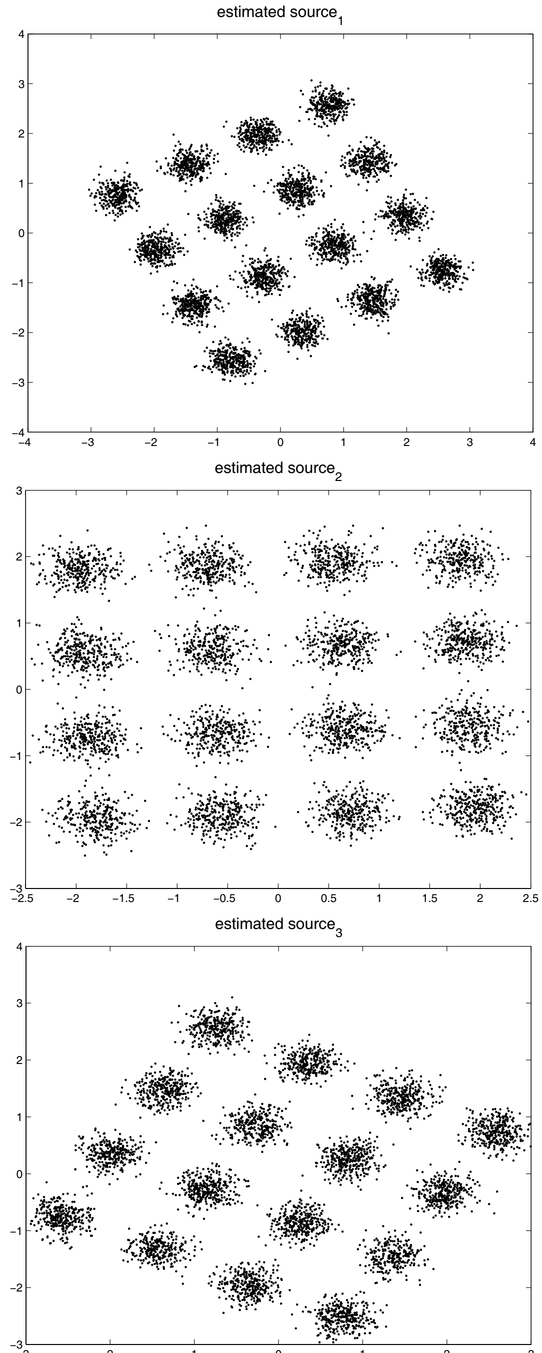

Fig. 1. Performance of the method for $n = 3$ 16-QAM sources, filter length $L = 6$ and $m = 22$ observations.