WHAT IS THE PRICE PAID FOR SUPERIMPOSED TRAINING IN OFDM? *

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) transmission with superimposed training is considered in this paper. The superimposed training scheme is promoted for its high bandwidth efficiency, low computational complexity, and possibly improved power amplifier (PA) efficiency. Channel equalization is also straightforward thanks to the OFDM structure. By analyzing the peak-to-average power ratio (PAR) of the superimposed OFDM signal and utilizing a peak power constraint, we demonstrate that it is possible to lose a little in the information signal power, but gain a lot in the power that is devoted to channel sounding.

1. INTRODUCTION

High-speed data transmission is a challenging problem in wireless communications. A major difficulty is the inter-symbol-interference (ISI) caused by multipath fading.

Orthogonal Frequency Division Multiplexing (OFDM) decomposes a wideband channel into a set of independent narrowband channels so that the frequency selective channel appears flat on each subcarrier. With respect to channel estimation, pilot-symbol assisted modulation (PSAM) is commonly employed in OFDM systems whereby training pilots are inserted in the frequency (and/or time) grids of the OFDM symbols. However, these training pilots consume valuable bandwidth and reduce the data rate. On the other hand, superimposed training has also been considered: training pilots are added onto the time-domain information signal and the channel is estimated without sacrificing the data rate.

The idea of superimposed training; i.e., simultaneous information transfer and channel sounding, was first described in a 1965 paper [1], albeit for analog communication. It was advocated for digital communication systems by Farhang-Boroujeny in 1995 [2] and more investigations followed in [3–6]. Recently, the superimposed training framework has been generalized to include precoding (affine precoding) and has attracted much interest [7, 8].

Superimposed training for OFDM has the following advantages: (i) no loss of information rate as compared to PSAM; (ii) very simple channel estimation; (iii) one-tap equalization thanks to the OFDM structure; (iv) possibility for improved power efficiency at the transmitter. Points (i)-(iii) are well documented in the literature [2–8]. The purpose of this paper is to bring about (iv), by examining the average transmit power of a power amplifier (PA) under a peak power constraint. We do so by deriving the complementary Cumulative Distribution Function (CCDF) of the peak-to-average power ratio (PAR) of the OFDM signal with superimposed training. We will show that by judiciously selecting the superimposed training sequence, we may sacrifice a little in the power that is dedicated to the information signal, but gain a lot in the power that is devoted to channel sounding, for the same amount of DC power consumed by a class A or light class AB PA. Such an "uneven" (and favorable!) trade-off is possible because with superimposed training, the average output power can be increased without increasing the DC or peak power, rendering the PA as more efficiently utilized.

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2. SUPERIMPOSED TRAINING

Consider a block fading channel $y(n) = x(n) * h(n) + v(n)$, where $x(n)$ is the transmitted signal, $y(n)$ is the received signal, $h(n)$ is the impulse response that is the convolution of the transmit filter, the frequency-selective channel, and the receive filter, $v(n)$ is the additive noise, and $*$ denotes convolution. We have omitted the block index in the above equation for notational simplicity. Without knowing $x(n)$, trying to estimate $h(n)$ from $y(n)$ constitutes a blind channel estimation problem. Interestingly, it is shown in [2–8] that by allowing a known $p(n)$ to become part of $x(n)$; i.e., $x(n) = s(n) + p(n)$, where the zero-mean $s(n)$ is still unknown, it is now possible to uncover $h(n)$ from $y(n)$. This is referred to as superimposed training. Writing

$$\begin{aligned} y(n) &= p(n) * h(n) + s(n) * h(n) + v(n) \\ &= p(n) * h(n) + u(n), \end{aligned} \quad (1)$$

we have $u(n) = s(n) * h(n) + v(n)$, which has zero-mean.

We realize from (1) that we can view $p(n)$ as the input, and $y(n)$ as the output of the same channel $h(n)$, and solve for $h(n)$ using linear least squares. Therefore, with superimposed training, we can estimate the channel "blindly" from $y(n)$ without complete knowledge about $x(n)$. An especially simple method was described in [3], where a periodic impulse train $p(n) = a \sum_l \delta(n - lP)$ was used, and when the period $P \geq L$ (the channel length), $\{h(n)\}_{n=0}^{L-1}$ can be estimated as $\frac{1}{aM} \sum_{l=0}^{M-1} y(n + lP)$ involving only a first-order statistic! Many $p(n)$ sequences can be utilized; $p(n)$ with a constant $|p(n)|$ is a popular choice.

One major advantage of the superimposed training method is that it incurs no loss of information rate. Since part of the transmitted power is diverted to the training signal $p(n)$, it is generally expected that for a fixed total average transmit power $\mathcal{P}_t$, superimposed training will perform much worse than traditional training in terms of BER, as a price paid for the higher information rate.

In this paper, we take a novel standpoint that, to compare various transmission methods, it is perhaps better to consider the DC power drawn by the PA as fixed. In that context, we can select a $p(n)$ such that $\mathcal{P}_t$ of the superimposed OFDM signal is larger than the $\mathcal{P}_t$ of the OFDM signal alone, for the same $\mathcal{P}_{dc}$. Therefore, some diversion of $\mathcal{P}_t$ to $p(n)$ can be offset by a larger increase in $\mathcal{P}_t$ itself, making superimposed training more appealing than what is already known in the literature.

3. THE DISTRIBUTION OF PAR

To start, let us consider the following question: For a fixed amount of power $\mathcal{P}_{dc}$ consumed by the PA, what is the average power $\mathcal{P}_t$ that is transmitted through the channel? Obviously, the more power that is delivered to the load for a given level of DC power (e.g., battery drain), the better. Fixing $\mathcal{P}_{dc}$ is equivalent to fixing the peak power $\mathcal{P}_{\max}$ of a class A or light class AB PA [9].

Consider for simplicity, a completely linear PA so the maximum output power $\mathcal{P}_{\max}$ can be reached without incurring nonlinear distortions. Although the PAR is a quantity that is used to characterize the PA input, whereas $\mathcal{P}_{\max}$ and $\mathcal{P}_t$ are PA output properties, in the case of a linear PA, $\mathcal{P}_{\max}/\mathcal{P}_t$ =PAR, where PAR is defined as

$$\text{PAR} = \frac{\max_n\{|x(n)|^2\}}{E\{|x(n)|^2\}}. \tag{2}$$

Therefore, for a given $\mathcal{P}_{\max}$, reducing the PAR is equivalent to increasing $\mathcal{P}_t$. In the PAR reduction literature, attention has been on peak power reduction methods. We argue that since the PA in the communication system is peak power limited, and its efficiency is determined by the average output power, it makes sense to investigate PAR reduction methods that aim at increasing the average power while keeping the peak power fixed.

In this paper, we will show that it is possible to achieve PAR reduction with superimposed training in OFDM. The possibility for PAR reduction, together with the simple channel estimation procedure offered by the superimposed training scheme, as well as simple channel equalization that is the hallmark of OFDM, make superimposed OFDM a promising transmission technique.

3.1. CCDF of PAR for Superimposed OFDM

Assume without loss of generality that $S(k)$ are i.i.d., drawn from a known constellation (e.g., QPSK) with variance σ_s^2. When the number of subcarriers N is large, we can use the Central Limit Theorem to show that the time-domain OFDM signal $s(n)$ is approximately i.i.d. complex Gaussian distributed with zero-mean and variance σ_s^2 [10]. It follows that $x(n) = s(n) + p(n)$ is independent complex Gaussian distributed with time-varying mean $p(n)$ and variance σ_s^2.

For simplicity, we do not consider the cyclic prefix in the PAR analysis since the cyclic prefix does not affect the peak or the average power. Denote by $r(n)$ the instantaneous power of $x(n)$; i.e., $r(n) = |x(n)|^2$. We infer that $r(n)$ has a noncentral chi-square distribution with 2 degrees of freedom and noncentrality parameter $|E[x(n)]|^2 = |p(n)|^2$.

The CDF of $r(n)$ is

$$F_{R_n}(r) = 1 - e^{-\frac{|p(n)|^2+r}{\sigma_s^2}} \sum_{k=0}^{\infty} (\frac{|p(n)|}{\sqrt{r}})^k I_k(\frac{|p(n)|\sqrt{r}}{\sigma_s^2/2}), \tag{3}$$

where $I_k(a)$ is the k-th order modified Bessel function of the first kind, which may be represented by the infinite series

$$I_k(a) = \sum_{n=0}^{\infty} \frac{(a/2)^{2n+k}}{n!\ (n+k)!}, \tag{4}$$

for $a \geq 0$.

The average power of $x(n)$ is given by $E\{|x(n)|^2\} = \sigma_s^2 + \sigma_p^2$, where $\sigma_p^2 = \frac{1}{N}\sum_{n=0}^{N-1} |p(n)|^2$ is the average power of the superimposed training sequence $p(n)$.

Therefore, the CCDF of the PAR of $x(n)$ can be expressed as

$$\begin{aligned} \Pr\{\text{PAR} > \gamma\} &= 1 - \Pr\{r(n) \leq \gamma(\sigma_s^2 + \sigma_p^2),\ \forall n\} \\ &= 1 - \prod_{n=0}^{N-1} F_{R_n}\{\gamma\ (\sigma_s^2 + \sigma_p^2)\}, \end{aligned} \tag{5}$$

by independence of $r(n)$ at different n's.

3.2. Periodic Pilot Sequence

Next, we investigate the CCDF of the PAR for the superimposed OFDM signal with a periodic $p(n)$. The idea of exploiting the cyclostationarity induced by the periodic $p(n)$ for simple channel estimation is discussed in [3–6].

Since the noncentrality parameter of $r(n)$ is $|p(n)|^2$, the CDF $F_{R_n}(r)$ is also periodic in n. Eq. (5) can thus be simplified to:

$$\Pr\{\text{PAR} > \gamma\} = 1 - \left(\prod_{n=0}^{P-1} F_{R_n}\{\gamma\ (\sigma_s^2 + \sigma_p^2)\}\right)^M, \tag{6}$$

where for simplicity, we have assumed that $M = N/P$ is an integer. Next, we specialize to the case where $p(n)$ is the periodic impulse train used in [3].

Consider frequency-selective block fading channels. It was shown in [8, 11, 12] that the optimal placement of pilot tones for PSAM is to modulate L pilots with equal power onto equally spaced subcarriers (i.e., insert pilots periodically in the frequency domain). Note that periodic insertion of pilot tones in the frequency domain with period M is equivalent to periodic superposition of an impulse sequence in the time domain with period $P = L$.

To utilize the bandwidth more efficiently, we proposed in [4] to modulate information sub-symbols onto each subcarrier, but superimpose the periodic impulse sequence in the time domain. In this section, we characterize the resulting PAR by its CCDF and show that it is a function of the period P, the block length N, and the power allocation factor

$$\beta = \frac{\sigma_p^2}{\sigma_s^2 + \sigma_p^2}. \tag{7}$$

Denote by $p_1(n)$ the periodic impulse sequence, $p_1(n) = \sqrt{P}\sigma_p \sum_l \delta(n - lP)$. Substituting it into (3), replacing $I_k(a)$ by its asymptotic form $e^a/\sqrt{2\pi a}$ for $k \ll a$, and simplifying the resulting equation for $0 < \beta < 1$ and $\gamma \gg \beta$ (i.e., PAR $\gg 1$, which is true for OFDM), we obtain an approximate close-form expression

$$\begin{aligned} \Pr\{\text{PAR} > \gamma\} \;=\; & 1 - \left(1 - e^{-\frac{\gamma}{1-\beta}}\right)^{N-M} \\ & \left(1 - \frac{\sqrt{(1-\beta)\sqrt{\gamma}/(2\pi\sqrt{P\beta})}}{\sqrt{\gamma} - \sqrt{P\beta}} e^{-\frac{P\beta+\gamma}{1-\beta}}\right)^M. \end{aligned} \tag{8}$$

A perhaps natural reaction is that adding $p_1(n)$ to $s(n)$ is bound to increase the PAR of the resulting $x(n) = s(n) + p_1(n)$. As we will see in Section 5.1., this is not necessarily the case. The PAR of $p_1(n)$ equals P (a constant), whereas the PAR of $s(n)$ increases with increasing N. For example, it can be shown that the median PAR of $s(n)$ is 6 (7.7 dB) for $N = 256$; thus with $P \leq 6$, the PAR of $p_1(n)$ is comparable with that of $s(n)$.

3.3. Constant Magnitude Pilot Sequence

For fast fading channels, it is reasonable to consider $p(n)$ with constant $|p(n)|$. It is shown in [5] and [6] that periodic $p(n)$ sequences with constant $|p(n)|$ provided low mean-squared error of the channel estimate and desirable receiver BER performance.

When $|p(n)|$ is constant, such as when $p(n)$ is a polynomial phase signal; e.g., $p(n) = \sigma_p \; e^{jan^m}$ (monomial phase signal here), $F_{R_n}(r)$ is not a function of n; thus, the CCDF is determined as:

$$\Pr\{\text{PAR} > \gamma\} = 1 - \left(1 - e^{-\frac{\gamma+\beta}{1-\beta}} \sum_{k=0}^{\infty} (\sqrt{\beta/\gamma})^k I_k(\frac{2\sqrt{\gamma\beta}}{1-\beta})\right)^N .$$

When $\beta = 0$; i.e., $p(n) = 0$, the above CCDF expression reduces to

$$\Pr\{\text{PAR} > \gamma\} = 1 - (1 - e^{-\gamma})^N, \quad (9)$$

which is commonly cited in the literature as the CCDF of the PAR of the OFDM signal. If $0 < \beta < 1$ and $\beta \ll \gamma$, we can replace $I_k(a)$ by its asymptotic form $e^a/\sqrt{2\pi a}$ for $k \ll a$ and simplify to obtain the close-form expression (proof is in [13], but is omitted here due to the space limitation)

$$\Pr\{\text{PAR} > \gamma\} = 1 - \left(1 - \sqrt{\frac{1-\beta}{4\pi}} \; e^{-\frac{(\sqrt{\gamma}-\sqrt{\beta})^2}{1-\beta}}\right)^N . \quad (10)$$

4. AVERAGE TRANSMIT POWER FOR SUPERIMPOSED OFDM

In this section, we limit ourselves to the case where $p(n)$ has constant magnitude. We will show that a single parameter β influences $\mathcal{P}_t$ and the power that is allocated to transmitting the information signal, $\mathcal{P}_s = \mathcal{P}_t(1-\beta)$.

Since PAR is a random quantity, we first determine a "representative" PAR value γ_0 based on the CCDF expression in (10). For a given probability p (e.g. $p = 10^{-2}$), solve for γ_0 by setting the right hand side (RHS) of (10) equal to p. We then obtain

$$\gamma_0 = \left(\sqrt{\beta} + \sqrt{(1-\beta)\ln\{\sqrt{(1-\beta)/4\pi}/(1-(1-p)^{\frac{1}{N}})\}}\right)^2 .$$

The average transmit power for superimposed OFDM with constant $|p(n)|$ can then be expressed as

$$\mathcal{P}_t = \frac{\mathcal{P}_{\max}}{\gamma_0}. \quad (11)$$

The power that is devoted to the information sequence is $\mathcal{P}_s = \mathcal{P}_t(1-\beta)$.

For the OFDM signal $s(n)$, by setting the RHS of (9) to p, we can also obtain a "representative" PAR γ_0' for the OFDM signal $s(n)$:

$$\gamma_0' = -\ln(1-(1-p)^{1/N}). \quad (12)$$

The average transmit power for $s(n)$ is then

$$\mathcal{P}_t' = \mathcal{P}_s' = \frac{\mathcal{P}_{\max}}{\gamma_0'}. \quad (13)$$

Since $\gamma_0 < \gamma_0'$ generally, we will have $\mathcal{P}_t > \mathcal{P}_t'$.

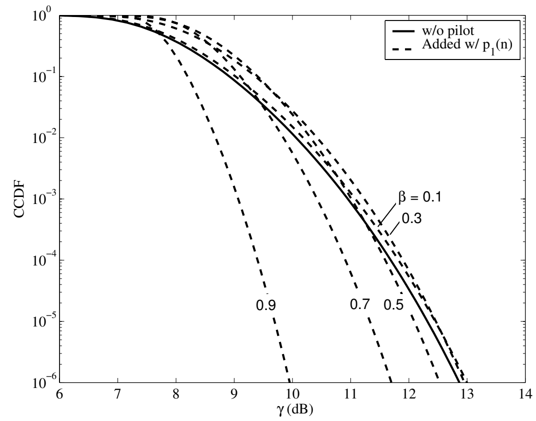

Figure 1. CCDF of PAR of $x(n) = s(n) + p_1(n)$.

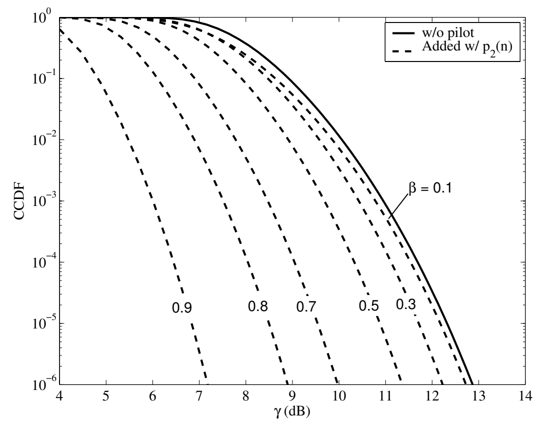

Figure 2. CCDF of PAR of $x(n) = s(n) + p_2(n)$.

5. NUMERICAL EXAMPLES

In this section, the number of sub-carriers in each OFDM block is $N = 256$, and the period of $p(n)$ is $P = 4$. We consider two superimposed training sequences:

$$p_1(n) = \sqrt{P}\sigma_p \sum_l \delta(n - lP), \quad 0 \le n \le N-1, \quad (14)$$

$$p_2(n) = \sigma_p \; e^{j\frac{\pi}{4}} \; e^{-j\frac{\pi n^2}{4}}, \quad 0 \le n \le N-1. \quad (15)$$

5.1. PAR and Power Allocation

First, we would like to compare the PAR of the OFDM signal $s(n)$ with that of $s(n) + p_i(n)$, $i = 1, 2$, for varying values of the power allocation factor β, by fixing $\sigma_s^2 = 2$.

Interestingly, Fig. 1 shows that the superposition of $p_1(n)$ onto $s(n)$ can reduce the PAR of the OFDM signal $s(n)$! This is because $p_1(n)$ has a PAR of $10\log_{10}(P) \approx 6$ dB, which can be smaller than the PAR of the OFDM signal $s(n)$.

Fig. 2 shows how the PAR is affected by the superposition of a $p(n)$ with constant magnitude; e.g., $p_2(n)$. We can see that the CCDF decreases as β is increased. This is expected since $p_2(n)$ has a PAR of 0 dB, whereas the PAR of the OFDM signal $s(n)$ is much larger.

5.2. Average Output Power and Power Allocation

Fig. 3 shows the increase in the average transmit power $\mathcal{P}_t$ due to the superposition of any constant-magnitude pilot sequence $p(n)$. We have set the baseline $\mathcal{P}_t' = \mathcal{P}_s' = 0$ dB for the average transmit power of the original OFDM signal.

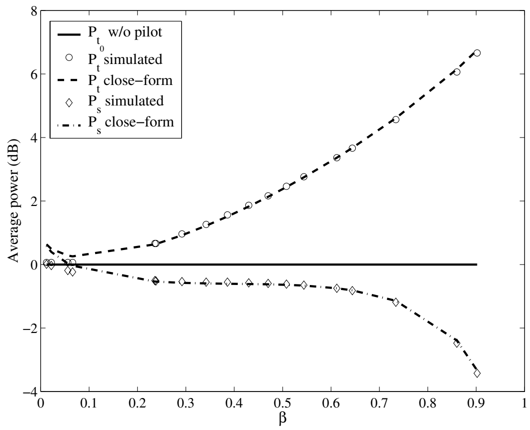

Figure 3. $\mathcal{P}_t$ **and** $\mathcal{P}_s = \mathcal{P}_t(1-\beta)$ **vs.** β.

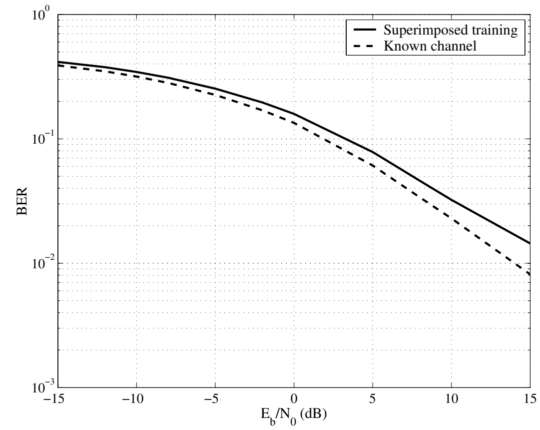

Figure 4. BER performance.

Setting $p = 10^{-2}$, we obtain γ_0 as a function of β only, as well as γ_0'. Since $\mathcal{P}_t\gamma_0 = \mathcal{P}_t'\gamma_0' = \mathcal{P}_{\max}$, we can determine $\mathcal{P}_t$ as a function of β only; $\mathcal{P}_s = \mathcal{P}_t(1-\beta)$ is subsequently determined.

Fig. 3 shows $\mathcal{P}_t$ and $\mathcal{P}_s$ vs. β. From Fig. 3, we observe that with a constant magnitude $p(n)$, $\mathcal{P}_s$ decreases while $\mathcal{P}_t$ increases rapidly as β increases. Moreover, we see that when $0 < \beta < 0.7$, $\mathcal{P}_s$ is reduced by < 1 dB, but $\mathcal{P}_t$ can be increased by up to 4 dB; the extra power can be used to estimate the channel. In addition, the close-form expression in (11) is shown to agree with simulated results using i.i.d. QPSK OFDM symbols.

5.3. BER Performance

In this example, we show the BER performance of the first-order channel estimator followed by a one-tap frequency-domain equalizer. The length of the FIR channel is $L = 4$. The channel coefficients were generated from a zero-mean unit variance complex Gaussian distribution, and 500 independent Monte Carlo runs were performed. The OFDM sub-symbols $\{S(k)\}$ were drawn from an i.i.d. BPSK constellation. The SNR is defined $E_b/N_0 = \mathcal{P}_t/(\sigma_v^2/2)$, where σ_v^2 is the average power of the noise. Fig. 4 shows the BER performance of the superimposed training scheme, followed by a one-tap frequency-domain equalizer. From Fig. 4, we can see that the BER performance is quite close to the known channel case. This example illustrates that a judicious choice of $p(n)$ can trade a small amount of information signal power $\mathcal{P}_s = \mathcal{P}_t(1-\beta)$ for much more training pilot power $\mathcal{P}_t\beta$.

6. CONCLUSIONS

We advocated in this paper, superimposed training with OFDM as an excellent combination that permits simple channel estimation as well as simple channel equalization. We presented a novel viewpoint of communication system performance evaluation with a peak power constraint. By analyzing the CCDF of the PAR of the superimposed OFDM signal and linking the PAR to the average transmit power, we demonstrated that the higher data rate that is inherent with the superimposed training scheme, does not necessarily come with a performance loss in terms of BER.

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