PERFORMANCE ANALYSIS OF NON-LINEARLY AMPLIFIED M-QAM SIGNALS IN MIMO CHANNELS

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ABSTRACT

MIMO systems using multiple transmit and receive antennas on both ends of a linear wireless communications link are by now well studied. Several axes of these schemes have been explored with the underlying MIMO channel assumed linear. In this work, we investigate the effect of Non-linearity in the MIMO channel. New results on symbol error rate performances of several M-QAM constellations in linear and non-linear MIMO channels are presented. The results show that for any MIMO configuration, performance degradation due to non-linearity reduces as the fading gets more severe, and for a particular fading channel, the degradation increases as the MIMO dimension is increased. Optimum ring-ratios for circular QAM constellations in MIMO channels are also reported, and compared with existing results where applicable.

1. INTRODUCTION

Multiple Input-Multiple output (MIMO) systems employing powerful coding and highly bandwidth-efficient modulation techniques is a candidate technology that can meet the High-speed and QoS requirements of future generations of wireless communications networks [2], [8]. To this end, performance behaviors of MIMO systems have been studied quite elaborately. However, to the best of Authors knowledge, all these studies have assumed that the transmit high-power amplifiers (HPA) are operated in the linear region so that the underlying MIMO channel is linear.

Amplifiers usually operate as a linear device under small signal conditions, but become more nonlinear and distorting with increasing drive level. There is therefore a tradeoff between power efficiency and the resulting distortion. In most commercial cellular systems, this tradeoff is constrained by allowable interference; thus, amplifier signal levels (operating points) are greatly reduced (backed off) from the peak efficiency operating point (saturation point). As more powerful signal processing techniques continue to emerge however, there are increasing needs to operate the HPA more efficiently. In the Satellite systems, on-board HPA must be operated at high efficiency close to the saturation point in order to raise the energy of the weak signal received by the transponder to an acceptable level before retransmission. Also, satellite diversity is a common solution for fading in the LEO satellites [5]. This results in a non-linear MISO (Multiple Input-Single Output) system – corresponding to the non-linear MIMO systems with one receive antenna.

***Contribution of paper*:** This paper investigates the performance degradations that will occur when HPA operating near their saturation points are used in a MIMO setup. The paper first extends the works in [3], [4], to the MIMO configurations; both linear and non-linear channels. This extension then enables us to report numerous new results on the system performance degradations that will occur, when the MIMO channel is non-linear. We also report optimum ring-ratios for Star-QAM constellations, as well as optimum operating points for non-linear HPA, in MIMO channels.

2. CHANNEL AND DATA MODEL

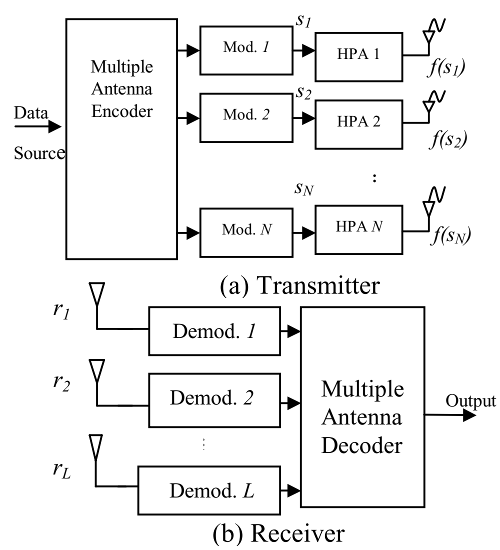

Figure 1: Non-linear MIMO Channel.

Fig.1 displays the block diagram of a non-linear MIMO channel. The received signal from this channel is given by:

$$\boldsymbol{r} = \mathbf{H}\, \boldsymbol{f(s)} + \boldsymbol{n} \quad , \qquad (1)$$

where $\mathbf{H}$ is the L x N MIMO channel matrix given by:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ . & . & \cdots & . \\ h_{L1} & h_{L2} & \cdots & h_{LN} \end{bmatrix},$$

and $\boldsymbol{n}$ is the L x 1 zero-mean additive white Gaussian noise vector. $\boldsymbol{f(s)}$ is a non-linear function of the modulated symbols $\boldsymbol{s}$, introduced by the HPA. For HPA nonlinearity, we have used the AM/AM and AM/PM model given by Saleh [7], and illustrated in Fig. 2 (a), where it is easy to see that the HPA must operate at very low drive level to maintain linear characteristics. To operate the amplifiers more efficiently therefore, some level of non-linearity must be tolerated. In Fig. 2(b), we have shown the distortive effect of non-linear amplifications on the optimum decision regions for 16 QAM constellations.

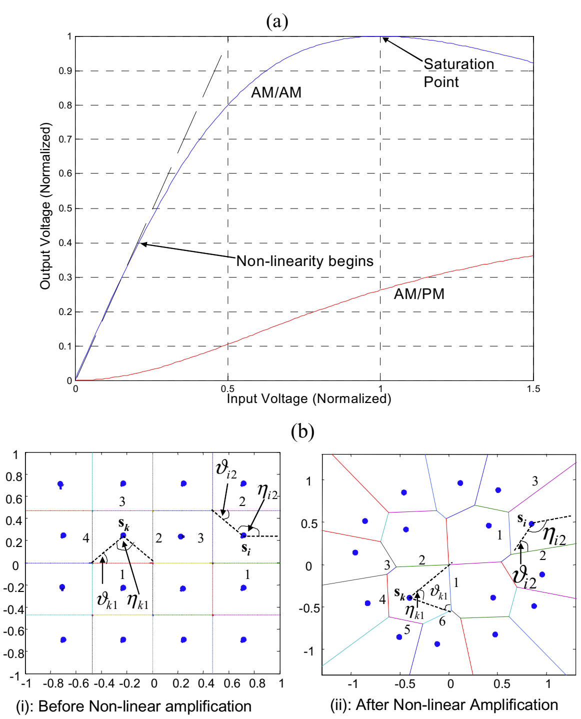

Figure 2: (a) HPA Characteristics (Saleh model [7]), (b) Decision regions for rectangular 16-QAM before and after non-linear amplifications.

3. PROBABILITY OF ERROR ANALYSIS

The error probability for reception in MIMO channel using the Craig's method can be expressed as [4]:

$$P_{MIMO} = \int_0^\infty f_{\gamma_{MIMO}}(\gamma) P_g(\gamma_s) \, d\gamma \ , \qquad (2)$$

where $f_{\gamma_{MIMO}}(\gamma)$ is the pdf of the output SNR from the MIMO channel, γ_{MIMO}. $P_g(\gamma_s)$ is the basic probability expression given by [3]:

$$P_g(\gamma_s) = \frac{1}{2\pi} \int_0^\eta \exp\left[\frac{-a\gamma_s \sin^2 \vartheta}{\sin^2(\theta + \vartheta)}\right] d\theta \qquad (3)$$

where γ_s is the average SNR defined respectively as $\gamma_s = \dfrac{E_{av}}{N_0}$ and $\gamma_s = \dfrac{E_{av-NL}}{N_0}$, for the linear and non-linear channels. E_{av} and E_{av-NL} are the respective average symbol energy before and after the HPA amplifications, and N_0 is the noise power. a is a scaling factor, while θ is a dummy variable of the integration. Parameters η and ϑ are determined by the decision region geometry [3], as illustrated in Fig. 2 (b).

The average symbol error probability (SEP) for M-QAM signals in MIMO channel is then given by:

$$P(E)_{MIMO} = \sum_{i=1}^{M} p(s_i) \sum_{j=1}^{N_{dec-reg-i}} P_{MIMO} \qquad (4)$$

where $p(s_i)$ is the *apriori* probability of sending symbol s_i, M is the constellation size, and $N_{dec\text{-}reg\text{-}i}$ is the total number of decision regions surrounding symbol s_i.

Substituting (3) in (2), and employing the MGF-based analysis [1], yields the following result, which is valid for both Linear and Non-linear MIMO channels:

$$P_{MIMO} = \frac{1}{2\pi} \int_0^\eta \Psi_{\gamma_{MIMO}}\left(\frac{-a\sin^2\vartheta}{\sin^2(\theta+\vartheta)}\right) d\theta, \qquad (5)$$

where $\Psi_{\gamma_{MIMO}}(j\omega)$ denotes the characteristic function of γ_{MIMO}. Next we illustrate the performance of the MIMO systems in the presence of non-linearity using the analysis of MIMO-MRC [6] as case study. For the case of Rayleigh fading, $\Psi_{\gamma_{MIMO}}(j\omega)$ can be derived as [6]:

$$\Psi_{\gamma_{MIMO}}(j\omega) = \frac{b^{NL}}{(b - j\omega)^{NL}}, \qquad (6)$$

where $b = \dfrac{1}{\bar{\gamma}_a}$, and $\bar{\gamma}_a = \gamma_s E\left[|h_{ij}|^2\right]$.

Using (6) and (5), the result for Rayleigh fading case is:

$$P_{MIMO} = \frac{1}{2\pi} \int_0^\eta \left(\frac{b\sin^2(\theta+\vartheta)}{\left(b\sin^2(\theta+\vartheta) + a\sin^2\vartheta\right)}\right)^{NL} d\theta. \ (7)$$

For Ricean fading, $\Psi_{\gamma_{MIMO}}(j\omega)$ is given by:

$$\Psi_{\gamma_{MIMO}}(j\omega) = \frac{b^{NL}}{(b - j\omega)^{NL}} \exp\left\{\frac{j\omega b S}{b - j\omega}\right\}, \quad (8)$$

where $S = \sum_{i=1}^{2NL} \mu_i^2$; μ_i is the mean of the i^{th} LOS component. Using (8) and (5) gives the result as:

$$P_{MIMO} = \frac{1}{2\pi}\int_0^{\eta}\left(\frac{b\sin^2(\theta+\vartheta)}{\left(b\sin^2(\theta+\vartheta) + a\sin^2\vartheta\right)}\right)^{NL} \cdot \exp\left(\frac{-Sab\ \sin^2\vartheta}{\left(a\sin^2\vartheta + b\sin^2(\theta+\vartheta)\right)}\right) d\theta. \quad (9)$$

For Nakagami-*m* fading, $\Psi_{\gamma_{MIMO}}(j\omega)$ is given by:

$$\Psi_{\gamma_{MIMO}}(j\omega) = \left[\frac{m}{m - j\omega\bar{\gamma}_a}\right]^{LNm}. \quad (10)$$

Again using (10) and (5), the following result is obtained for the case of Nakagami-*m* fading:

$$P_{MIMO} = \frac{1}{2\pi}\int_0^{\eta}\left(\frac{\sin^2(\theta+\vartheta)}{\left(\sin^2(\theta+\vartheta) + \frac{\bar{\gamma}_a}{m} a\sin^2\vartheta\right)}\right)^{LNm} d\theta. \quad (11)$$

Using Eq. (7), (9), and (11) in Eq. (4) gives the solutions for the SEP performances of M-QAM signals in linear and nonlinear MIMO channels, for the cases of Rayleigh, Ricean and Nakagami fading respectively. Notice that these results are easily evaluated numerically [3], [4].

4. RESULTS AND DISCUSSIONS

Fig. 3 displays the results for the SEP performance of rectangular 16-QAM signals in MIMO channels for various fading scenarios. It can be observed from these results that given any MIMO configuration (e.g. N=2, L=2), system performance degradation, due to nonlinearity in the MIMO channel, reduces as the fading level gets more severe. The reason for this important result is that at high fading level, fading becomes the dominant factor in the performance deterioration and the effect of nonlinearity becomes less significant. This therefore builds a strong motivation for operating HPA at high efficiency in severe fading scenario. It is also observed from this figure that, for any particular fading channel (e.g. Nakagami *m=0.5*), the performance degradation will increase as the MIMO dimensions are increased. This is somehow expected.

Next we present results on the performances of circular QAM constellations. Noting that SEP performances depend on the average energy of the constellation, which inturn vary as the ring-ratio varies [4], we first used the derived expressions for SEP to numerically search for the optimum ring-ratios for the circular QAM constellations. These results are summarized in Table I. The entries in the first row of Table I(a) correspond to the results in [4], and our results here agree well with theirs. Apart from these entries, other results for the linear channel here are, to the best of Authors knowledge, new. The results in Table I(b) are entirely new. Our results indicate that the optimum ring-ratio only depends on the order (N x L), of the MIMO system and independent of the specific configuration used.

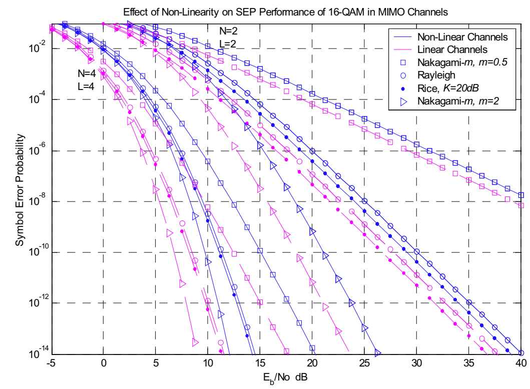

Figure 3: SEP Performance of 16-QAM in Linear and Non-Linear MIMO Channels.

Fig. 4 makes comparisons among the SEP of 16 rectangular-QAM and several circular QAM constellations in linear and non-linear MIMO Channels, for the case of Rayleigh fading. All the results presented in this figure for the circular constellations have been computed using the respective optimum value of the ring-ratio. It is observed from the results that: for the non-linear channels, the performances of the circular QAM constellations are roughly similar. However, their performances are distinctly better than that of the counter-part rectangular QAM constellation. This shows that circular constellations are more suited for use in non-linear channels than the rectangular constellation. For the linear channels however, performance gaps among all the constellations are not as much distinct, especially at high MIMO order.

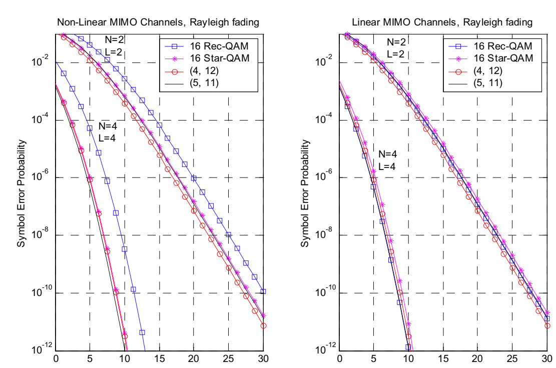

Figure 4: Comparing SEP of rectangular 16-QAM, and circular QAM constellations in MIMO Channels.

A typical performance measure for quantifying the effect of nonlinear distortion is total degradation defined by:

$$TD_{dB} = OBO_{dB} + \Delta_{dB}(SNR), \qquad (12)$$

where OBO (Output Back Off) is the difference between the saturation power and the average power of the transmitted signal: $OBO = 10\log_{10}(E_{sat}) - 10\log_{10}(E_{av-NL})$, and $\Delta_{dB}(SNR)$ represents the SNR degradation due to the non-linear distortions.

Fig. 5 plots the total degradations for various 16-QAM constellations in MIMO channels. It can be observed from these results that optimum operating point (optimum OBO) exists for each constellation and each MIMO configurations. This is the point at which minimum degradation is suffered and highly efficient use of the HPA is achieved. For example for N=2, L=2, the optimum operating points, in dB, are 1.36 and 2.77 for 16 rectangular-QAM and Star-QAM respectively. A comprehensive table, similar to Table I, was developed for these results but has not been included due to space limits.

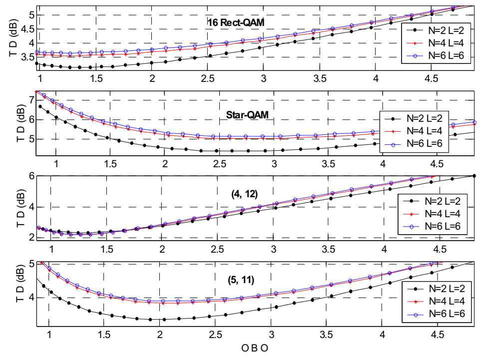

Figure 5: Total Degradation, due to Non-Linearity, in MIMO Channels.

5. CONCLUSIONS

This work investigates the effect of non-linearity in MIMO communication channels. Analytical expressions are derived for evaluating the SEP of several QAM constellations in both linear and nonlinear MIMO channels. The expressions are then used for predicting system performance degradations, due to non-linearity, for various MIMO configurations. Our results indicate that performance degradations due to non-linearity will increase as the fading severity reduces, given any MIMO configuration. However, for a given fading channel, the degradations increase with increasing MIMO dimensions. Results on optimum operating points for non-linear amplifiers in MIMO channels are then shown. Optimum ring-ratios for circular 16-QAM constellations are as well presented.

Acknowledgement: This work was supported by the Premier's Research Excellence Award (PREA), ON, Canada.

Table I: Asymptotically Optimum Ring Ratios for 16 Star-QAM in MIMO Channels, with Rayleigh or Ricean fading.

MIMO CONFIGU-RATIONS	*L = 1*	*L = 2*	*L = 3*	*L = 4*	*L = 5*	*L = 6*
N = 1	1.953	1.878	1.845	1.828	1.818	1.810
N = 2	1.878	1.828	1.810	1.800	1.793	1.790
N = 3	1.845	1.810	1.795	1.790	1.785	1.783
N = 4	1.828	1.800	1.790	1.784	1.780	1.778
N = 5	1.818	1.793	1.785	1.780	1.778	1.778
N = 6	1.810	1.790	1.783	1.778	1.778	1.775

(a) Linear Channels.

MIMO CONFIGU-RATIONS	*L = 1*	*L = 2*	*L = 3*	*L = 4*	*L = 5*	*L = 6*
N = 1	3.533	3.363	3.283	3.240	3.215	3.198
N = 2	3.363	3.240	3.198	3.175	3.160	3.150
N = 3	3.283	3.198	3.165	3.150	3.140	3.133
N = 4	3.240	3.175	3.150	3.138	3.130	3.125
N = 5	3.215	3.160	3.140	3.130	3.123	3.120
N = 6	3.198	3.150	3.133	3.125	3.120	3.115

(b) Non-Linear Channels.

6. REFERENCES

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