ON THE NUMBER OF PILOTS FOR OFDM SYSTEM IN MULTIPATH FADING CHANNELS

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) system has been demonstrated to be effective in a frequency-selective multipath fading environment. To compensate the deleterious effects of the fading channels, pilot symbol assisted modulation (PSAM) technique has been used to obtain the channel estimate. In order to have an accurate estimate, a high percentage of pilot symbols is usually needed and thus reducing the spectrum efficiency. Therefore, the number of pilot symbols is a tradeoff between channel estimation accuracy and bandwidth efficiency. In this paper, we propose a novel method of deciding the optimal number of pilots in OFDM transmission. The method is based on the derivation and analysis of bit error rate (BER) of a PSAM-OFDM system in multipath fading channel in terms of the pilot spacing for a given block size.

1. INTRODUCTION

To compensate the deleterious effects of frequency-selective multipath fading distortion, an accurate channel estimate is necessary for coherent demodulation in OFDM system [1]. Among the various channel estimation methods for OFDM, the PSAM technique has been widely used [2][3]. Pilot symbols are first uniformly inserted into the transmitted data stream. Upon receiving the corrupted pilot symbols at the receiver, the channel state information (CSI) at pilot locations is estimated. The CSI at data locations can then be obtained through interpolation with the pilot channel estimates.

One important problem in PSAM-OFDM system is to determine how many pilot symbols should be used the least for a given BER at a certain channel signal-to-noise ratio (SNR). More pilots will result in better channel estimation, but less bandwidth efficiency. Recently, some researchers studied this problem by maximizing the capacity of the system [4], which provided a useful framework for analyzing the capacity achievability by pilot-based scheme in general. In this paper, we consider the problem from a different viewpoint, namely for a desired BER and a given SNR. We propose a novel method to determine the optimal number of pilots. It is based on the derivation and analysis of BER for a given channel SNR. Through examination of the analytical BER curves with different pilot spacing, the smallest number of pilots can be easily found.

The rest of the paper is organized as follows. In Section 2, the conventional pilot-based OFDM channel estimation is introduced. In Section 3, an analytical BER of PSAM-OFDM system in a fading channel is given. The search of the optimal number of pilots to be used is described in Section 4. Section 5 gives some numerical examples based on the analysis. Finally, Section 6 summarizes the main results.

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2. CHANNEL ESTIMATION FOR OFDM

A typical pilot symbol assisted OFDM channel estimation method is reviewed as follows. Through multipath fading channel, the received OFDM signal can be written as

$$y(t) = \sum_{k=1}^{N_p} h_k(t)x(t-\tau_k) + n_k(t) \tag{1}$$

where $x(t)$ is the transmitted signal, N_p is the total number of transmission paths and $n_k(t)$ is the zero mean additive white Gaussian noise (AWGN). The channel impulse response of the kth path is given by

$$h_k(t) = c_k r_k(t)\delta(t-\tau_k) \tag{2}$$

where c_k and τ_k denote the normalized amplitude and propagation delay for the kth path respectively, and $r_k(t)$ are independent stationary complex zero-mean Gaussian process with unit variance for the kth path.

After removing the guard interval and performing DFT on $y(t)$, we obtain [1]

$$Y(m,n) = H(m,n)X(m,n) + N(m,n) \tag{3}$$

where $X(m,n)$, $Y(m,n)$ and $N(m,n)$ are the transmitted symbol, received symbol and AWGN at the mth subchannel and the nth OFDM symbol interval, respectively. Furthermore, the corresponding frequency response of the channel is given by

$$H(m,n) = \sum_{k=1}^{N_p} h_k(n) \exp \frac{-j2\pi m \tau_k(n)}{T_s} \tag{4}$$

where T_s denotes the duration of one OFDM symbol.

The pilot subchannel can be estimated using the least square method [3]:

$$\hat{H}_p(j,n) = Y_p(j,n)/X_p(j,n) \tag{5}$$

where subscript p denotes the pilot position and j denotes the index of pilot subchannel at the nth OFDM symbol.

The data subchannel information can be estimated by taking interpolation between the pilot subchannel estimates. Here, linear interpolation is used. In linear interpolation, the data subchannel estimate is given by

$$\hat{H}_d(i,n) = (1 - \frac{l}{L})\hat{H}_p(j,n) + \frac{l}{L}\hat{H}_p(j+1,n) \tag{6}$$

where subscript d denotes the data position, i denotes the index of data subchannel at the nth OFDM symbol, L is the pilot spacing and l is the distance between the jth pilot subchannel and the ith data subchannel.

Thus, the transmitted data symbol at the ith data subchannel and the nth OFDM symbol can be estimated by

$$\hat{X}_d(i,n) = Y_d(i,n)/\hat{H}_d(i,n) \tag{7}$$

3. ANALYTICAL BER OF OFDM SYSTEM

3.1. Frequency-selective fading case

First, we assume the channel is a wide sense stationary uncorrelated scattering (WSSUS) frequency-selective Rayleigh fading channel with a Jakes power spectrum density (PSD). The average bit error rate for an OFDM-QPSK system is given by [5]

$$P_b(E) = \tfrac{1}{2}[1 - \tfrac{1}{2}\frac{\frac{\rho_1+\rho_2}{\sqrt{2}}}{\sqrt{1+\frac{1}{2\bar{\gamma}_b}-\frac{(\rho_1-\rho_2)^2}{2}}} - \tfrac{1}{2}\frac{\frac{\rho_1-\rho_2}{\sqrt{2}}}{\sqrt{1+\frac{1}{2\bar{\gamma}_b}-\frac{(\rho_1+\rho_2)^2}{2}}}] \tag{8}$$

where $\bar{\gamma}_b$ denotes the average SNR per bit, and

$$\rho_1 = \frac{\mu_1}{\sigma_1\sigma_2}, \rho_2 = \frac{\mu_2}{\sigma_1\sigma_2} \tag{9}$$

$$\mu_1 + j\mu_2 = \frac{E\{\hat{H}_d H_d^*\}}{2} \tag{10}$$

$$\sigma_2^2 = \frac{E\{|\hat{H}_d|^2\}}{2} \tag{11}$$

Here, linear interpolation is used. Hence (10) and (11) can be easily obtained from (6). To evaluate (10) and (11), we need to know the channel's frequency correlation function (FCF). Assuming a 6-tap typical urban (TU) channel model [6], the resulting channel's FCF in one OFDM symbol is given by

$$E\{H(m)H^*(n)\} = \sum_{k=1}^{6} c_k^2 \exp \frac{-j2\pi(m-n)\tau_k}{T_s} \tag{12}$$

where m and n denote the index of the mth and nth frequency subchannel respectively and the asterisk represents the conjugate.

3.2. Frequency-selective and time-selective fading case

Considering the Doppler spread induced by the mobile receiver, the resultant time-varying channel is approximated as a time-invariant channel and the approximation error is considered as Gaussian noise [7]. The variance of the approximation error can be denoted by [see Appendix]

$$\sigma_\zeta^2 = \frac{1}{N}\sum_{k=1}^{N}\{J_0(0) - J_0(2\pi f_m T(k-1))\} \tag{13}$$

where $J_0(x)$ denotes the 0th order first class Bessel function, f_m denotes the maximum Doppler shift and T is the sampling interval. Therefore, the total mean noise power equals to

$$\sigma_n^2 = \sigma_\eta^2 + \sigma_\zeta^2 \tag{14}$$

where η denotes the additive white Gaussian noise.

3.3. Multipath correlated fading case

In wide-band radio-wave propagation, the correlation coefficients between transmission paths may have values ranging from 0 to larger than 0.8 [8]. Thus the FCF depends on the actual frequencies under consideration, rather than just the frequency spacing. Here, the channel's FCF can be expressed as

$$E\{H(m)H^*(n)\} = F_m \Lambda \Phi \Lambda^* F_n^* \tag{15}$$

where, F_m and F_n represent the mth and nth row of Fourier coefficient matrix, Λ denotes the diagonal matrix with the elements of c_k referred in (2) and Φ denotes the correlation matrix of the six paths. Note that if Φ is a diagonal matrix that indicates the multiple paths are uncorrelated, (15) will lead to (12).

4. OPTIMAL NUMBER OF PILOTS

As mentioned earlier that the number of pilots is a tradeoff between channel estimation accuracy and bandwidth efficiency, the optimal number of pilots needs to be found. One problem, usually, confronted by engineers working on the design of OFDM system is to use the least pilots while keeping the BER lower than a desired value at a predetermined SNR. Essentially, the problem is equivalent to finding the maximal pilot spacing to satisfy the BER requirement at the given SNR.

The detail process of searching the optimal number of pilots is described as follows. First, substitute an initial pilot spacing, L_0, and other OFDM system and channel parameters into the analytical BER formula. So the BER in terms of SNR at pilot spacing L_0 can be obtained and represented by $P_b(L_0)$. Next, increase the pilot spacing by one, i.e. $L_1 = L_0 + 1$, and obtain $P_b(L_1)$ similarly. Repeat the process iteratively and a set of $P_b(L)$ $(L = L_0, L_1, \cdots, L_P)$ can be obtained. Then, plot these BER curves in one figure, and compare them with the desired BER at the given SNR. The curve having the maximal pilot spacing yet satisfying the requirement is identified and from which the minimal number of pilot symbols can be found.

5. SIMULATION RESULTS

An uncoded PSAM-OFDM system is simulated. The duration of one OFDM symbol is $224\mu s$ (guard interval excluded). The sampling interval is $7/64\mu s$. The bandwidth is $8MHz$ and the center carrier frequency is assumed to be $470MHz$. The length of cyclic prefix is $1/16$ of 2048, the number of carriers. The 6-tap TU channel model is considered and the uncorrelated multipath fadings are simulated with Jakes PSD. Modulation of the subcarrier is assumed to be QPSK.

Fig.1 shows the comparison of the simulated BER with the analytical BER in uncorrelated multipath fading channel with different pilot spacing and speed. It demonstrates that the analytical BER curves give a good approximation to the simulation results. This will provide us an easy way to analyze the PSAM-OFDM.

In Fig.2, the BER results of PSAM-OFDM system in the correlated multipath fading channels are illustrated. Suppose that the correlation coefficient between adjacent complex gains is 0.8. The multiple correlated fadings can be generated from the uncorrelated multipath fadings multiplied by the lower triangular matrix of the Cholesky decomposition of the correlation matrix [9]. Also, the analytical BER approximates the simulation BER well.

One example is given here to illustrate the validity of the proposed method. Suppose that a mobile receiver is moving with 100 km/h in an uncorrelated multipath fading channel,

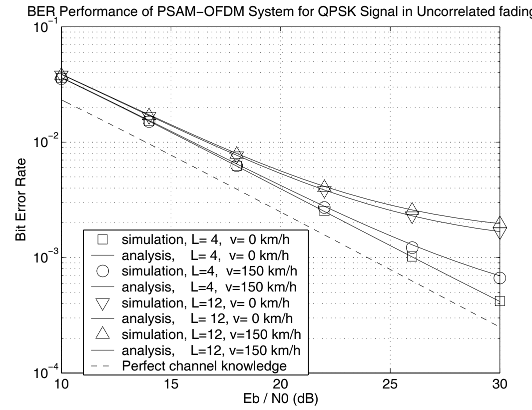

Fig. 1. BER performance of PSAM-OFDM System for QPSK signal in uncorrelated multipath fading (pilot spacing $L = 4, 12$ and speed $v = 0, 150$km/h, respectively)

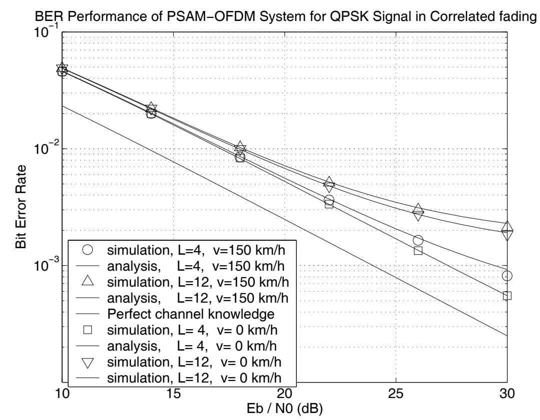

Fig. 2. BER performance of PSAM-OFDM System for QPSK signal in correlated multipath fading (pilot spacing $L = 4, 12$ and speed $v = 0, 150$km/h, respectively)

how many pilots should at least be used in a PSAM-OFDM system in order to achieve a BER lower than 10^{-3} at 30 dB SNR? Firstly, by considering (12) and (14), a series of analytical BER curves can be evaluated from (8) and plotted with various pilot spacing as shown in Fig.3. Next, by finding the SNR corresponding to the BER of 10^{-3} in each curve, we can draw the pilot spacing in terms of SNR, as shown by the circles on the curve depicted in Fig.4. By noting the pilot spacing at 30 dB SNR, we see that the minimal spacing that can satisfy the BER requirement is 9. Correspondingly, the desired optimal number of pilot symbols is 229 per OFDM symbol.

6. CONCLUSION

In this paper, a new and simple method is proposed to find the optimal number of pilot symbols for practical OFDM system. Analytical BER of PSAM-OFDM in uncorrelated

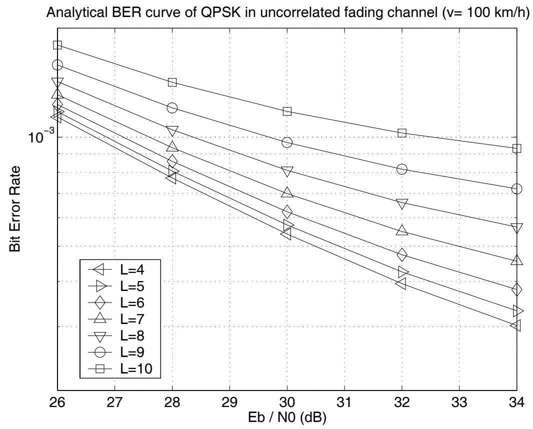

Fig. 3. BER vs. SNR with various pilot spacing for QPSK in uncorrelated multipath fading channel (TU, COST207, pilot spacing L = $4, 5, 6, 7, 8, 9, 10$ and $v = 100$ km/h)

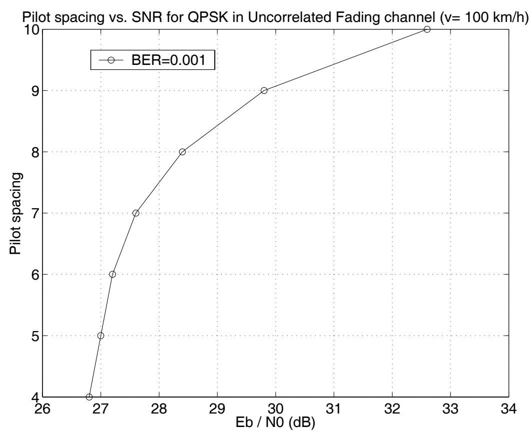

Fig. 4. Pilot spacing vs. SNR for QPSK in uncorrelated multipath fading channel (TU, COST207, $v = 100$km/h, BER= 10^{-3})

or correlated multipath fading is studied to search for the minimal number of pilot symbols. Typical example is used to validate the efficacy of the proposed method, which can be generalized for all different scenarios.

7. APPENDIX

The tme-variant ISI channel is modelled as

$$y(n) = \sum_{l=1}^{L} h_l(n)x(n-l) + \eta(n) \tag{16}$$

where $h_l(n)$ is the lth channel impulse response and $\eta(n)$ is the additive white Gaussian noise.

We use the first channel value $h_l(1)$ as the approximation value of $h_l(n), n = 1, 2, \cdots, N$, then

$$y(n) = \sum_{l=1}^{L} h_l(1)x(n-1) + \zeta(n) + \eta(n) \tag{17}$$

where the approximation error is given by

$$\zeta(n) = \sum_{l=1}^{L} (h_l(n) - h_l(1))x(n-l) \tag{18}$$

After the normalization of the path power, the variance of $\zeta(n)$ will reduce to (13).

8. REFERENCES

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