KALMAN-FILTER CHANNEL ESTIMATOR FOR OFDM SYSTEMS IN TIME AND FREQUENCY-SELECTIVE FADING ENVIRONMENT

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ABSTRACT

We present a Kalman-filter method for the estimation of time-frequency-selective fading channels in OFDM systems. Based on the Jakes model, an auto-regressive (AR) model of the channel dynamics is built. To reduce the complexity of the high-dimensional Kalman filer for joint estimation of the subchannels, we propose to use a low-dimensional Kalman filter for the estimation of each subchannel. Then, a minimum mean-square-error (MMSE) combiner is used to refine the Kalman estimates. The per-subchannel Kalman estimator explores the time-domain correlation of the channel, while the MMSE combiner explores the frequency-domain correlation. This two-step solution offers a performance comparable to the much more complicated joint Kalman estimator.

1. INTRODUCTION

Orthogonal frequency division multiplex (OFDM) is an effective technique for combating frequency-selective fading channels in wireless communication systems, e.g., [1]. Being a multicarrier modulation scheme, OFDM divides the overall frequency band into a number of subband and transmits a low-rate data stream in each subband. This way, a wideband frequency-selective channel is converted to a number of parallel narrow-band flat-fading subchannels which are free of intersymbol interference (ISI). In addition, OFDM allows overlap of the subchannels but keeps the orthogonality of the subcarriers. Therefore, high spectral efficiency is achieved.

For coherent detection of the information symbols, reliable estimation of the gain of each subchannel in the OFDM system is crucial. This problem is further complicated by the time-varying nature of the channel fading and the correlation between the subchannels due to Doppler frequencies. A minimum mean-square-error (MMSE) channel estimator has been proposed in [2]. It uses only the correlation of the channel in frequency domain (i.e., the correlation between subchannels) and fails to address the time-domain dynamics. Instead, it assumes a quasi-static channel over at least tens of OFDM symbols. Li, Cimini and Sollenberger [3] have proposed another MMSE channel estimator in which coarse channel estimates from several successive OFDM symbols are further combined optimally in the MMSE sense to get an updated channel estimate. In that method, the time-domain together with the frequency-domain correlation of the channel is used to get the optimal combining coefficients. However, the initial coarse estimates are obtained independently from each OFDM symbol to another without taking advantage of the time dynamics of the channel. There are also many blind methods for estimating OFDM channels [4–9]. They either work in a symbol-by-symbol manner or need statistics over a block of OFDM symbols. Though the former ones can deal with fast time-varying channels, the information of the time-domain correlation is not utilized.

In this paper, we develop a state-space model for the OFDM system in a time-frequency-selective fading channel environment based on Jakes' channel fading model [10]. We propose to use Kalman filter to estimate and track the channel. To reduce the dimension of the Kalman filter, we further propose a structure which uses a (low-dimensional) Kalman-filter estimator for each subchannel and a linear combiner to refine the estimate of each subchannel. The per-subchannel Kalman-filter only explores the time-domain correlation of each subchannel, while the linear combiner is optimized in the MMSE sense based on the frequency-domain correlation between subchannels.

2. SIGNAL MODEL

We consider a standard OFDM system in which the information symbols are grouped into blocks and inverse discrete Fourier transform (IDFT) is performed on each block and cyclic prefix (CP) added before they are fed into the modulator and transmitted. At the receiver, DFT is performed on each received OFDM symbol after the CP is removed. With proper CP extensions, carrier synchronization and sample

This work was supported by ONR (Grant N00014-03-1-0123).

timing of tolerable leakage, the sample from the kth subcarrier for the nth OFDM symbol is

$$y_k[n] = H_k[n]s_k[n]+w_k[n], k = 1,\ldots,N, -\infty < n < \infty, \quad (1)$$

where $s_k[n]$ is the kth information symbol of the nth OFDM symbol, $H_k[n]$ is the gain of the kth subchannel during the nth OFDM symbol, $w_k[n]$ is the noise, and N is the total number of the subcarriers. In our studies, we assume that $s_k[n]$ is drawn from a BPSK constellation $\{-1,+1\}$ independently for different k and n, and $w_k[n]$ is a circular Gaussian random variable with zero mean and variance σ_w^2 and i.i.d for different k and n. We also assume that the channel is Rayleigh fading and we use Jakes' model [10] for the power spectral density and Doppler spectrum of the fading process. Specifically, we have the correlation of the channel gain $H_k[n]$s as

$$\begin{aligned} r_{k,l}[m] &= \mathrm{E}\{H_k[n]H_l^*[n-m]\} \\ &= J_0(2\pi f_d mT)\frac{1-j2\pi(l-k)\sigma_t/T}{1+4\pi^2(l-k)^2\sigma_t^2/T^2}, \end{aligned} \quad (2)$$

where f_d is the maximum Doppler frequency, σ_t the maximum delay spread of the channel and T the OFDM symbol duration, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

It is known that the dynamics of $H_k[n]$s can be well modeled by an auto-regressive (AR) process. Define $\boldsymbol{h}[n] := [H_1[n],\ldots,H_N[n]]^T$. An pth-order AR model for $\boldsymbol{h}[n]$ is presented as

$$\boldsymbol{h}[n] = -\sum_{i=1}^{p}\boldsymbol{A}[i]\boldsymbol{h}[n-i] + \boldsymbol{Q}\boldsymbol{u}[n], \quad (3)$$

where $\boldsymbol{A}[1],\ldots,\boldsymbol{A}[p]$ and $\boldsymbol{Q}$ are $N\times N$ matrices and $\boldsymbol{u}[n]$ is an $N\times 1$ vector white Gaussian process. $\boldsymbol{A}[1],\ldots,\boldsymbol{A}[p]$ and $\boldsymbol{Q}$ are the model parameters which are obtained by solving a Yule-Walker equation using $r_{k,l}[m]$ in (2).

Based on the AR model of the channel, a state-space model for the OFDM system can be built. By defining $\boldsymbol{x}[n] := [\boldsymbol{h}^T[n],\ldots,\boldsymbol{h}^T[n-p+1]]^T$, we have

$$\boldsymbol{x}[n] = \boldsymbol{C}\boldsymbol{x}[n-1] + \boldsymbol{G}\boldsymbol{v}[n] \quad (4)$$

which is the state equation with $\boldsymbol{v}[n]$ being the white Gaussian process noise. The matrices $\boldsymbol{C}$ and $\boldsymbol{G}$ are defined as

$$\begin{aligned} \boldsymbol{C} &:= \begin{bmatrix} -\boldsymbol{A}[1] & -\boldsymbol{A}[2] & \ldots & -\boldsymbol{A}[p] \\ \boldsymbol{I}_N & \boldsymbol{0}_N & \ldots & \boldsymbol{0}_N \\ & \ddots & & \\ \boldsymbol{0}_N & \ldots & \boldsymbol{I}_N & \boldsymbol{0}_N \end{bmatrix}, \\ \boldsymbol{G} &:= [\boldsymbol{Q},\boldsymbol{0}_K,\ldots,\boldsymbol{0}_K]^T, \end{aligned}$$

and $\boldsymbol{I}_N$ and $\boldsymbol{0}_N$ are $N\times N$ identity matrix and all-zero matrix, respectively. The observation equation of the state-space model is a vector version of (1):

$$\boldsymbol{y}[n] = \boldsymbol{D}[n]\boldsymbol{x}[n] + \boldsymbol{w}[n], \quad (5)$$

where $\boldsymbol{y}[n] := [\boldsymbol{y}_1[n],\ldots,\boldsymbol{y}_N[n]]^T$, $\boldsymbol{D} := [\boldsymbol{S}[n],\boldsymbol{0}_N,\ldots,\boldsymbol{0}_N]$, $\boldsymbol{w}[n] := [w_1[n],\ldots,w_N[n]]^T$ and $\boldsymbol{S}[n]$ is an $N\times N$ diagonal matrix with $s_k[n]$ being its kth diagonal entry.

3. VECTOR KALMAN CHANNEL ESTIMATOR

The state-space model of (4) and (5) allows us to use Kalman filter to adaptively track the channel gain $H_k[n]$. The algorithm is standard and is given below.

1. Initialize the Kalman Filter with $\boldsymbol{x}[0] = \boldsymbol{0}_{pN}$ and $\boldsymbol{\Sigma}_0 = \boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}$ is the stationary covariance of $\boldsymbol{x}[n]$ and can be computed analytically from (2).

2. For each n, do the Kalman Filter update according to

$$\begin{aligned} \mathbf{M}_n &= \boldsymbol{C}\Sigma_{n-1}\boldsymbol{C}^H + \boldsymbol{G}\boldsymbol{G}^H, \\ \boldsymbol{\Gamma}_t &= \boldsymbol{D}[n]\mathbf{M}_n\boldsymbol{D}^H[n] + \sigma_w^2\boldsymbol{I}_N, \\ \mathbf{K}_n &= \mathbf{M}_n\boldsymbol{D}^H[n]\boldsymbol{\Gamma}_n^{-1}, \\ \boldsymbol{x}[n] &= \boldsymbol{C}\boldsymbol{x}[n-1] + \mathbf{K}_n(\boldsymbol{y}[n] - \boldsymbol{D}[n]\boldsymbol{C}\boldsymbol{x}[n-1]), \\ \boldsymbol{\Sigma}_n &= (\boldsymbol{I}_{pN} - \mathbf{K}_n\boldsymbol{D}_n)\mathbf{M}_{n-1}. \end{aligned}$$

3. Channel estimate at instance n is

$$\hat{\boldsymbol{h}}[n] = [\boldsymbol{I}_N,\boldsymbol{0}_N,\ldots,\boldsymbol{0}_N]\boldsymbol{x}[n]. \quad (6)$$

Noted that the algorithm needs the information symbol $s_k[n]$s, so is working in the training or decision-feedback mode.

The vector Kalman-filter algorithm gives the optimal linear estimate of the channel. Its drawback is the high complexity. Considering that the dimension of the state vector is pN which can be significantly high when there are large number of subcarriers.

4. PER SUBCARRIER KALMAN ESTIMATOR WITH MMSE COMBINER

One solution to reduce the complexity of the Kalman-filter channel estimator is to implement it at a per-subchannel fashion. Consider the kth subchannel gain $H_k(n)$. It can be modeled as an one-dimensional AR process:

$$H_k[n] = -\sum_{i=1}^{p} a_i H_k[n-i] + \sigma u_k[n], \quad (7)$$

where the model parameter $a_1,\ldots,a_p$ and σ can be computed from the correlation $r_{k,k}(0)$ of (2) according to a Yule-Walker equation. Noted that these parameters does not have index k. This is because all the subcarriers has the same statistics and fit in the same AR model. This may greatly simplify the channel estimator for many components are shared by the estimator for each subchannel.

Follow the similar procedure as that of the previous section, we obtain the state-space model for the kth subchannel as

$$\begin{aligned} \boldsymbol{x}_k[n] &= \boldsymbol{C}\boldsymbol{x}_k[n-1] + \boldsymbol{g}\boldsymbol{v}_k[n] \,, & (8) \\ y_k[n] &= \boldsymbol{d}_k[n]\boldsymbol{x}[n] + w_k[n] \,, & (9) \end{aligned}$$

where $\boldsymbol{x}_k[n] = [H_k[n], \ldots, H_k[n-p+1]]^T$, $\boldsymbol{g} = [\sigma, 0, \ldots, 0]$, $\boldsymbol{d}_k = [s_k[0], 0, \ldots, 0]$, and

$$\boldsymbol{C} = \begin{bmatrix} -a_1 & -a_2 & \ldots & -a_p \\ 1 & 0 & \ldots & 0 \\ & \ldots & & \\ 0 & \ldots & 1 & 0 \end{bmatrix}.$$

In the equation above, we have recycled the variables $\boldsymbol{C}$ from (4) just for sake of uniformity.

The Kalman filter for the model of (8) is a p-dimensional one which is much simpler. We present it as follows.

1. Initialize the Kalman Filter with $\boldsymbol{x}_k[n] = \mathbf{0}_p$ and $\boldsymbol{\Sigma}_{k,0} = \boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}$ is the stationary covariance of $\boldsymbol{x}_k[n]$ and can be computed analytically from (2).

2. For each n, do the Kalman Filter update according to

$$\begin{aligned} \mathbf{M}_n &= \boldsymbol{C}\Sigma_{n-1}\boldsymbol{C}^H + \boldsymbol{g}\boldsymbol{g}^H, \\ \gamma_{k,n} &= \boldsymbol{d}_k[n]\mathbf{M}_n\boldsymbol{d}_k^H[n] + \sigma_w^2, \\ \mathbf{K}_{k,n} &= \mathbf{M}_n\boldsymbol{d}_k^H[n]/\gamma_{k,n}, \\ \boldsymbol{x}_k[n] &= \boldsymbol{C}\mathbf{x}_k[n-1] + \mathbf{K}_{k,n}(y_k[n] - \boldsymbol{d}_k[n]\boldsymbol{C}\boldsymbol{x}_k[n-1]), \\ \boldsymbol{\Sigma}_{,n} &= (\boldsymbol{I}_p - \mathbf{K}_{k,n}\boldsymbol{d}_k[n])\mathbf{M}_{n-1}. \end{aligned}$$

3. Channel estimate at instance n is

$$\hat{H}_k[n] = [1, \mathbf{0}, \ldots, \mathbf{0}]\boldsymbol{x}_k[n]. \qquad (10)$$

However, the per-subcarrier Kalman filter only explores the time-domain correlation of the channel fading and fails to take advantage of the frequency-domain correlation. Therefore, further improvement of the estimates of (10) is possible. Our proposal is to introduce a linear combiner to combine $\hat{H}_1[n], \ldots, \hat{H}_N[n]$ to refine them. Specifically, the refined channel estimate is

$$\check{h}[n] = \boldsymbol{T}\tilde{\boldsymbol{h}}[n] \,, \qquad (11)$$

where $\tilde{\boldsymbol{h}}[n] = [\hat{H}_1[n], \ldots, \hat{H}_N[n]]^T$ and $\boldsymbol{T}$ is the combining matrix. We optimize $\boldsymbol{T}$ in an MMSE sense, that is

$$\begin{aligned} \boldsymbol{T} &= \arg\min \mathrm{E}[\|\boldsymbol{h}[n] - \check{\boldsymbol{h}}[n]\|^2] \\ &= \arg\min \mathrm{E}[\|\boldsymbol{h}[n] - \boldsymbol{T}\tilde{\boldsymbol{h}}[n]\|^2] \\ &= \mathrm{E}[\boldsymbol{h}[n]\tilde{\boldsymbol{h}}^H[n]]\{\mathrm{E}[\tilde{\boldsymbol{h}}[n]\tilde{\boldsymbol{h}}^H[n]]\}^{-1} \,. \qquad (12) \end{aligned}$$

Since $\tilde{\boldsymbol{h}}$ is the estimate of $\boldsymbol{h}[n]$ from the Kalman filter in (10), we can write

$$\tilde{\boldsymbol{h}}[n] = \boldsymbol{h}[n] + \boldsymbol{e}[n] \,, \qquad (13)$$

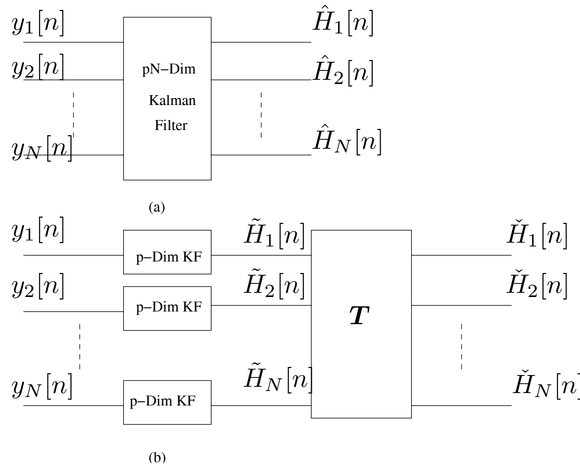

Fig. 1. Block diagram of Kalman-filter channel estimator for OFDM systems: (a) Vector Kalman-filter; (b) Per-subcarrier Kalman-filter with MMSE combiner

where $\boldsymbol{e}[n]$ is the estimation error which is zero-mean Gaussian and independent of $\boldsymbol{h}[n]$. The covariance matrix of $\boldsymbol{e}[n]$ (also the covariance matrix $\boldsymbol{h}[n]$) $\boldsymbol{P} = \mathrm{E}[\boldsymbol{e}[n]\boldsymbol{e}[n]^H]$ is a diagonal matrix with the kth diagonal entry being the covariance of $\hat{H}_k[n]$ in (10), which can be obtained from the Kalman filter updating procedure:

$$\boldsymbol{P}(i,i) = [1, 0, \ldots, 0]\Sigma_{k,n}[1, 0, \ldots, 0]^T \,, \qquad (14)$$

i.e, $\boldsymbol{P}(i,i)$ is the $(1,1)$st entry of $\Sigma_{k,n}$. Consequently, we have

$$\begin{aligned} \mathrm{E}[\boldsymbol{h}[n]\tilde{\boldsymbol{h}}^H[n]] &= \mathrm{E}[\boldsymbol{h}[n]\boldsymbol{h}^H[n]] = \boldsymbol{R}[0] \,, \\ \mathrm{E}[\tilde{\boldsymbol{h}}[n]\tilde{\boldsymbol{h}}^H[n]] &= \boldsymbol{R}[0] + \boldsymbol{P} \,, \qquad (15) \end{aligned}$$

where $\boldsymbol{R}[0]$ is an $N \times N$ matrix with its (l,k) being $r_{l,k}(n)$ of (2). The expression

$$\boldsymbol{T} = \boldsymbol{R}[0](\boldsymbol{R}[0] + \boldsymbol{P})^{-1} \qquad (16)$$

then follows.

The block diagram of the per-subcarrier Kalman estimator with MMSE combiner is presented in Fig. 1(b), as against the vector Kalman-filter estimator in (a).

5. SIMULATIONS

In this section, we provide computer simulation results to demonstrate the performance of the algorithms discussed above. We consider an OFDM system with the parameters below:

Number of sub-carriers	16	
Frequency separation Δf	1.76	KHz
Maximum delay spread σ_T	25	μs
Maximum Doppler shift f_D	80	Hz
AR model order for OFDM System	2	
AR model order for each subcarrier	2	

Note that $\sigma_T/T = 0.044$, $f_D T = 0.045$. This OFDM system satisfies the assumptions made before.

Figure 2 shows the mean square error (MSE) of the channel estimation of the two Kalman-filter channel estimator versus the received signal-to-noise ratio (SNR). To drive the Kalman filter, 10 training OFDM symbols are used; and after that the system switches to decision-feedback mode. The MSE is the average of the channel estimation for 100 OFDM symbols after the training. Figure 3 shows the bit-error-rate (BER) versus the received SNR when the estimated channel is supplied to the coherent detector. The BER is again the average of the transmission of 100 OFDM symbols after 10 trainings.

From the figures we see that the proposed per-subcarrier Kalman filter with MMSE combiner offers comparable performance to the much more complicated vector Kalman filter

6. CONCLUSION

We proposed a Kalman-filter solution for the estimation of OFDM channels in a time-frequency-selective fading environment. Though the method is a two-step solution: filtering in time and frequency domain successively, the performance of it is comparable to the much more complicated joint Kalman estimator. The behind reason may be that the time and frequency components of the Jakes' model in (2) is separable in nature.

7. REFERENCES

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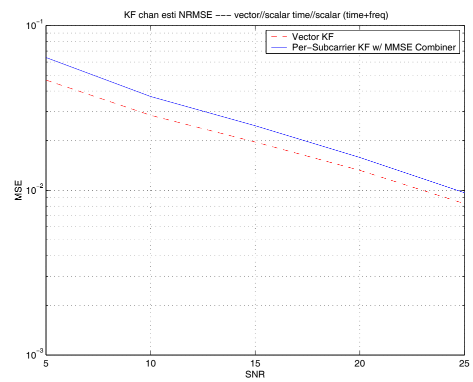

Fig. 2. Normalized channel estimate error of the Kalman-filter Estimators

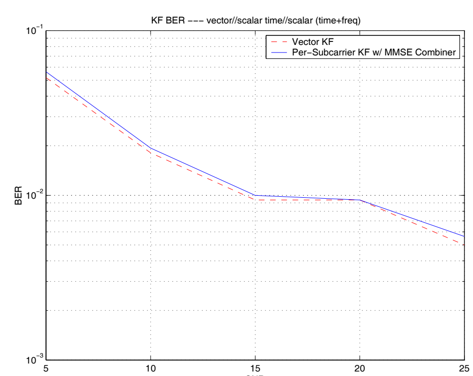

Fig. 3. Bit-error rate of the Kalman-filter Estimators