UNIFIED FRAMEWORK FOR A CLASS OF FREQUENCY-OFFSET ESTIMATION TECHNIQUES FOR OFDM

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ABSTRACT

Due to the high sensitivity of OFDM to carrier frequency-offset, accurate estimation algorithms are required in order to achieve high performance. We show that many of the existing methods share the same underlying approach, which is the exploitation of null subcarriers. Based on this, a novel computationally simple estimator, named the approximate non linear squares estimator (ANLS), is developed. We show that the performance of the ANLS estimator is very close to that of the computationally more demanding NLS estimator, and superior to existing low-complexity estimators.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become the standard choice for many wireless communications systems. The popularity of OFDM stems from its ability to transform a wideband frequency selective channel to a set of parallel flat-fading narrowband channels, which substantially simplifies the channel equalization problem. However, carrier synchronization turns out to be a critical issue for OFDM systems: CFO destroys the orthogonality of the sub-carriers leading to degraded SNR and BER. Here, we address the frequency synchronization problem. Clearly, issues such as timing recovery and channel estimation are important, but will not be treated in this paper. Here, we assume perfect frame and time synchronization and focus on carrier frequency offset (CFO) estimation.

CFO estimation techniques may be classified as time-domain (pre-FFT) or frequency-domain (post-FFT) techniques. The latter are usually used to estimate the integer part of the CFO (normalized by the subcarrier spacing) after the fractional part has been identified and corrected. Time-domain methods are used to estimate the fractional part of the CFO, although some of these techniques can also estimate the integer part. Here, we focus on time-domain methods.

Time-domain methods can be classified into those that exploit the time-diversity provided by the cyclic prefix (CP) and/or oversampling, those that ignore the CP and rely on pilots or null subcarriers (NSC), and those based on joint channel and CFO estimation.

Some CFO estimation methods are classified as being data-aided although they do not use the known pilot block. This is the case for methods based on structuring the OFDM symbol as a repetition of $J \geq 2$ identical slots, [1, 3]. These methods are, in fact, NSC-based techniques, since a repetition of $J \geq 2$ identical slots can be generated by zeroing all subcarriers whose frequencies are not multiple of $J\Delta f$. In [2], two identical OFDM symbols were used to estimate the CFO. Even though this method is not NSC-based, it can be mathematically described as one by considering the two symbols as a double-sized OFDM symbol (i.e., $2M$ subcarriers) generated by setting the subcarriers with odd frequencies to zero. These repetitive slot-based techniques require the number of NSC to be larger or equal to half the total number of subcarriers. Since the number of training zeros is large, it is appropriate to call them data-aided. When the only NSCs are those imposed by system design requirements, i.e. the virtual subcarriers, the method is referred to as blind [4]. If in addition to these NSC, extra NSC are inserted, the technique is classified as semi-blind since the receiver knows the locations of the NSC. Here, we avoid this confusion and refer to the approach simply as the NSC-based approach. A nice property of the NSC-based methods is that CFO and channel estimations are decoupled, which implies that no model is required for the channel *during* CFO estimation. In [6], we presented a general framework for this approach and derived the best placement of NSC in terms of CFO estimation accuracy. Here, we show that exploiting the repetitive structure of the preamble is a simplified version of the NSC-based method. We also present a novel low-complexity CFO estimator.

2. SIGNAL MODEL

We consider a standard OFDM system with M subcarriers spaced at Δf. Subcarriers at the band-edges are usually unmodulated in order to prevent interference with adjacent OFDM channels; such subcarriers are often called virtual subcarriers (VSC). In addition to the VSC, other subcarriers may be switched off; e.g., if the transmitter has CSI information, subcarriers subject to deep fades or NBI may be left inactive. Further, synchronization preambles are often made by zeroing a large number of subcarriers. Indeed, a preamble consisting of a repetition of two identical slots is obtained by zeroing all the odd subcarriers [1]. The set of NSC includes the VSC, whose placement and number are imposed by system design; the number and placement of the remaining NSC are controlled by the system user, and could vary across the OFDM symbols. Let $\mathcal{M} = \{0, ..., M-1\}$ denote the entire set of subcarriers, and let $\mathcal{K}_i$ (resp. $\mathcal{Z}_i$) denote the subsets of $\mathcal{M}$ that contain the K_i (resp. Z_i) modulated (resp. null) subcarriers during the ith OFDM block.

The vector modulating the entire set of subcarriers during the ith block can then be expressed as $\boldsymbol{s}(i) := \mathbf{V}_i \boldsymbol{s}_{\mathcal{K}}(i)$, where $\boldsymbol{s}_{\mathcal{K}}(i)$ is the K_i-element vector of symbols transmitted on the activated subcarriers, and $\mathbf{V}_i$ is the $M \times K_i$ matrix whose (m, n) entry is one if the nth symbol is transmitted on the

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m-th subcarrier during the ith OFDM block, and is zero otherwise. We assume without loss of generality that the symbols are zero mean and have unit variance, i.e., $E|s(i,m)|^2 = 1$ for $m \in \mathcal{K}_i$.[1] The $(M \times 1)$ data block $\boldsymbol{s}(i)$ is first precoded by the IFFT matrix $\mathbf{F}^{\mathcal{H}}$ with (k,m)th entry $\frac{1}{\sqrt{M}}\exp(j2\pi km/M)$. The resulting $(M \times 1)$ vector $\boldsymbol{u}(i) = \beta_i \mathbf{F}^{\mathcal{H}} \boldsymbol{s}(i)$ is called the time-domain block vector, or the time domain OFDM symbol. We have also introduced a normalization parameter $\beta_i := \sqrt{M/K_i}$ to ensure that the transmitted power is kept constant regardless of K_i, the number of active subcarriers. Next, a CP of length L_{cp} is inserted by replicating the last L_{cp} elements of each block in the front. The $P(= M + L_{cp})$ samples of each block are then pulse shaped, upconverted to the carrier frequency, and transmitted sequentially through the channel.

We model the frequency-selective channel as a FIR filter with channel impulse response (CIR) $\boldsymbol{h} = [h_0, ..., h_L]^T$ where L is the channel order, with $L \leq L_{cp} \leq M$. We assume that the CIR is time-invariant over $N \geq 1$ consecutive symbol blocks, but could vary from one set of N blocks to the next.

The received signal is downconverted to baseband and sampled at the rate of P samples per extended OFDM symbol. We will assume that time synchronization has been achieved. Let ν (a real number) denote the CFO normalized by $\Delta_f = 1/T$, where T is the duration of the useful OFDM symbol. In the presence of CFO, the m-th sample of the i-th received OFDM symbol will experience a phase shift equal to $2\pi\nu(iP + m)/M$. After discarding the CP, the m-th sample will have phase shift $2\pi\nu(iP + L_{cp} + m)/M$, $m = 0, ..., M-1$. The noise and CFO corrupted symbol blocks are obtained as

$$\boldsymbol{x}(i) = \beta_i e^{j2\pi\nu(iP+L_{cp})/M} \mathbf{D}(\nu) \mathbf{H}_c \mathbf{F}^H \boldsymbol{s}(i) + \boldsymbol{n}(i) \quad (1)$$

where $\mathbf{D}(\nu) = \text{diag}\,(1, ..., \exp(j2\pi\nu(M-1)/M))$, $\mathbf{H}_c$ is the $(M \times M)$ circulant matrix with first row $[h_0, 0, ..., 0, h_L, ..., h_1]$ and $\boldsymbol{n}(i)$ is the $M \times 1$ noise vector, which is assumed to be zero mean complex Gaussian with covariance matrix $\sigma^2\mathbf{I}$. The circulant matrix $\mathbf{H}_c$ is diagonalized by the FFT matrix, so that $\boldsymbol{x}(i)$ can also be expressed as

$$\boldsymbol{x}(i) = \beta_i e^{j2\pi\nu(iP+L_{cp})} \mathbf{D}(\nu) \mathbf{F}^H \mathbf{D}(H) \boldsymbol{s}(i) + \boldsymbol{n}(i) \quad (2)$$

where $H_k := \sum_{l=0}^{L} h_l e^{-j2\pi kl/M}$ and $\mathbf{D}(H) = \text{diag}\,(H_0, ..., H_{M-1})$.

3. REPETITIVE SLOTS-BASED CFO ESTIMATION

In OFDM systems, pilot symbols are usually transmitted prior to the information frame. For example in IEEE802.11a, the preamble is a series of identical slots. Using this preamble structure, a nonlinear least square (NLS) CFO estimator is proposed in [5]. Correlation-based estimation methods were also proposed in [2] and [3]. In this section, we show the links between the NLS and the correlation-based techniques.

[1]Notation: We use the Matlab notation $A(1:R, 1:S)$ to denote the submatrix formed by the first R rows and first S columns of the matrix A; when all columns are included '$1:S$' is replaced simply by':'. $s(i,m)$ denotes the mth element of $\boldsymbol{s}(i)$. Superscripts T and $\mathcal{H}$ will denote transpose and conjugate transpose. $\mathcal{R}[\cdot]$ denotes the real part operator; $\arg\{\cdot\}$ the argument operator, and $[x]_{2\pi}$ modulo-2π operation.

In order to compare the repetitive slots-based approach with the null-subacarrier-based approach, we assume that the preamble is a single OFDM block made of J identical sub-blocks of length $Q = M/J$ each; we assume that Q is an integer. A CP is also used with this preamble. The case where the preamble is made up of a sequence of identical OFDM blocks can be treated similarly. For example, two identical OFDM symbols can be thought of as two half symbols of a $2M$-point OFDM block. In this case, a guard interval is not needed between the identical blocks.

In this section, we use a single OFDM symbol; hence we may set $i = 0$. Using eq. (1), the received samples can be expressed as

$$\begin{aligned} x(k + \ell Q) &= z(k)\, e^{j2\pi\nu\ell/J} + n(k + \ell Q) \\ &\quad k = 0, ..., Q-1;\ \ell = 0, ..., J-1 \end{aligned}$$

where $z(k) = e^{j2\pi\nu(L_{cp}+k)/M} H_c(k,:)\boldsymbol{u}$; the last equality follows from the slot structure $u(k + \ell Q) = u(k)$, $k = 0, ..., Q-1$, $\ell = 0, ..., J-1$. Although $z(k)$ depends upon ν, the ν dependent factor can be absorbed into $\boldsymbol{u}$, and hence ignored. The vector $\boldsymbol{z} = [z(0), ..., z(Q-1)]^T$ may be modelled as an unknown non-random vector. Note that the acquisition range increases with J and is given by $[-J/2, J/2)$.

3.1. Nonlinear Least Squares Method

The NLS estimators of $\boldsymbol{z}$ and ν are obtained by minimizing [5]

$$\sum_{i=0}^{J-1} \sum_{k=0}^{Q-1} \left| x(k + \ell Q) - z(k) e^{j2\pi\nu\ell/J} \right|^2$$

Since this criterion is quadratic in the $z(k)$'s, the NLS of ν is found to be

$$\hat{\nu}_{REP} = \arg\max_{\nu} \sum_{k=0}^{Q-1} \frac{1}{J} \left| \sum_{\ell=1}^{J-1} e^{-j2\pi\ell\nu/J} x(k + \ell Q) \right|^2 .$$

This estimator can also be expressed as

$$\hat{\nu}_{REP} = \arg\max_{\nu} \sum_{m=1}^{J-1} \mathcal{R}\left[r(mQ) e^{-j2\pi m\nu/J} \right] \quad (4)$$

where $r(\tau)$ is the sample correlation

$$r(\tau) = \sum_{k=0}^{M-\tau-1} x^*(k) x(k+\tau) .$$

When $J = 2$, the estimator is given in closed-form as

$$\hat{\nu}_{REP} = \frac{1}{\pi} \arg\{r(M/2)\} . \quad (5)$$

The estimator in eq. (5) was proposed in [2] and [1]. If $J > 2$, no closed-form solution is available for the optimization problem in eq. (4). The estimator can be initialized or even replaced by the following simpler estimators.

3.2. Computationally simpler estimators

The expected value of $r(mQ)$ is given by

$$E\{r(mQ)\} = (J-m)\|\boldsymbol{z}\|^2 e^{j2\pi m\nu/J}, \quad m = 1, ..., J-1,$$

where $\|\boldsymbol{z}\|$ is the L_2-norm of $\boldsymbol{z}$. Therefore, the phase of any correlation coefficient $r(mQ)$ can be used to estimate ν. This implies that the estimator in eq. (5) is valid even when $J > 2$. In order to improve accuracy, the phases of the $r(mQ)$'s, $m = 0, ..., J-1$, can be judiciously combined. Next, we present two ways of combining these phases.

3.2.1. Approximate NLS estimator

Let ϕ_m denote the unwrapped phase of $r(mQ)$ for $m = 1 \ldots J-1$. The NLS criterion can be rewritten as

$$\sum_{m=1}^{J-1} |r(mQ)| \cos(\phi_m - 2\pi m\nu/J) .$$

Setting the derivative of this criterion with respect to ν to zero, we obtain

$$\sum_{m=1}^{J-1} m|r(mQ)| \sin(\phi_m - 2\pi m\nu/J) = 0 .$$

Under the small error approximation, i.e., $\sin(\phi_m - j2\pi m\nu/J) \approx (\phi_m - j2\pi m\nu/J)$, the approximate NLS estimator is obtained as

$$\tilde{\nu}_{ANLS} = \frac{J}{2\pi} \frac{\sum_{m=1}^{J-1} m|r(mQ)|\phi_m}{\sum_{m=1}^{J-1} m^2|r(mQ)|} . \tag{6}$$

This technique requires phase unwrapping. This task is not too demanding given the fact that the lags of the few correlations to be computed are quite apart from each other. In our simulations, this phase unwrapping has always been carried out successfully.

3.2.2. BLUE estimator [3]

A technique of combining the individual phases without phase unwrapping was developed in [3]. It is based on the best linear unbiased estimator (BLUE) concept. Let $\varphi(m) = [\arg\{r(mQ)\} - \arg\{r((m-1)Q)\}]_{2\pi}$. The BLUE estimator of ν was then expressed as

$$\check{\nu}_{REP} = \frac{J}{2\pi} \sum_{m=1}^{p} w(m)\varphi(m) \tag{7}$$

where p is a design parameter and the weighting coefficients are given by [3]

$$w(m) = 3\frac{(J-m)(J-m+1) - p(J-p)}{p(4p^2 - 6pJ + 3J^2 - 1)} .$$

It was stated in [3] that the variance of the above estimator is minimum when $p = J/2$.

4. NSC-BASED CFO ESTIMATION

Here, we first review the general framework for the NSC-based approach that we developed in [6]. Then, we establish the link between the NLS and NSC-based estimators. Let $\mathbf{D}(H_{\mathcal{K}_i}) = \mathrm{diag}\,(H_n,\ n \in \mathcal{K}_i)$, and denote the ith block of CFO-rotated and faded symbols by

$$\boldsymbol{\alpha}(i) = e^{j2\pi\nu(iP+L_{cp})/M}\mathbf{D}(H_{\mathcal{K}_i})\boldsymbol{s}_{\mathcal{K}}(i) .$$

Then, the signal model in (2) can be rewritten as

$$\boldsymbol{x}(i) \quad = \quad \beta_i \mathbf{D}(\nu)\mathbf{F}^H\mathbf{V}_i\boldsymbol{\alpha}(i) + \boldsymbol{n}(i) . \tag{8}$$

Frequency synchronization is often required prior to channel estimation. Thus, at this stage, the channel coefficients, the H_k's, may be regarded as unknown non-random parameters. Therefore, the $\boldsymbol{\alpha}(i)$'s will be modelled as unknown non-random vectors. We adopt a deterministic maximum likelihood (ML) approach.

Our objective is to estimate the CFO using N symbols, $\boldsymbol{x}(i)$, $i = 1, \ldots, N$. If a reference symbol is used for frequency synchronization, $N = 1$ and a large number of NSC is usually deployed. For blind and semi-blind method, N is usually relatively larger and the number of NSC is significantly lower. For example, in the blind CFO estimation proposed in [4], the only NSC are the VSC, which are imposed by system design.

The ML estimator was shown to be [6]

$$\hat{\nu}_{NSC} = \arg\min_{\nu} \sum_{i=1}^{N} \sum_{k \in \mathcal{Z}_i} |X(i, \nu + k)|^2 \tag{9}$$

where $X(i, f) = \sum_{\ell=0}^{M-1} x(i, \ell) \exp(-j2\pi f\ell/M)$. We can therefore interpret the estimator as follows: in the absence of CFO, the subcarriers are orthogonal, and the energy of the received signal at the NSC should be zero. We estimate the CFO as the frequency shift that minimizes the energy at the NSC or maximizes the energy at the active carriers ([4, 6]).

The estimator can also be written in terms of the time-domain correlation function as in [6]:

$$\hat{\nu}_{NSC} = \arg\max_{\nu} \sum_{\tau=1}^{M-1} \mathcal{R}\left[\sum_{i=1}^{N} [r_i(\tau)\psi^*_{\mathcal{K}_i}(\tau)] e^{-j2\pi\tau\nu/M}\right] \tag{10}$$

where

$$\psi_{\mathcal{K}_i}(\tau) = \frac{1}{M} \sum_{k \in \mathcal{K}_i} e^{j2\pi k\tau/M} ,$$

and

$$r_i(\tau) = \sum_{\ell=0}^{M-1-\tau} x^*(i, \ell)x(i, \ell + \tau) ,$$

and $x(i, \ell)$ was defined earlier as the ℓth entry of $\boldsymbol{x}(i)$.

4.1. Special case: repetition of slots

Here, we assume $N = 1$ and we drop the time index i from all vectors and matrices defined above. As mentioned previously, an OFDM symbol structured as a repetition of J identical slots can be generated by nulling the subcarriers whose normalized frequencies are not multiples of J. Some of the remaining subcarriers could also be nulled depending on whether VSC are present or not. Next, we consider how the presence of VSC impacts the estimator.

4.1.1. Virtual subcarriers absent

The elements of $\mathcal{K}$ are now $n_m = mJ$, $m = 0, \ldots, Q-1$, where $Q = M/J$ is assumed an integer, and $J \geq 2$. Now $\psi_{\mathcal{K}}(\tau)$ is nonzero only if τ is a multiple of Q, i.e.,

$$\psi_{\mathcal{K}}(\tau) = \frac{K}{M}\delta(\tau - mQ) \qquad m = 0, \pm 1, \pm 2, \ldots \ . \tag{11}$$

The estimator in eq. (10) thus reduces to the repetition slot-based NLS estimator in eq. (4). Therefore, we have shown that the latter estimator is also the NSC-based ML estimator when the odd subcarriers are switched off and no virtual subcarriers are present.

4.1.2. Virtual subcarriers present

In the more realistic case where some of the subcarriers at the edge of the spectrum are nulled to avoid interference between adjacent OFDM systems, the ML estimator is different from that in eq. (4). Let $K = (2I + 1)$, the number of active subcarriers. The useful subcarriers are then $\{0, \ldots, I, M-I, \ldots, M-1\}$ with $I < M/2$. The subcarriers of this set whose frequencies are not multiple of J are also nulled in order for the OFDM symbol to have a repetitive structure. The function $\psi_{\mathcal{K}}(\tau)$ is still real-valued but different from that in eq. (11). An example of this function is displayed in Figure 1. Here, most of the correlation

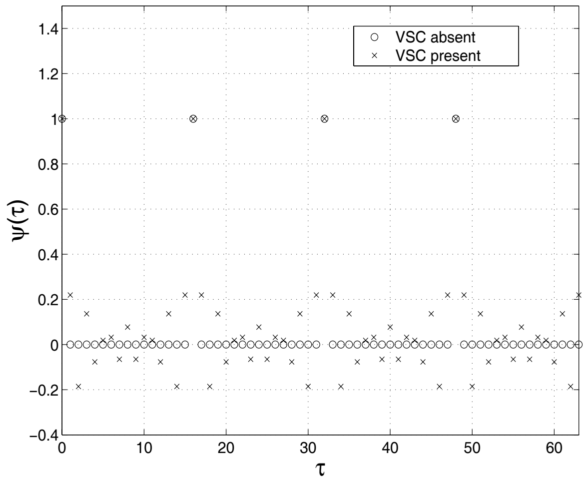

Figure 1. $\psi_{\mathcal{K}}(\tau)$ **when** $M = 64$**,** $J = 4$ **and** $I = 24$**.**

coefficients contribute to the ML estimator. The estimator in (4) is still consistent but is no longer ML. The estimator in (4) now consists of using only the $(J-1)$ highest correlation coefficients, and could therefore be seen as an approximate ML estimator. In the simulation section, we investigate the difference in performance of the two estimators. We will see that the performances are very similar for practical values of J and/or for moderate-to-high SNR.

5. SIMULATION RESULTS

We compare the performance of the various techniques developed in this paper. We assume a preamble is available for CFO estimation. This preamble consists of one OFDM symbol structured as a repetition of $J \geq 2$ identical slots. We consider an OFDM system with a total of 64 subcarriers. There are 11 virtual subcarriers at the edges of the spectrum. The useful part of an OFDM symbol, which contains 64 samples, is preceded by a cyclic prefix of length 16. QPSK modulation is used. The channel has 15 paths, with path delays $0, 1, 2, ..., 14$ samples. The magnitudes of the channel coefficients, the h_i's, are Rayleigh distributed with exponential power delay profile (the decay factor of the exponential is taken to be $1/5$) , while their phases are uniformly distributed over $[-\pi, \pi)$. Further, the channel coefficients are independent of each other. We consider the scenario where the channel is static over an OFDM symbol. The comparison is based on the mean square error (MSE) in CFO estimation which is calculated using 2000 runs.

Figures 2-3 display the MSEs vs. SNR for different values of J. It is seen that the proposed approximate NLS (ANLS) estimator has approximately the same accuracy as the NLS and the ML estimators. This accuracy is close to the CRB (derived in [6]). This suggests that ANLS should be preferred to the other estimators as it is computationally simpler. Indeed, no numerical optimization is required for the ANLS method. Furthermore, the ANLS estimator performs better than the BLUE estimator in eq. (7) where $p = J/2$ [3]. For high SNR, all the techniques have identical accuracy.

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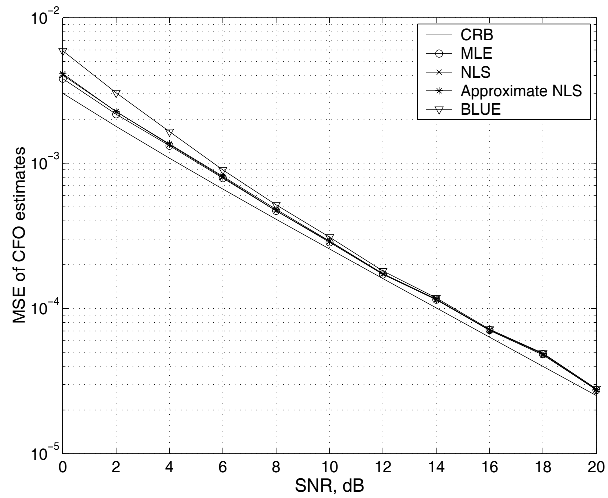

Figure 2. MSE of CFO estimates; $J = 4$**;** $M = 64$**;** $L = 15$**.**

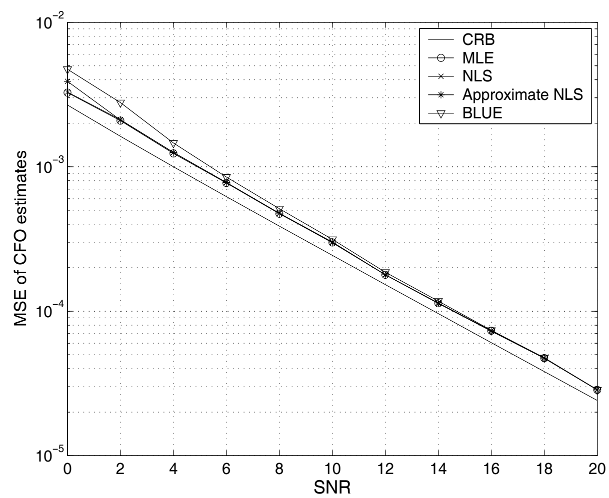

Figure 3. MSE of CFO estimates; $J = 8$**;** $M = 64$**;** $L = 15$**.**