PERFORMANCE ANALYSIS OF AN RLS-LMS ALGORITHM FOR LOSSLESS AUDIO COMPRESSION

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ABSTRACT

In this paper, we analyze the effect of the input signal correlation on the performance of an RLS-LMS based adaptive linear prediction algorithm for lossless audio compression. We show that the mis-adjustment is same for both correlated and uncorrelated input samples. However the variance of the residual error power of the algorithm increases as the signal variance increases, which may causes the numerical instability or stalling problem of the RLS algorithm. For real audio signals quantized at 16, 24 bit, experimental results show that the algorithm is capable of modelling audio signals quantized at 16 bit, but it can yield reduced performance coding gain for audio signals quantized at 24 bit, which confirm our theoretical analysis.

1. INTRODUCTION

Recently, more and more interests are focus on lossless coding of high quality audio signal as the broadband services emerge rapidly. Various compression techniques have been proposed for digital audio waveforms. All of the techniques remove firstly redundancy from signal and then code the resulting signal with an efficient digital coding scheme.

For bandwidth constraint applications, the compression ratio is emphasized over complexity. Considering the characterization of audio signals, abundant tonal and harmonic components and non-stationary, a higher order adaptive FIR predictor possessing good tracking capabilities is an attractive candidate for modelling of the audio signal.

There are some research work using adaptive linear prediction for speech prediction [2] and lossless audio coding [3, 4]. An RLS-LMS based adaptive linear prediction algorithm has been reported in good coding gain, resulting in a better overall compression performance than some widely used lossless audio codecs for audio clips sampling at 44.1 kHz and quantized at 16 bit [4]. However, the challenge for lossless audio coding is that it should provide high compression ratio for storage of audio signals at higher resolution (e.g., 16, 20 and 24 bit) and sampling rates (e.g.,48, 96 and 192 kHz) in MPEG-4 standard. Simulation results shows that the predictive coding gain of RLS-LMS codec degrades for audio clips quantized at 20 and 24 bit. Since the performance of the RLS is degraded for audio clips quantized at 20, 24 bit, we carried out an analysis of the effect of correlated input on the performance of the RLS predictor. It was shown that the mis-adjustment is same for both correlated and uncorrelated input samples. However the variance of the residual error power of the algorithm increases as variance of the signal increases. As the output of RLS is nonstationary in strict sense, its large variance output may increase the eigenvalues spread of the input to the LMS predictor, which results in slow convergence and larger residual error of the whole RLS-LMS predictor.

The organization of paper is as follows. In Section II, we review RLS-LMS algorithm. In Section III, we study the effects of input correlation on the performance of the RLS algorithm. In Section IV, we show the simulation results to confirm our analysis. Finally, we point out our conclusion and indicate our future work direction.

2. CASCADED RLS-LMS ALGORITHM

The structure of the cascaded RLS-LMS predictor proposed in the paper [4] is shown in Fig.1. With $x(n)$ denoting the input to the predictor, the residual error $u(n)$ of the RLS predictor is given by

$$u(n) = x(n) - \mathbf{w}^T(n)\mathbf{x}(n) \tag{1}$$

where T denotes matrix transposition and $\mathbf{x}(n) = [x(n-1), x(n-2), \cdots, x(n-L_{rls})]^T$. The filter tap weights $\mathbf{w}(n) = [w_1(n), w_2(n), \cdots, w_{l_{rls}}(n)]^T$ is updated using the RLS algorithm as follows:

$$K(n) = \frac{\lambda^{-1}Q(n-1)\mathbf{x}(n)}{1+\lambda^{-1}\mathbf{x}^T(n)Q(n-1)\mathbf{x}(n)} \tag{2}$$

$$\mathbf{w}(n) = \mathbf{w}(n-1) + K(n)u(n) \tag{3}$$

and

$$Q(n) = \lambda^{-1}Q(n-1) - \lambda^{-1}K(n)\mathbf{x}^T(n)Q(n-1) \tag{4}$$

where λ is a positive value that is slightly smaller than 1 with initialization

$$Q(0) = \delta I, (\delta \ll 1) \tag{5}$$
$$\mathbf{p}(n) = \mathbf{0} \tag{6}$$

The $u(n)$ is feed to the LMS predictor to get the prediction error $e(n)$:

$$e(n) = u(n) - \mathbf{p}^T(n)\mathbf{u}(n)$$

where $\mathbf{u}(n) = [u(n-1), u(n-2), \cdots, u(n-L_{lms})]^T$. The filter tap weights $\mathbf{p}(n) = [p_1(n), p_2(n), \cdots, p_{L_{lms}}(n)]^T$ is updated using the normalized LMS algorithm:

$$\mathbf{p}(n+1) = \mathbf{p}(n) + \mu\frac{\mathbf{u}(n)}{\| \mathbf{u}^T(n)\mathbf{u}(n) \|}e(n)$$

where $0 < \mu < 2$ is the adaptation step sizes of the LMS algorithm.

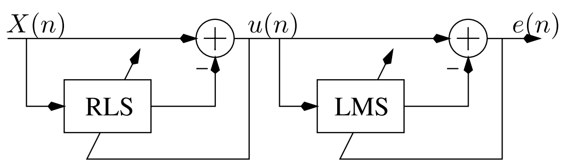

Fig. 1. RLS-LMS predictor.

The RLS-LMS predictor features fast convergence and successfully modelling of audio signals quantized at 16 bit. However, its behavior exhibits a degraded performance for highly colored signal with high resolution. In this paper, we try to explain the phenomena through the stability analysis of RLS algorithm with correlated inputs and white noise.

3. STABILITY ANALYSIS OF THE RLS ALGORITHM WITH CORRELATED INPUTS

In lossless audio compression, the residual error powers are taken as performance measures: the smaller the residual error, the higher the coding gain. The stability of the RLS algorithm can be analyzed through the study the sensitivity of the RLS algorithm to perturbations in the filter coefficient from the optimum error power, which involves in expanding the deviation from the optimum error power due to random perturbation in the filter coefficient in a Taylor series with second order partial derivatives since the first order derivatives are zero for the optimal RLS Filter. This method has been applied to a system identification problem [5] for RLS algorithm with $\lambda = 1$. Here we use it to RLS adaptive linear prediction problem with $\lambda < 1$. We review the method firstly, then to develop further for RLS ($\lambda < 1$) algorithm and use the results to explain the phenomenon accounted in the simulations.

Consider the adaptive linear prediction problem of estimating the desired response $x(n)$ by a linear combination of the current and past input samples $x(n)$ to produce the residual error $u(n)$ in Eq.1. The vector $\mathbf{x}_n$ presents all samples of $x(n)$ as

$$\mathbf{x}_n = [x(n), x(n-1), \cdots, x(0)]^T \tag{7}$$

and the input vector

$$\mathbf{x}_n = [x(n), x(n-1), \cdots, x(0)]^T \tag{8}$$

Define the $n \times N$ matrix, $X_{N,n}$

$$X_{N,n} = [\mathbf{x}_n, \mathbf{x}_{n-1}, \cdots, \mathbf{x}_{n-N+1}] \tag{9}$$

where

$$\mathbf{x}_{n-1} = z\mathbf{x}_n = [x(n-1), x(n-2), \cdots, x(0)]^T \tag{10}$$

and z^{-1} is the delay unit. The weight matrix $\mathbf{w_{N,n}}$ can be used to estimate $\hat{x}(n)$ of $x(n)$ by a linear combination of the current and past input samples. Then the prediction residual error is

$$\mathbf{u}_n = \mathbf{x}_n^T - X_{N,n}\mathbf{w}_{N,n} \tag{11}$$

We define $\epsilon_N(n)$ - the residual error power of the RLS predictor as a performance measure,

$$\epsilon_N(n) = \mathbf{u}_n^T\mathbf{u}_n \tag{12}$$

The optimal RLS filter minimizing the optimum error power is given by [5]

$$\mathbf{w}_{N,n} = \mathbf{x}_n^T X_{N,n}(X_{N,n}^T X_{N,n})^{-1} \tag{13}$$

As the first order derivatives of $\epsilon_N(n)$ with respective to the weight vector are zero for the optimal RLS filter, therefore, consider the cost function $J(w_o, w_1, \cdots, w_{N-1})$ where $\partial J/\partial w_i = 0, i = 0, \cdots, N-1$. Then, the second order derivatives due to small perturbations Δw_i is

$$\Delta J \approx \frac{1}{2}\sum_{i=0}^{N-1}\sum_{i=0}^{N-1}(\frac{\partial^2 J}{\partial w_i \partial w_j} \mid_\infty \Delta w_i \Delta w_j) \tag{14}$$

Assume that the perturbations Δw_i are zero mean, independent random variables. Hence the mean value $E\{dJ\}$ is

$$E\{\Delta J\} = \frac{1}{2}\sum_{i=0}^{N-1}(\frac{\partial^2 J}{\partial^2 w_i} \mid_\infty)E\{(\Delta w_i)^2\} \neq 0 \tag{15}$$

Define $\varepsilon_i = (\Delta w_i)^2$ and $E\{\varepsilon_i\} = \bar{\varepsilon}$. Thus

$$E\{\Delta J\} = \frac{1}{2}\sum_{i=0}^{N-1}(\frac{\partial^2 J}{\partial^2 w_i} \mid_\infty)\bar{\varepsilon} \tag{16}$$

Define $E\{\varepsilon_i^2\} = \bar{\varepsilon^2}$, the variance of ΔJ is [5]

$$\sigma^2_{\Delta J} = \frac{\bar{\varepsilon}^2}{4}\sum_{i=0}^{N-1}(\frac{\partial^2 J}{\partial^2 w_i}\mid_\infty)^2 + \bar{\varepsilon}^2\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}(\frac{\partial^2 J}{\partial^2 w_i}\mid_\infty)^2 \tag{17}$$

Assume that $\epsilon_N(n)$ is the performance measure of the optimal RLS algorithm, which is minimized, we define the deviation from $\epsilon_N(n)$ as there is the value $\triangle_\epsilon(n)$ due to perturbations:

$$\epsilon'_N(n) = \epsilon_N(n) + \triangle_\epsilon(n) \tag{18}$$

Now we try to derive the mean value and variance of this deviation from the performance measure since the perturbations are random variables, $\triangle_\epsilon(n)$ is random. From (11) and (12), we can get

$$\frac{\partial^2 \epsilon_N(n)}{\partial \mathbf{w}^2_{N,n}} = 2X^T_{N,n}X_{N,n} \tag{19}$$

Define the matrix $\Phi(n)$

$$\Phi(n) = X^T_{N,n}X_{N,n} \tag{20}$$

which determines the sensitivity of the algorithm to perturbation in the weight coefficients. For our adopted RLS algorithm

$$\Phi(n) = X^T_{N,n}X_{N,n} = \sum_{i=0}^{n}\lambda^{n-i}x(i)x^T(i) \tag{21}$$

For n becomes large, we have

$$E\{x(i)x^T(i)\} = R_x \tag{22}$$

where R_x is the sample autocorrelation matrix. Since $\lambda < 1$, hence

$$E\{\Phi(n)\} = \sum_{i=0}^{n}\lambda^{n-i}R_x = \frac{1}{1-\lambda}R_x \tag{23}$$

If $x(n)$ is a white random process

$$R_x = \sigma_x^2 I \tag{24}$$

The mean value of the error power deviation due to random perturbation $\Delta_i(n)$ in the weights $w_i(n)$ was derived as [5] for n large, since $\lambda < 1$:

$$E\{\Delta_\epsilon(n)\} = \frac{1}{1-\lambda}N\bar{\varepsilon}\sigma_x^2 \tag{25}$$

where $\varepsilon = E[\Delta_i^2(n)]$ and σ_x^2 is the variance of the input samples, N is the filter order. The variance of the deviation increases for white signals as

$$\sigma^2_{\Delta_\epsilon} = \frac{N\sigma_x^4\bar{\varepsilon}^2}{(1-\lambda)^2} \tag{26}$$

For correlated signals R_x,

$$R_x = \begin{bmatrix} R_x(0) & r_x(1) & \cdots & r_x(N-1) \\ r_x(1) & r_x(0) & \cdots & r_x(N-2) \\ \vdots & \vdots & \ddots & \vdots \\ r_x(N-1) & r_x(N-2) & \cdots & r_x(0) \end{bmatrix} \tag{27}$$

of which each element $r_x(i)$ is

$$r_x(i) = E\{x(n)x(n-i)\} \tag{28}$$

The $r_x(0)$ is

$$r_x(0) = \sigma_x^2 \tag{29}$$

It is well known that the scalar function of square matrix R can be expressed as $g(R)$ such that

$$g(R_x) = \underbrace{\sum\sum}_{i\neq j} r_{ij}^2 \tag{30}$$

Since R_x is symmetric, from (15), (23) and (30), we have the mean value of and variance of the deviation from the performance measure (16) and (17) , respectively

$$E\{\Delta_\epsilon(n)\} = \frac{1}{1-\lambda}N\bar{\varepsilon}\sigma_x^2 \tag{31}$$

and

$$\sigma^2_{\Delta_\epsilon} = \frac{N\sigma_x^4\bar{\varepsilon}^2}{(1-\lambda)^2} + \frac{4N\bar{\varepsilon}^2\sum_{i=1}^{N-1}r_i^2(i)}{(1-\lambda)^2} \tag{32}$$

$$\ll -\frac{3N\sigma_x^4\bar{\varepsilon}^2}{(1-\lambda)^2} + \frac{4N\bar{\varepsilon}^2\lambda_{max}}{(1-\lambda)^2} \tag{33}$$

During the derivation process, the mean of the deviation is the same for equal power of uncorrelated and correlated signals Eq.(25) and Eq.(31). From the formula (32), we can see that the the variance of deviation increases with the variance of the input σ_x^2 and λ_{max} the maximum eigenvalue of the input autocorrelation matrix. In order to get lossless audio signals, the algorithm can be implemented only in fixed point rather than floating point. On the other hand, the large variance of the input data may make the stalling phenomenon occur when the tap weights in the RLS algorithm stop adapting [1]. This phenomenon occurs when the matrix $Q(n)$ become very small. Consider the correlation matrix $\Phi(n)$ that the expectation of $\Phi(n)$ is given in Eq.(23). For λ is close to 1, we get

$$E[Q(n)] = E[\Phi^{-1}(n)] \simeq (E[\Phi(n)])^{-1} \tag{34}$$

We have, using Eq.(23)in Eq.(34) for large n

$$E[Q(n)] \simeq (1-\lambda)R_x^{-1} \tag{35}$$

This equation reveals that the RLS algorithm may stall if the exponential weighting for λ is close to 1 and/or the input data variance σ_x^2 is large. From these two aspects, it is

Table 1. The resulting average bitrate in bits/sample for different lossless codec

Item	Monkey 3.97	RLS-LMS	TUB
Blackandtan	8.67	8.62	8.96
Dcymbals	9.47	9.25	9.83
Fouronsix	7.21	7.15	7.49
Mfv	4.52	4.44	4.85
Unfo	7.95	7.86	8.31
Waltz	8.26	8.16	8.56

easy to explain that the performance of the RLS algorithm for audio clips quantized at 24 bit degrades compared to 16 bit signals, since the audio signals quantized at 24 bit has large variance. It leads to the output of the RLS with large variance, which may increase the eigenvalues spread of inputs to the LMS algorithm. The final residual error becomes large due to the slow convergence of the LMS algorithm.

4. SIMULATION RESULTS

We carried out some experimental work to evaluate the performance of the RLS-LMS predictor. In the first work, we built up a lossless audio codec to evaluate the practical performance of the RLS-LMS predictor. In this codec, the audio signal in PCM or wav format is segmented into frames with 4096 samples and then is feed to a cascaded RLS-LMS predictor to produce the residual error. The residual error is coded using a Rice code and LZARI algorithm. In the decoder, the signal flow chart is reversed to obtain the exact copy of the original audio signal. We use 6 audio files in our test, which are stereo, sampled at 48 kHz and quantized at 16 bit. We compare the compression performance of the RLS-LMS codec with the state-of-the-art audio codecs of the Technical University of Berlin (TUB, MPEG-4 RM0) and Monkey's audio compression (the benchmark codec) under their highest compression ratio setting. The results are given in Table 1. Clearly, the best compression performance is achieved by the RLS-LMS codec, which outperforms the Monkey's audio codec with average reduced 0.15 bit/sample in all the music files.

In the second experimental work, we evaluate the prediction gain for audio signals sampled at 48 kHz and quantized at $16, 20, 24$ bit respectively. The results are shown in Table 2. From the table, we observe that the predictive coding gain of signals decreases as long as the resolution of signals increases. Our theoretical analysis can give a reasonable and logical explanation.

5. CONCLUSION

This paper analyzes the effect of the correlated input on the performance of the RLS-LMS predictor for lossless audio

Table 2. The prediction gain of RLS-LMS for different resolution audio signals

Track (48 kHz)	16 bit	20 bit	24 bit
RLS-LMS	32.2463 dB	29.5968 dB	29.5296 dB

compression. The analysis shows that the mis-adjustment (steady-state error) is the same for equal power correlated and white signals. However, the variance of the deviation increases for correlated signal and signal with large variance, which may lead to the degraded performance of the RLS algorithm compared to white signals or signal with smaller variance. The signals with large variance results in larger output of the RLS algorithm compared to that with low variance. It increases the eigenvalues spread of inputs to the LMS predictor, which results in slow convergence for high resolution signals. Experimental results of the RLS-LMS predictor for audio clips at sampling rate 48 kHz and quantized at 16, 20, 24 bit verified our theoretical analysis. The performance bound of the RLS-LMS algorithm is under the way of studying.

6. REFERENCES

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