TOWARDS OPTIMAL QUANTIZATION IN MULTISTAGE AUDIO CODING

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ABSTRACT

In this work, we develop a new method for quantization in multistage audio coding. We consider the case of a two-stage sinusoidal/waveform coder. Given a distortion measure and a bit-rate constraint, we analytically derive the optimal rate distribution between subcoders (stages) and the corresponding optimal quantizers, which allows the coder to easily adapt to changes in bit-rate requirements. We verify that the performance, both in terms of signal-to-noise ratio (SNR) and perceptual quality, is higher if the input to the second stage is obtained by subtracting the quantized first-stage reconstruction from the original signal, as opposed to subtracting the unquantized reconstruction.

1. INTRODUCTION

Multistage (residual) coding has found extensive use for low bit-rate representations of speech and audio signals [1–5]. In a multistage coder, each subcoder (stage) contributes to the total reconstruction by approximating a residual signal, i.e., the difference between the original input signal and the reconstruction obtained with the previous stages, and forms a new residual for the next stage. Each subcoder aims at representing those features of the input for which it is most efficient. Thus, first stages are typically model-based subcoders, and the last stage is often a waveform subcoder. In [1,3], a CELP coder is used at the first stage to efficiently model clean speech signals, and then a waveform (MDCT respectively adaptive tree) subcoder is applied to the reconstruction error of CELP to enhance performance for noisy speech and general audio signals. In [4, 5], a sinusoidal subcoder models the tonal part of an original audio signal, and then a wavelet transform subcoder is applied to represent the resulting residual.

Conventionally, the available bit rate is distributed between subcoders heuristically, based on an informal judgment of perceptual importance of the corresponding signal features. In our work, given a distortion measure, we derive the optimal rate distribution between the subcoders and the corresponding optimal quantizers. The quantizers minimize the final distortion at the output of the complete coder under a given bit-rate constraint. We consider the case of a sinusoidal subcoder, followed by a waveform subcoder.

We apply high-rate theory to analytically derive the optimal quantizers (no iterative or ad-hoc steps are used in quantizer design). High-rate theory assumes small quantization errors, or, more formally, that the probability densities of quantized variables are accurately approximated as being constant within quantization cells. In high-rate theory, quantizers are described with quantization point densities (the reciprocals of quantization step sizes), without exactly specifying quantization points. In [6, 7], high-rate theory was used to analyze performance of multistage vector quantization. Here, we generalize the approach to multistage audio coding, where successive stages are subcoders of various types.

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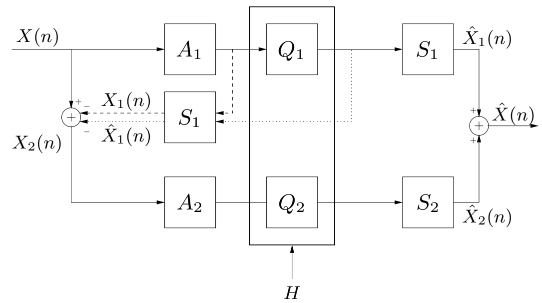

Fig. 1. Two-stage coder. A - analysis, Q - quantization, S - synthesis. The dashed arrows illustrate parallel coding, while the dotted arrows illustrate sequential coding.

We will analyze and compare two methods, which we refer to as 'parallel' and 'sequential' coding, cf. Fig. 1. The input to the second stage (the residual signal), $X_2(n)$, is obtained by subtracting the unquantized first-stage reconstruction, $X_1(n)$, (parallel method) or quantized first-stage reconstruction, $\hat{X}_1(n)$, (sequential method) from the original signal, $X(n)$.

In the examples presented in this paper we consider the mean-square error (MSE) distortion measure. However, the framework can be easily generalized to a weighted distortion measure that accounts for properties of human auditory perception. The bit-rate constraint is formulated as a given average rate (entropy constraint), as opposed to a given fixed rate (resolution constraint). This is advantageous for statistical communication networks.

The remainder of the paper is organized as follows. Section 2 presents a theoretical analysis for a synthetic input signal and a two-stage sinusoidal/PCM coder. Section 3 presents a practical case of real audio signals and a two-stage sinusoidal/MDCT coder. Section 4 summarizes the paper and outlines the future work.

2. SYNTHETIC SIGNAL, SINUSOIDAL/PCM CODER

The simple example of this section provides an insight into quantization in a two-stage sinusoidal/waveform coder and introduces a framework that is generalized and applied to quantization of audio signals in Section 3. We define the original input signal as

$$X(n) = \sum_{l=0}^{L-1} A_l \cos(\Omega_l n + \Phi_l) + M(n), \quad n = 0, \ldots, N-1, \quad (1)$$

where $\{A_l, \Omega_l, \Phi_l\}_{l=0}^{L-1}$ are amplitudes, frequencies and phases of L sinusoids, and $M(n)$ are noise samples. We use the frequency

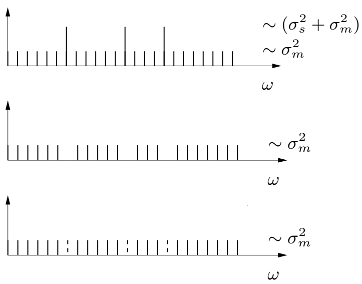

Fig. 2. A schematic illustration of the discrete power spectrum at positive frequencies: input signal (upper plot), input to the second stage in the parallel method (middle plot), input to the second stage in the sequential method (lower plot).

domain to define the signal statistics. The real and imaginary parts of the discrete Fourier transform (DFT) of a sinusoid, noise, and sinusoid plus noise are Gaussian variables, $\mathcal{N}(0,\sigma_s)$, $\mathcal{N}(0,\sigma_m)$, and $\mathcal{N}(0,\sqrt{\sigma_s^2+\sigma_m^2})$, respectively, cf. Fig. 2 upper plot. We assume that at the sinusoidal frequencies the entire contribution of sinusoids and noise is captured by the sinusoidal subcoder.

The sinusoidal subcoder (the first stage) uses an unrestricted polar quantizer (UPQ) [8] to quantize the sinusoidal amplitudes and phases (the sinusoidal frequencies are not quantized), and the PCM subcoder (the second stage) uses a scalar quantizer (SQ) to quantize the time-domain residual signal. The goal is to find the optimal rate distribution between the two stages and the corresponding optimal quantizers, which minimize the MSE distortion evaluated over a segment of the input signal of length N:

$$D = E[\frac{1}{N}\sum_{n=0}^{N-1}(X(n)-\hat{X}(n))^2], \qquad (2)$$

where $\hat{X}(n)$ is the reconstructed signal and $E[\cdot]$ denotes expectation. The constraint is that the sum of sinusoidal rate, H_1, and PCM rate, H_2, does not exceed a given entropy: $H_1+H_2\le H$.

2.1. Parallel coding

In parallel coding, the input to the second stage is $X_2(n) = X(n)-X_1(n)$. The spectrum has gaps at the sinusoidal frequencies, cf. Fig. 2 middle plot. The distortion (2) is

$$D_{pc} = E[\frac{1}{N}\sum_{n=0}^{N-1}((X_1(n)-\hat{X}_1(n))^2+(X_2(n)-\hat{X}_2(n))^2 \\ +2(X_1(n)-\hat{X}_1(n))(X_2(n)-\hat{X}_2(n)))]. \qquad (3)$$

The distortion is a contribution of a sinusoidal error term, a PCM error term, and a cross-term. Assuming that the sinusoidal and PCM errors are uncorrelated (the cross-term is zero), and that the quantization errors of individual sinusoids are uncorrelated (in practice, these assumptions are justified), and using $\sum_{n=0}^{N-1}\cos^2(\Omega_l n)/N = \sum_{n=0}^{N-1}\sin^2(\Omega_l n)/N \approx 1/2$, $\sum_{n=0}^{N-1}\cos(\Omega_l n)\sin(\Omega_l n)/N \approx 0$ we can rewrite the distortion:

$$D_{pc} \approx \frac{L}{2}E[A^2+\hat{A}^2-2A\hat{A}\cos(\Phi-\hat{\Phi})]+E[(X_2-\hat{X}_2)^2], \qquad (4)$$

where the subscript l and the index n are omitted since the same UPQ is used for all sinusoids and the same SQ is used for all time-domain residual samples.

Given a sinusoidal rate H_1 (H_1/L per sinusoid) and a PCM rate H_2 (H_2/N per time-domain sample), distortion (4) can be minimized by optimizing the sinusoidal and PCM quantizers individually. We use high-rate theory results for entropy-constrained UPQ (in UPQ, phase quantization depends on amplitude) [8] and SQ [9] to write the optimal quantization point densities:

$$g_A = 2^{\frac{1}{2}(H_1/L-h(A)-h(\Phi)-b(A))}, \qquad (5)$$

$$g_\Phi(a) = a2^{\frac{1}{2}(H_1/L-h(A)-h(\Phi)-b(A))}, \qquad (6)$$

$$g_{X_2} = 2^{H_2/N-h(X_2)}, \qquad (7)$$

where $h(A)=0.5\log_2(0.5e^{2+\gamma}(\sigma_s^2+\sigma_m^2))$, $h(\Phi)=\log_2(2\pi)$, $h(X_2) = (N/2-L)/N\log_2(4\pi e\sigma_m^2)$ are the differential entropies of sinusoidal amplitude and phase, and the time-domain residual sample, respectively, $b(A) = \int f_A(a)\log_2(a)da = 0.5\log_2(2e^{-\gamma}(\sigma_s^2+\sigma_m^2))$ is an auxiliary variable, γ is the Euler constant ($f_A(a)$ is the probability density function (pdf) of amplitude, a is a realization of the random variable A). Using the results of [8, 9], we find that the corresponding minimum distortion is

$$D_{pc} \approx \frac{L}{12}2^{-(H_1/L-h(A)-h(\Phi)-b(A))}+\frac{1}{12}2^{-2(H_2/N-h(X_2))}. \qquad (8)$$

The goal is to find the optimal rate distribution H_1 and H_2 ($H_1+H_2=H$). We define the rate-distribution parameter $\alpha\in[0,1]$, such that $H_1=\alpha H$, $H_2=(1-\alpha)H$. Substituting this into (8), we find the optimal value for α by solving $\partial D/\partial\alpha=0$:

$$\alpha_{pc} = \frac{NL(2H/N+\log_2(N/2)+h(A)+h(\Phi)+b(A)-2h(X_2))}{(N+2L)H}. \qquad (9)$$

2.2. Sequential coding

In sequential coding, the input to the second stage is $X_2(n) = X(n)-\hat{X}_1(n)$. At the sinusoidal frequencies, the spectrum is defined by the sinusoidal quantization error, cf. Fig. 2 lower plot. The distortion (2) is equal to the error of the second stage:

$$D_{sc} = E[\frac{1}{N}\sum_{n=0}^{N-1}(X_2(n)-\hat{X}_2(n))^2]. \qquad (10)$$

We use the high-rate theory results for SQ [9] to write the minimum distortion for given sinusoidal rate H_1 and PCM rate H_2:

$$D_{sc} \approx \frac{1}{12}2^{-2(H_2/N-h(X_2(H_1/L)))}, \qquad (11)$$

where we show that the residual signal, $X_2(n)$, and its differential entropy, $h(X_2)$, depend on the sinusoidal quantization error and, thus, the sinusoidal rate, H_1 (H_1/L per sinusoid).

To find the optimal rate distribution H_1 and H_2, we again use a parameter α. Since $h(X_2)$ depends on α (recall that $H_1 = \alpha H$), at this point we cannot solve for the optimal α analytically. We evaluate the differential entropy $h(X_2)$ and the distortion (11) for a grid of α (for each α, $h(X_2)$ is evaluated numerically) and save the value of α that results in minimum distortion. Table 1 shows the intermediate results for different input settings. The right-most column shows the resulting variance of the real part of the sinusoidal quantization error: $E[(A\cos(\Phi)-\hat{A}\cos(\hat{\Phi}))^2] = 2^{-(\alpha H/L-h(A)-h(\Phi)-b(A))}/12$. The last expression follows from [8]. An important result is that the variance of the sinusoidal quantization error is very close to the variance of the original noise: $2^{-(\alpha H/L-h(A)-h(\Phi)-b(A))}/12 \approx \sigma_m^2$. This result

		Input			Result	
N	L	σ_s^2	σ_m^2	H	α	$E[(A\cos(\Phi) - \hat{A}\cos(\hat{\Phi}))^2]$
512	10	10	1	1000	0.040	0.98
256	10	10	1	1000	0.040	0.98
512	25	10	1	1000	0.100	0.98
512	10	10	1	700	0.058	0.94
512	10	100	1	1000	0.071	1.05
512	10	10	0.1	1000	0.072	0.10

Table 1. Optimal rate distribution in the sinusoidal/PCM coder (sequential method).

is rather intuitive: the optimal sinusoidal subcoder tries to flatten the spectrum of the input to the PCM subcoder. We use this result to select the rate-distribution parameter:

$$\alpha_{sc} = \frac{L(h(A) + h(\Phi) + b(A) - \log_2(12\sigma_m^2))}{H}. \quad (12)$$

At any total entropy, H, the average rate per sinusoid is fixed: $\alpha_{sc}H/L = h(A) + h(\Phi) + b(A) - \log_2(12\sigma_m^2)$. Given α, the optimal quantizers are again of the form (5)-(7).

3. AUDIO SIGNALS, SINUSOIDAL/MDCT CODER

We now generalize the framework of Section 2 to coding of audio signals. We consider a two-stage coder, where the first stage is a sinusoidal subcoder, and the second stage is an MDCT subcoder. The quantizers are derived for a segment of an audio signal, a typical segment length being 100-200 ms. Within the segment, the sinusoidal and MDCT subcoders apply individual subsegmentations, which are in general not equal, cf. Fig. 3. We find the optimal rate distribution between the two subcoders, between subsegments in the individual subcoders, and the corresponding optimal quantizers for sinusoidal and MDCT parameters. The unquantized sinusoidal reconstruction is

$$X_1(n) = \sum_{m_1} e_{m_1}(n - M_{m_1}) \sum_{l=0}^{L_{m_1}-1} A_{m_1 l} \cos(\Omega_{m_1 l}(n - M_{m_1}) + \Phi_{m_1 l}), \quad (13)$$

where m_1 is the sinusoidal subsegment index, $e_{m_1}(n - M_{m_1})$ is the corresponding overlap-add synthesis window shifted by M_{m_1}, and $\{A_{m_1 l}, \Omega_{m_1 l}, \Phi_{m_1 l}\}_{l=0}^{L_{m_1}-1}$ are amplitudes, frequencies and phases of L_{m_1} sinusoids in subsegment m_1. In the parallel method the residual signal is $X_2(n) = X(n) - X_1(n)$, while in the sequential method it is $X_2(n) = X(n) - \hat{X}_1(n)$, where $\hat{X}_1(n)$ is the sinusoidal reconstruction after quantizing the amplitudes and phases. The residual signal is represented with the MDCT [10]:

$$\begin{aligned} X_2(n) = \sum_{m_2} & \frac{e_{m_2}(n - M_{m_2})}{L_{m_2}} \\ & (\cos(\frac{m_2\pi}{2}) \sum_{l=0}^{L_{m_2}/2} C_{m_2 l} \cos(\frac{2\pi l}{L_{m_2}}(n + n_{m_2} - M_{m_2})) + \\ & \sin(\frac{m_2\pi}{2}) \sum_{l=0}^{L_{m_2}/2} C_{m_2 l} \sin(\frac{2\pi l}{L_{m_2}}(n + n_{m_2} - M_{m_2}))), \quad (14) \end{aligned}$$

where m_2 is the MDCT subsegment index, L_{m_2} is the subsegment length, $e_{m_2}(n - M_{m_2})$ is a perfect-reconstruction analysis/synthesis window shifted by M_{m_2}, n_{m_2} is a shift to avoid time aliasing, and $\{C_{m_2 l}\}_{l=0}^{L_{m_2}/2}$ are unique transform coefficients in subsegment m_2. Sinusoidal frequencies are selected from a redundant dictionary, while MDCT is an orthogonal transform.

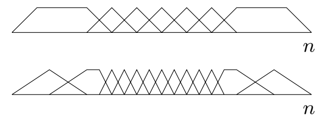

Fig. 3. An example of subsegmentation in a sinusoidal/MDCT coder.

We quantize the sinusoidal amplitudes and phases using UPQ and MDCT coefficients using SQ. The sinusoidal frequencies are assumed to be quantized to their original values and we count frequency quantization index entropies $H(I_{\Omega,m_1})$ in the total rate. The goal is to minimize MSE distortion of the form (2) where N is the segment length, under the constraint that the total rate of the sinusoidal and MDCT subcoders does not exceed a given entropy: $H_1 + H_2 \leq H$. The derivations are generalizations of those in Section 2. Therefore, we only summarize the results.

3.1. Optimal quantizers

In both parallel and sequential method, the optimal quantization point densities can be expressed as

$$g_{A,m_1} = (P_{m_1} 2^{-F_1/G_1} 2^{\tilde{H}_1/G_1})^{\frac{1}{2}}, \quad (15)$$

$$g_{\Phi,m_1}(a) = (a^2 P_{m_1} 2^{-F_1/G_1} 2^{\tilde{H}_1/G_1})^{\frac{1}{2}}, \quad (16)$$

$$g_{C,m_2} = (\frac{L_{m_2} + 2}{L_{m_2}^3} P_{m_2} 2^{-F_2/G_2} 2^{\tilde{H}_2/G_2})^{\frac{1}{2}}, \quad (17)$$

where $F_1 = \sum_{m_1} L_{m_1} \log_2(P_{m_1})$, $G_1 = \sum_{m_1} L_{m_1}$, $\tilde{H}_1 = H_1 - \sum_{m_1} L_{m_1}(H(I_{\Omega,m_1}) + h(A) + h(\Phi) + b(A))$, $F_2 = \sum_{m_2} \frac{L_{m_2}}{4} \log_2(\frac{L_{m_2}+2}{L_{m_2}^3} P_{m_2})$, $G_2 = \sum_{m_2} \frac{L_{m_2}}{4}$, $\tilde{H}_2 = H_2 - \sum_{m_2} \frac{L_{m_2}}{2} h(C)$, P is the energy of a synthesis window normalized by the segment length, $h(A)$, $h(\Phi)$, $h(C)$ are the differential entropies of sinusoidal amplitude and phase, and MDCT coefficient, respectively, and $b(A) = \int f_A(a) \log_2(a) da$. In both methods, the parameters $h(A)$, $h(\Phi)$ and $b(A)$ can be estimated a-priori using an audio database and stored in memory. The optimal rate distribution described by the parameter $\alpha \in [0, 1]$ ($H_1 = \alpha H$, $H_2 = (1 - \alpha)H$) is different for the two methods.

In the parallel method, the differential entropy of MDCT coefficient, $h(C)$, does not depend on sinusoidal rate. The optimal value for α can be found in a closed form:

$$\begin{aligned} \alpha_{pc} = \frac{1}{(G_1 + G_2)H} & (G_1 H + F_1 G_2 - G_1 F_2 - G_1 \sum_{m_2} \frac{L_{m_2}}{2} h(C) \\ & + G_2 \sum_{m_1} L_{m_1}(H(I_{\Omega,m_1}) + h(A) + h(\Phi) + b(A))). \quad (18) \end{aligned}$$

In the sequential method, the differential entropy of MDCT coefficient, $h(C)$, depends on the sinusoidal rate, $H_1 = \alpha H$, which makes it difficult to find α analytically. However, similarly to the results of Section 2.2, using an audio database we found that, independently of the total rate H, the optimal equivalent average rate for amplitude and phase of one sinusoid is fixed (we define the equivalent average rate as $H(I_A, I_\Phi) = H(I_{A,m_1}, I_{\Phi,m_1}) - \log_2(P_{m_1})$, i.e., we take into account that longer subsegments are assigned more rate). Thus, we can estimate the optimal equivalent rate, $H(I_A, I_\Phi)$, in advance and store it in memory. Then, the value for α is

$$\alpha_{sc} = \frac{\sum_{m_1} L_{m_1}(H(I_{\Omega,m_1}) + H(I_A, I_\Phi) + \log_2(P_{m_1}))}{H}. \quad (19)$$

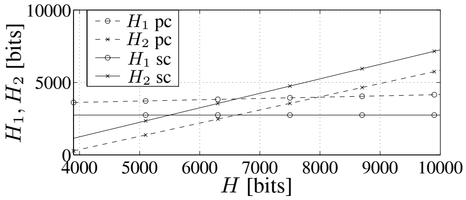

Fig. 4. Rate distribution in the sinusoidal/MDCT coder (pc - parallel coding, sc - sequential coding).

3.2. Experimental results

We have applied the sinusoidal/MDCT coder to speech and audio signals. In the experiments reported here, the lengths of the sinusoidal and MDCT subsegments are both 1024 samples, hop-sizes are 512 samples, and the length of the audio segments is 4608 samples (ca. 105 ms at sampling frequency 44.1 kHz). The sinusoidal analysis applies a matching pursuit algorithm [11] to find 25 sinusoids within each subsegment. The size of the sinusoidal frequency dictionary is 2048, and the estimated entropy of the sinusoidal quantization indices is $H(I_{\Omega,m_1}) = H(I_\Omega) = 8.5$ bits.

Fig. 4 shows the optimal rate distribution. For both parallel and sequential methods, we observe the expected result that at low total rates most rate is assigned to the sinusoidal (model-based) subcoder, while at higher total rates most bits are assigned to the MDCT (waveform) subcoder. The sinusoidal rate in the sequential method is the same at all total rates. The MDCT subcoder is assigned more rate in the sequential method than in the parallel method. Fig. 5 shows the signal-to-noise ratios (SNR) predicted by the high-rate theory and obtained in practice with quantizers that are based on the theory. The distortion-rate performance is higher for the sequential method. For both methods, we observe that the difference between the theoretical and practical performance is quite small (e.g., at segment rate $H = 8000$ bits, the theoretical and practical SNR are 27.3 and 25.1 dB in the parallel method, and 28.4 and 26.2 dB in the sequential method). The difference between the theoretical and practical distortion decreases with increasing rate.

We have also performed informal listening tests, which revealed a slight preference for the sequential method. We have identified two perceptual effects. First, the burbling artifact typical for sinusoidal coders is less audible in the sequential method. This is due to the fact that the sinusoidal quantization error is frequency-spread by the MDCT subcoder. Second, a lowpass-filtering effect was stronger in the parallel method. This is due to the fact that high-frequencies are mainly represented by the MDCT subcoder, which receives less bits in the parallel method.

4. SUMMARY

In this paper, we developed a new framework for quantization in multistage audio coding. The framework is applied to a two-stage sinusoidal/waveform coder. For a given bit-rate constraint, we analytically derive optimal rate distribution between subcoders and optimal quantizers. Thus, the coder can easily adapt to changes in bit-rate requirements. Despite the simple structure of the presented coder, initial listening tests indicate promising results, especially for the sequential method. Including a perceptual distortion measure, differential coding of sinusoidal parameters and vector quantization of MDCT coefficients into the proposed analytical framework should result in a practical competitive audio coder.

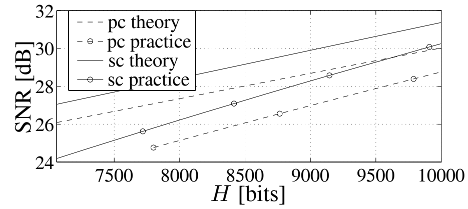

Fig. 5. Theoretical and practical performance of the sinusoidal/MDCT coder (pc - parallel coding, sc - sequential coding).

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