A LOW-COST AND FAST CONVERGENCE GAUSS-SEIDEL PSEUDO AFFINE PROJECTION ALGORITHM FOR MULTICHANNEL ACTIVE NOISE CONTROL

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ABSTRACT

It is well known that fast affine projection algorithms can produce a good trade-off between convergence speed and computational complexity. In this paper, we propose a new pseudo fast affine projection algorithm based on the Gauss-Seidel scheme for multichannel active noise control (ANC) systems. It is shown that the proposed algorithm has a lower complexity than previously published fast affine projection algorithms, with similar convergence properties and better numerical stability.

1. INTRODUCTION

Active noise control systems are being increasingly researched and developed [1]. In such systems, an adaptive controller is used to optimally cancel unwanted acoustic noise. The delay compensated modified filtered-x structure [2] for active noise control systems using FIR adaptive filtering is presented in Fig. 1. Fast affine projection (FAP) algorithms suitable for active noise control were introduced in [3]-[5]. For multichannel active noise control, the delay-compensated modified filtered-x FAP algorithm with sliding window RLS (or MFXFAP-RLS) algorithm was derived in [4], providing the expected trade-off between performance and complexity, and also providing an improved performance when noisy plant models (Fig. 1) are used, as is often the case in practice. In [5], the delay-compensated modified filtered-x Gauss-Seidel FAP algorithm (or MFX-GSFAP) for multichannel active noise control was derived from the GSFAP algorithm recently introduced in [6], providing exactly the same performance as the MFXFAP-RLS in [4], but with a lower complexity. The algorithm introduced in this new paper is based on a Pseudo Affine Projection (or PAP) algorithm [7], derived from the original affine projection algorithm and using a Levinson-Durbin recursion under some realistic hypothesis. Starting from the same hypothesis and replacing the Levinson-Durbin recursion with the Gauss-Seidel method, a simpler algorithm was recently derived in [8], called the Gauss-Seidel Pseudo Affine Projection algorithm (or GSPAP). The complexity of the GSPAP is lower than the GSFAP, with a similar performance.

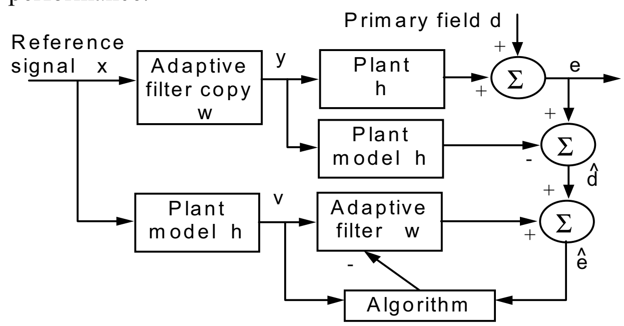

Fig. 1. A delay compensated modified filtered-x structure for active noise control

In Section 2, we propose the adaptation of the GSPAP to multichannel active noise control systems. The objective is to have good performance (similar to the one reported in [4], [5]), at a lower cost and with good numerical stability. In Section 3, the results of simulations comparing the new proposed algorithm with previously published algorithms are presented, and the computational complexity of the proposed algorithm is evaluated in Section 4. Section 5 concludes this work.

2. THE MULTICHANNEL MODIFIED FILTERED-X GAUSS-SEIDEL PSEUDO AFFINE PROJECTION ALGORITHM

To describe the multichannel delay-compensated modified filtered-x Gauss-Seidel Pseudo Affine Projection algorithm (or MFXGSPAP), the following notation is defined:

- I number of reference sensors in an ANC system
- J number of actuators in an ANC system
- K number of error sensors in an ANC system
- L length of the adaptive FIR filters
- N affine projection order
- M length of (fixed) FIR filters modeling the plant in an ANC system
- $x_i(n)$ value at time n of the i^{th} reference signal

$y_j(n)$ value at time n of the j^{th} actuator signal

$d_k(n)$ value at time n of the primary sound field at the k^{th} error sensor

$e_k(n)$ value at time n of the k^{th} error sensor

$\hat{d}_k(n)$ estimate of $d_k(n)$, computed in delay-compensated or modified filtered-x structures

$\hat{e}_k(n)$ value at time n of an alternative error signal for the k^{th} sensor, computed in delay-compensated or modified filtered-x structures

$h_{j,k,m}$ value of the m^{th} coefficient in the (fixed) FIR filter modeling the plant between $y_j(n)$ and $e_k(n)$

$v_{i,j,k}(n)$ value at time n of the filtered reference signal

$w_{i,j,l}(n)$ value at time n of the l^{th} coefficient in the adaptive FIR filter linking $x_i(n)$ and $y_j(n)$

$\boldsymbol{R}(n)$ $KN \times KN$ auto-correlation matrix. $\boldsymbol{R}(n)$ is initialized as an identity matrix multiplied by the regularization factor δ_1.

$\overline{\boldsymbol{R}}(n)$ the top left $K(N-1)\times K(N-1)$ values of $\boldsymbol{R}(n)$

$\boldsymbol{P}(n)$ an inverse correlation matrix of size $KN \times K$

$\overline{\boldsymbol{P}}(n)$ the top $K \times K$ components of $\boldsymbol{P}(n)$

$\boldsymbol{b}$ a matrix of size $KN \times K$ whose elements are zeros, except for the top $K \times K$ values which are set to an identity matrix

$\boldsymbol{r}(n)$ correlation matrix of size $K(N-1)\times K$, initialized with zero values

$\bar{\boldsymbol{r}}(n)$ correlation matrix of size $K \times K$, initialized with zero values

$\underline{\boldsymbol{a}}(n)$ the last $K(N-1)$ rows of $\boldsymbol{a}(n)$ (defined below)

$\boldsymbol{u}(n)$ decorrelated filtered reference signals of size $IJ \times K$, computed by the PAP algorithm

$\boldsymbol{M}(n)$ inverse matrix of size $K \times K$, recurrently computed by the PAP algorithm, initialized with an identity matrix multiplied by the regularization factor δ_2

$\boldsymbol{h}_{j,k} = \left[h_{j,k,1}, h_{j,k,2}, \cdots h_{j,k,M}\right]^T$ (size $M \times 1$)

$\boldsymbol{x}_i(n) = \left[x_i(n), x_i(n-1), \cdots x_i(n-L+1)\right]^T$ (size $L \times 1$)

$\boldsymbol{x}'_i(n) = \left[x_i(n), x_i(n-1), \cdots x_i(n-M+1)\right]^T$ (size $M \times 1$)

$\boldsymbol{y}_j(n) = \left[y_j(n), y_j(n-1), \cdots y_j(n-M+1)\right]^T$ (size $M \times 1$)

$\boldsymbol{v}_{i,j,k}(n) = \left[v_{i,j,k}(n), v_{i,j,k}(n-1), \cdots v_{i,j,k}(n-L+1)\right]^T$ (size $L \times 1$)

$\hat{\boldsymbol{d}}(n) = \left[\hat{d}_1(n), \hat{d}_2(n), \cdots \hat{d}_K(n)\right]$ (size $1 \times K$)

$\hat{\boldsymbol{e}}(n) = \left[\hat{e}_1(n), \hat{e}_2(n), \cdots \hat{e}_K(n)\right]$ (size $1 \times K$)

$$\boldsymbol{v}(n) = \begin{bmatrix} v_{1,1,1}(n) & \cdots & v_{1,1,K}(n) \\ \vdots & \ddots & \vdots \\ v_{I,J,1}(n) & \cdots & v_{I,J,K}(n) \end{bmatrix} \text{ (size } IJ \times K\text{)}$$

$$\boldsymbol{V}(n) = \begin{bmatrix} \boldsymbol{v}(n) \\ \vdots \\ \boldsymbol{v}(n-L+1) \end{bmatrix} \text{ (size } IJL \times K\text{)}$$

$\boldsymbol{a}(n) = \left[\boldsymbol{v}(n) \ \cdots \ \boldsymbol{v}(n-N+1)\right]^T$ (size $KN \times IJ$)

$\boldsymbol{w}(n) = \left[\left[w_{1,1,1}(n) \ \ldots \ w_{I,J,1}(n)\right] \ \cdots \ \left[w_{1,1,L}(n) \ \cdots \ w_{I,J,L}(n)\right]\right]^T$ (size $IJL \times 1$)

$$\boldsymbol{R}(n) = \begin{bmatrix} \bar{\boldsymbol{r}}(n) & \boldsymbol{r}^T(n) \\ \boldsymbol{r}(n) & \overline{\boldsymbol{R}}(n-1) \end{bmatrix} \text{ (size } KN \times KN\text{).}$$

$$\boldsymbol{U}(n) = \begin{bmatrix} \boldsymbol{u}(n) \\ \vdots \\ \boldsymbol{u}(n-L+1) \end{bmatrix} \text{ (size } IJL \times K\text{)}$$

The MFXGSPAP algorithm can be described by equations (1)-(10):

$$y_j(n) = \sum_{i=1}^{I} \boldsymbol{w}_{i,j}^T(n)\boldsymbol{x}_i(n) \quad (1)$$

$$v_{i,j,k}(n) = \boldsymbol{h}_{j,k}^T \boldsymbol{x}'_i(n) \quad (2)$$

$$\hat{d}_k(n) = e_k(n) - \sum_{j=1}^{J} \boldsymbol{h}_{j,k}^T \boldsymbol{y}_j(n) \quad (3)$$

$$\boldsymbol{r}(n) = \boldsymbol{r}(n-1) + \underline{\boldsymbol{a}}(n)\boldsymbol{v}(n) - \underline{\boldsymbol{a}}(n-L)\boldsymbol{v}(n-L) \quad (4)$$

$$\bar{\boldsymbol{r}}(n) = \bar{\boldsymbol{r}}(n-1) + \boldsymbol{v}^T(n)\boldsymbol{v}(n) - \boldsymbol{v}^T(n-L)\boldsymbol{v}(n-L) \quad (5)$$

$$\boldsymbol{R}(n)\boldsymbol{P}(n) = \boldsymbol{b} \quad \text{(to solve with Gauss-Seidel method)} \quad (6)$$

$$\hat{\boldsymbol{e}}^T(n) = \hat{\boldsymbol{d}}^T(n) + \boldsymbol{V}^T(n)\boldsymbol{w}(n) \quad (7)$$

$$\boldsymbol{u}(n) = \boldsymbol{a}^T(n)\boldsymbol{P}(n)\left(\overline{\boldsymbol{P}}(n)\right)^{-1} \quad (8)$$

$$M(n) = M(n-1) + u^T(n)v(n) - u^T(n-L)v(n-L) \qquad (9)$$

$$w(n+1) = w(n) - \mu U(n)\left(M(n)\right)^{-1} \hat{e}^T(n) \qquad (10)$$

Equation (6), to be solved with one Gauss-Seidel iteration, can be computed at a reduced rate. In (10), μ is a normalized convergence gain with $0 \le \mu \le 1$ (typically close to 1). Even though $P(n)$ is computed recurrently in each Gauss-Seidel iteration, the MFXGSPAP (and also the MFXGSFAP in [5]) computes $P(n)$ directly from the correlation matrix $R(n)$, unlike the MFXFAP-RLS [4] or other RLS-based algorithms [9], [10]. Therefore, it has the potential for an inherently better numerical stability, and this is verified in the simulations of Section 3.

The MFXGSPAP algorithm computes directly in (10) the time domain coefficients needed in (1) between the reference sensors and the actuators, unlike the previously published FAP algorithms for active noise control [3]-[5]. This can be interesting for several reasons. For example, the only equation that must absolutely be computed in real time in ANC applications is the computation of the actuator values in (1), all the other equations could possibly be computed offline, at a reduced rate, using recorded blocks of data (at the cost of having reduced tracking capabilities). Equation (1) is simpler in the case of the MFXGSPAP. Also, to have direct access to the time domain coefficients can be interesting because it provides more physical insights into the control system, for example to understand at the design stage the physical meaning of the solution found by the controller, to observe its causality and the number of required coefficients, etc..

3. SIMULATIONS

The MFXGSPAP, MFXGSFAP [5], MFXFAP-RLS [4], and the multichannel modified filtered-x LMS (MFXLMS, [9]) and RLS (MFXRLS, [10]) algorithms for ANC were simulated with acoustic transfer functions experimentally measured in a duct. The impulse responses used for the multichannel acoustic plant had 64 samples each ($M = 64$), while the adaptive filters had 100 coefficients each ($L = 100$). The convergence is defined as the ratio of the sum of the error signals power over the sum of primary field (i.e. the disturbance signals) power. It was found by simulations that a projection order of size $N = 10$ was sufficient for PAP/FAP algorithms to get a significantly improved convergence performance over a projection order of $N = 1$ (i.e. a normalized LMS algorithm). For the MFXGSPAP and MFXGSFAP algorithms, the update of $P(n)$ (i.e. solving equation (6) using the Gauss-Seidel method) was performed at a slower rate, with an update period of $p = 20$.

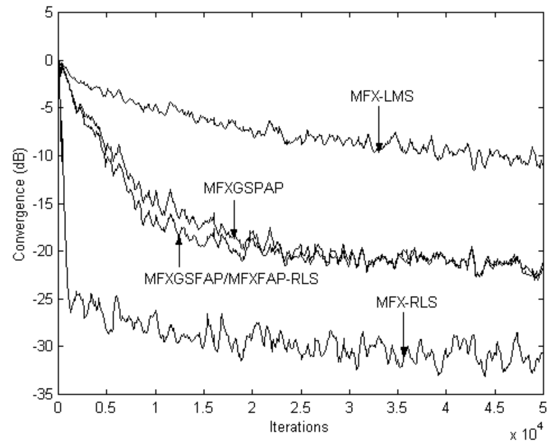

Fig. 2. Convergence curves of multichannel delay-compensated modified filtered-x algorithms for ANC, with ideal plant models (I=1, J=3, K=2, L=150, M=64).

Simulation results have shown that updating $P(n)$ at such a reduced rate did not affect significantly the convergence performance of the MFXGSPAP and MFXGSFAP algorithms, while reducing their complexity. Figure 2 compares the performance of the selected algorithms, with ideal plant models, for a multichannel system (I=1, J=3, K=2), and from Matlab™ simulations (double precision 64 bits floating point format). It can be seen that the MFXGSPAP has almost the same performance as the previously published FAP algorithms (with a lower complexity). As expected, the convergence performance of the PAP/FAP algorithms is found between the convergence of the LMS-based algorithm and the RLS-based algorithm. Although the curves from [4] for simulations using noisy plant models are not reproduced here, the same superior performance of all PAP/FAP-based algorithms over the more complex RLS-based algorithms was observed for noisy plant models. The behavior of the algorithms using simulated 16-bits fixed-point (no saturation) and 32-bits floating-point arithmetic was also investigated. Simulations have shown that, if proper regularization factors are chosen, the finite precision simulation of the MFXGSPAP algorithm outperforms that of the other considered FAP algorithms, especially in the 16-bits fixed-point format. Fig. 3 shows the simulation results for a monochannel system (I=1, J=1, K=1). The 16-bits simulation of MFXFAP-RLS was unstable after 500 iterations, while the algorithms based on the Gauss-Seidel method were stable and showed better numerical stability. The results of the 32-bits floating-point simulations of all PAP/FAP algorithms were very close to the results with double precision 64 bits.

4. COMPUTATIONAL COMPLEXITY

The computational complexity of the different algorithms considered was estimated by the number of multiplies required per iteration. Matrix inversions were assumed to be performed with standard LU decomposition: $O\{X^3/2\}$ multiplies, where X is the size of a square matrix. As mentioned earlier, the complexity of the MFXGSPAP algorithm (and also the MFXGSFAP algorithm) can be reduced by updating less frequently the $\boldsymbol{P}(n)$ inverse correlation matrix in (6), with an update period p. Similar reductions of the update rate for $\boldsymbol{P}(n)$ can not be done in purely RLS-based algorithms or in the MFXFAP-RLS algorithm, where $\boldsymbol{P}(n)$ needs to be exactly computed at each iteration. The number of multiplications per iteration for the MFXGSPAP algorithm is:

$$IJK(M+2L+3KN+2K+1)+IJL+JKM+K^3(\frac{1}{2}+\frac{N^2}{p})+K^2 \quad (11)$$

Table 1 evaluates the complexity of the considered FAP/PAP algorithms, with previously published LMS, fast-RLS and RLS-based algorithms for multichannel ANC systems [9],[10]. It can be seen that the complexity of the MFXGSPAP is lower than the MFXGSFAP, significantly less than the MFXFAP-RLS, and much less than algorithms based on fast-RLS or RLS algorithms. The complexity of the MFXGSPAP is actually close to the complexity of the LMS-based MFX-LMS algorithm.

5. CONCLUSION

The MFXGSPAP algorithm was introduced for multichannel ANC, and it was shown that it provides a significant improvement of the convergence speed over a the MFX-LMS algorithm, for similar computational complexities. For noisy plant models, the performance of PAP/FAP algorithms can be better than the more complex RLS-based algorithms. Also, the MFXGSPAP showed better numerical properties than other FAP algorithms in finite precision simulations. Therefore, this algorithm could be an attractive algorithm for practical real-time implementations.

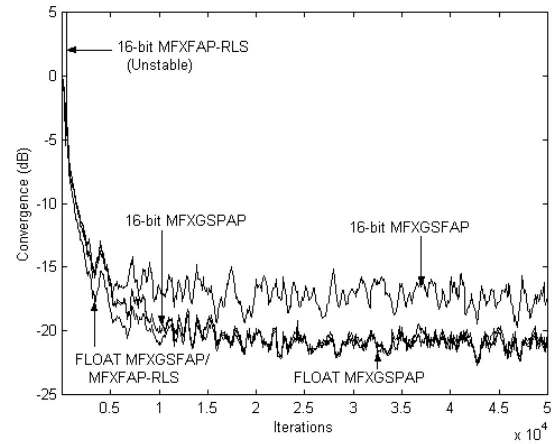

Fig. 3. Convergence curves of the 16-bits fixed-point and 32-bits floating-point results for the PAP/FAP algorithms, with ideal plant models (I=1, J=1, K=1, L=100, M=64).

Algorithm for multichannel ANC L=100, M=64, N=10 (for FAP/PAP)	Multiplies per iteration for I=1, J=1, K=1	Multiplies per iteration for I=1, J=3, K=2
MFX-LMS [9]	428	2,268
MFXGSPAP	468	2,706
MFXGSFAP[5]	510	3,016
MFXFAP-RLS[4]	943	8,505
MFX-FTF[9]	1,226	16,007
MFX-QRD-LSL [10]	4,752	54,616
MFX symmetry-preserving RLS [10]	20,729	320,122

Table 1. Comparison of the computational load of multichannel delay-compensated modified filtered-x algorithms for ANC

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