TRACKING OF MULTIPLE MANEUVERING TARGETS USING MULTISCAN JPDA AND IMM SMOOTHING

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ABSTRACT

We consider the problem of tracking multiple maneuvering targets in the presence of clutter using switching multiple target motion models. A novel suboptimal fixed-lag smoothing algorithm is developed by applying the basic interacting multiple model (IMM) approach and joint probabilistic data association (JPDA) technique to a state augmented system. But unlike the standard single scan JPDA approach, we exploit a multiscan JPDA (Mscan-JPDA) approach to solve the data association problem. The algorithm is illustrated via a simulation example.

1. INTRODUCTION

We consider the problem of tracking multiple maneuvering targets in presence of clutter using switching multiple target motion models. The switching multiple model approach has been found to be very effective in modeling maneuvering targets [1]-[4],[9]. While tracking multiple targets in the presence of clutter, one has to solve the problem of measurement origin uncertainty, i.e. how to associate the data available at the sensor(s) with various targets or clutter (false measurements). In the Bayesian framework the standard JPDA algorithm uses only a single (latest) scan data available at the sensors. To use more information to solve the data association problem, the idea of using multiple scans of data (current and past scans) seems to have been initially proposed by Drummond [8]. Roecker [6] has extended Drummond's ideas where he has discussed problem formulation and solution in some detail. In [6] only non-maneuvering targets (i.e. one model per target) have been considered. In this paper, we extend Roecker's approach to highly maneuvering targets where we allow multiple kinematic motion models per target. A novel suboptimal fixed-lag smoothing algorithm is developed by applying the basic IMM approach and multiple scan JPDA technique.

2. MULTISCAN JPDA

A disadvantage of JPDA is that it uses only the data present in the current scan; in multiscan JPDA, we use multiple scans. A marginal association event $\theta_{ir}(k)$ is said to be effective at time scan k when the validated measurement $y_k^{(i)}$ is associated with (i.e. originates from) target r ($r = 0, 1, \cdots, N$ where $r = 0$ means that the measurement is caused by clutter). Assuming that there are no unresolved measurements, a joint association event Θ_k is said to be effective when a set of marginal events $\{\theta_{ir}(k)\}$ holds true simultaneously. That is, $\Theta_k = \bigcap_{i=1}^m \theta_{ir_i}(k)$ where r_i is the index of the target to which measurement $y_k^{(i)}$ is associated in the event under consideration, ($i = 1, 2, \cdots, m$). In the multiscan case with a scan window size L (L-scan-back) and $k_s = k - L + s$, we define multiscan joint events $\Theta_{kL} = \bigcap_{s=1}^L \bigcap_{i=1}^m \theta_{ir_{is}}(k_s)$ where $\theta_{ir_{is}}(k_s)$ is the marginal association event that at time scan k_s, ith the validated measurement $y_{k_s}^{(i)}$ is associated with target r_{is}. In JPDA we use Θ_k whereas in multiscan JPDA, we exploit Θ_{kL}, $L > 1$.

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3. PROBLEM FORMULATION

Assume that there are total N targets with the target set denoted as $\mathcal{T}_N := \{1, 2, \cdots, N\}$. Assume that the dynamics of each target can be modeled as one of the n hypothesized models. The model set is denoted as $\mathcal{M}_n := \{1, 2, \cdots, n\}$. For target r ($r \in \mathcal{T}_N$), the event that model i is in effect during the sampling period $(t_{k-1}, t_k]$ will be denoted by $M_k^i(r)$. For the j-th model (mode), the state dynamics and measurements of target r ($r \in \mathcal{T}_N$) are modeled as

$$x_k(r) = F_{k-1}^j(r)x_{k-1}(r) + G_{k-1}^j(r)v_{k-1}^j(r), \tag{1}$$

$$z_k(r) = h^j(x_k(r)) + w_k^j(r) \tag{2}$$

where $x_k(r)$ is of dimension n_x, $z_k(r)$ is of dimension n_z, $F_{k-1}^j(r)$ and $G_{k-1}^j(r)$ are the system matrices when model j is in effect over the sampling period $(t_{k-1}, t_k]$ for target r and h^j is the nonlinear transformation of $x_k(r)$ to $z_k(r)$ for model j. A first-order linearized version of (2) is given by

$$z_k(r) = H_k^j(r)x_k(r) + w_k^j(r). \tag{3}$$

The noise processes $v_{k-1}^j(r)$ and $w_k^j(r)$ are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices Q_{k-1}^j (same for all targets) and R_k^j (same for all targets), respectively. At the initial time t_0, the initial conditions for the system state of target r under each model j are assumed to be Gaussian random variables with the known mean $\bar{x}_0^j(r)$ and the known covariance $P_0^j(r)$. The probability of target r in model j at t_0, $\mu_0^j(r) = P\{M_0^j(r)\}$, is also assumed to be known. The switching from model $M_{k-1}^i(r)$ to model $M_k^j(r)$ is governed by a finite-state stationary Markov chain (same for all targets) with known transition probabilities $p_{ij} = P\{M_k^j(r)|M_{k-1}^i(r)\}$. Henceforth, t_k will be denoted by k.

The measurement set (not yet validated) generated at time k is denoted as $Z_k := \{z_k^{(1)}, z_k^{(2)}, \cdots, z_k^{(m)}\}$ where m is the number of measurements generated at time k. Variable $z_k^{(i)}$ ($i = 1, \cdots, m$) is the ith measurement within the set. The validated set of measurements at time k will be denoted by Y_k, containing $\overline{m}$ ($\leq m$) measurement vectors. The cumulative set of validated measurements up to time k is denoted as $\mathcal{Z}^k = \{Y_1, Y_2, \cdots, Y_k\}$..

The goal is to find the fixed-lag smoothing state estimate for target r ($r \in \mathcal{T}_N$) and some fixed lag d ($k \geq d$) $\widehat{x}_{k-d|k}(r) = E\{x_{k-d}(r)|\mathcal{Z}^k\}$ and the associated error covariance matrix. We will use $x_k(r)'$ to denote the transpose of $x_k(r)$.

4. IMM/MSCAN-JPDA SMOOTHING

4.1. The State Augmented System

For each target r, augment the state variable $x_k(r)$ to $\widetilde{x}_k(r)$ as

$$\widetilde{x}_k(r)' = \left[\widetilde{x}_k^{(0)}(r)',\ \widetilde{x}_k^{(1)}(r)',\ \ldots,\ \widetilde{x}_k^{(d)}(r)'\right] \quad (4)$$

where $\widetilde{x}_k^{(j)}(r) := x_{k-j}(r)$. Suppose that for the augmented system, we obtain the filtered state estimate $\widehat{\widetilde{x}}_{k|k}(r) := E\left\{\widetilde{x}_k(r)|\mathcal{Z}^k\right\}$ It therefore follows that $\widehat{\widetilde{x}}_{k|k}^{(i)}(r) := E\left\{\widetilde{x}_k^{(i)}(r)|\mathcal{Z}^k\right\} = \widehat{x}_{k-i|k}(r)$. Similar comments apply to the associated covariance. Using the above definitions and the measurement equation (3), the augmented system can be written as

$$\widetilde{x}_k(r) = \widetilde{F}_{k-1}^j \widetilde{x}_{k-1}(r) + \widetilde{G}_{k-1}^j v_{k-1}^j(r), \quad (5)$$

$$z_k(r) = \widetilde{H}_k^j(r)\widetilde{x}_k(r) + w_k^j(r) \quad (6)$$

where the matrices $\widetilde{F}_{k-1}^j, \widetilde{G}_{k-1}^j$ and $\widetilde{H}_k^j(r)$ for the augmented system are defined in an obvious manner (see [11]).

4.2. Fixed-Lag Smoothing Algorithm

We now extend the single scan IMM/JPDA fixed-lag smoothing algorithm of [5] to apply to the multiscan case. As in [6] we will follow a sliding window multiscan approach. We assume that the scan window size is two. Given state estimate at time $k-1$ based on data up to time $k-1$, in Sec. 4.2.1 we provide first scan steps (using data up to time k) and in Sec. 4.2.2 we provide the second scan steps (using data up to time $k+1$). Sec. 4.2.1 mimics [5] for the most part with a few exceptions, hence, it is only briefly discussed. **Assumed available:** Given the state estimate $\widehat{\widetilde{x}}_{k-1|k-1}^j(r) = E\left\{\widetilde{x}_k(r)|M_k^j(r), \mathcal{Z}^{k-1}\right\}$, the associated covariance $\widetilde{P}_{k-1|k-1}^j(r)$ and the conditional mode probability $\mu_{k-1}^j(r) = P[M_{k-1}^j(r)|\mathcal{Z}^{k-1}]$ at time $k-1$ for each mode $j \in \mathcal{M}_n$ and each target $r \in \mathcal{T}_N$.

4.2.1. FIRST SCAN STEPS:

Step 1.1. Interaction – mixing of the estimate from the previous time $(\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N)$: The expressions for the predicted mode probability $\mu_k^{j-}(r) := P\{M_k^j(r)|\mathcal{Z}^{k-1}\}$ and the mixing probability $\mu^{i|j}(r) := P\{M_{k-1}^i(r)|M_k^j(r), \mathcal{Z}^{k-1}\}$ are as in [5, Sec. 4.2]. Similarly, the expressions for the mixed estimate $\widehat{\widetilde{x}}_{k-1|k-1}^{0j}(r) := E\{\widetilde{x}_{k-1}(r)|M_k^j(r), \mathcal{Z}^{k-1}\}$ and the associated covariance $\widetilde{P}_{k-1|k-1}^{0j}(r) := E\{[\widetilde{x}_{k-1}(r) - \widehat{\widetilde{x}}_{k-1|k-1}^{0j}(r)][\widetilde{x}_{k-1}(r) - \widehat{\widetilde{x}}_{k-1|k-1}^{0j}(r)]'|M_k^j(r), \mathcal{Z}^{k-1}\}$ are as in [5, Sec. 4.2].

Step 1.2. Predicted state $(\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N)$:
State prediction:

$$\widehat{\widetilde{x}}_{k|k-1}^j(r) := E\{\widetilde{x}_k(r)|M_k^j(r), \mathcal{Z}^{k-1}\} = F_{k-1}^j \widehat{\widetilde{x}}_{k-1|k-1}^{0j}(r). \quad (7)$$

State prediction error covariance:

$$\widetilde{P}_{k|k-1}^j(r) = \widetilde{F}_{k-1}^j \widetilde{P}_{k-1|k-1}^{0j}(r)\widetilde{F}_{k-1}^{j'} + \widetilde{G}_{k-1}^j Q_{k-1}^j \widetilde{G}_{k-1}^{j'}. \quad (8)$$

The mode-conditioned predicted measurement of target r, $\widehat{z}_k^j(r)$, and the covariance $S_k^j(r)$ of the mode-conditioned residual $\nu_k^{j(i)}(r) := z_k^{(i)} - \widehat{z}_k^j(r)$ are as in [5].

Step 1.3. Measurement validation: This is exactly as in Step 3.3 of [5]. Denote the volume of validation region for the whole target set by $V_k = \sum_{r=1}^N V_k(r)$.

Step 1.4. State estimation with validated measurements $(\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N)$: From among all the raw measurements at time k, i.e., $Z_k := \{z_k^{(1)}, z_k^{(2)}, \cdots, z_k^{(m(k))}\}$, define the set of validated measurement for sensor 1 at time k as $Y_k := \{y_k^{(1)}, y_k^{(2)}, \cdots, y_k^{(\overline{m}(k))}\}$ where $\overline{m}(k)$ is the total number of validated measurement at time k and $y_k^{(i)} := z_k^{(l_i)}$ where $1 \le l_1 < l_2 < \cdots < l_{\overline{m}(k)} \le m(k)$ when $\overline{m}(k) \ne 0$. Define the validation matrix

$$\Omega = [\omega_{ir}] \quad i = 1, \cdots, \overline{m}(k), \quad r = 0, \cdots, N \quad (9)$$

where $\omega_{ir} = 1$ if the measurement i lies in the validation gate of target r, else it is zero. A joint association event Θ_k is represented by the event matrix

$$\widehat{\Omega}(\Theta_k) = [\widehat{\omega}_{ir}(\Theta_k)] \quad i = 1, \cdots, \overline{m}(k), \quad r = 0, \cdots, N \quad (10)$$

where $\widehat{\omega}_{ir}(\Theta_k) = 1$ if $\theta_{ir}(k) \subset \Theta_k$, else it is 0. A feasible association event is one where a measurement can have only one source, i.e. $\sum_{r=0}^N \widehat{\omega}_{ir}(\Theta_k) = 1 \quad \forall i$, and where at most one measurement can originate from a target, i.e. $\delta_r(\Theta_k) := \sum_{i=0}^{\overline{m}(k)} \widehat{\omega}_{ir}(\Theta_k) \le 1$ for $r = 1, \cdots, N$. The above joint events Θ_k are mutually exclusive and exhaustive. Define the binary measurement association indicator $\tau_i(\Theta_k) := \sum_{r=1}^N \widehat{\omega}_{ir}(\Theta_k), \quad i = 1, \cdots, \overline{m}(k)$, to indicate whether the validated measurement $y_k^{(i)}$ is associated with a target in event Θ_k. Furthermore, the number of false (unassociated) measurements in event Θ_k is $\phi(\Theta_k) = \sum_{i=1}^{\overline{m}(k)}[1 - \tau_i(\Theta_k)]$. We will limit our discussion to nonparametric JPDA [2],[5]. One can evaluate the likelihood that the target r is in model j_r as [5] $\Lambda_k^{j_r}(r) :=$

$$p[Y_k|M_k^{j_r}(r), \mathcal{Z}^{k-1}] = \sum_{\Theta_k} p[Y_k|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}]P\{\Theta_k\}. \quad (11)$$

The first term in the last line of (11) can be written as

$$p[Y_k|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}] = \sum_{j_1=1}^n \cdots \sum_{j_{r-1}=1}^n \sum_{j_{r+1}=1}^n \cdots \sum_{j_N=1}^n$$

$$p[Y_k|\Theta_k, M_k^{j_1}(1), \cdots, M_k^{j_N}(N), \mathcal{Z}^{k-1}] \times P\{M_k^{j_1}(1), \cdots, M_k^{j_{r-1}}(r-1), M_k^{j_{r+1}}(r+1), \cdots, M_k^{j_N}(N)|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}\}. \quad (12)$$

The second term in the last line of (11) turns out to be [5]

$$P\{\Theta_k\} = \frac{\phi(\Theta_k)!\,\epsilon}{\overline{m}(k)!} \prod_{s=1}^N (P_D)^{\delta_s(\Theta_k)}(1-P_D)^{1-\delta_s(\Theta_k)} \quad (13)$$

where P_D is the detection probability (assumed to be the same for all targets) and $\epsilon > 0$ is a "diffuse" prior (for nonparametric modeling of clutter) whose exact value is irrelevant. We assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then the first term in (12) is

$$p[Y_k|\Theta_k, M_k^{j_1}(1), \cdots, M_k^{j_N}(N), \mathcal{Z}^{k-1}]$$

$$\approx \prod_{i=1}^{\overline{m}(k)} p[y_k^{(i)}|\theta_{ir_i}(k), M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}], \quad \theta_{ir_i}(k) \subset \Theta_k \quad (14)$$

where the conditional pdf of the validated measurement $y_k^{(i)}$ given its origin and target mode, is given by $p[y_k^{(i)}|\theta_{ir_i}(k), M_k^{jr_i}(r_i), \mathcal{Z}^{k-1}] = \mathcal{N}(y_k^{(i)}; \widehat{z}_k^{jr_i}(r_i), S_k^{jr_i}(r_i))$ if $\tau_i(\Theta_k) = 1$, else it equals $1/V_k$ where

$$\mathcal{N}(x; y, P) := |2\pi P|^{-1/2}\exp[-\frac{1}{2}(x-y)'P^{-1}(x-y)]. \quad (15)$$

The second term on the right-side of (12) is given by $\prod_{s=1, s\neq r}^{N} \mu_k^{j_s-}(s)$. Moreover, $P\{\Theta_k|M_k^j(r), \mathcal{Z}^{k-1}, Y_k\}$

$$= \frac{1}{c}p[Y_k|\Theta_k, M_k^j(r), \mathcal{Z}^{k-1}]P\{\Theta_k\} =: \beta_k^j(r, \Theta_k)$$

where c is such that $\sum_{\Theta_k} \beta_k^j(r, \Theta_k) = 1$. The following updates are done for each target r ($r \in \mathcal{T}_N$). Calculate $\Lambda_k^{j_r}(r)$ (needed in Step 1.5 later) via (11)-(15). Define the target and mode-conditioned innovations $\nu_k^j(r, \Theta_k) := y_k^{(i)} - \widehat{z}_k^j(r)$ if $\theta_{ir}(k) \subset \Theta_k$, else 0. Using $\widehat{\widetilde{x}}_{k|k-1}^j(r)$ and its covariance $\widetilde{P}_{k|k-1}^j(r)$, one computes the state update $\widehat{\widetilde{x}}_{k|k}^j(r)$ and its covariance $\widetilde{P}_{k|k}^j(r)$ according to the standard PDAF [5].

Step 1.5. Update of mode probabilities ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$): $\mu_k^j(r) := P[M_k^j(r)|\mathcal{Z}^k]$

$$= P[M_k^j(r)|\mathcal{Z}^{k-1}]p[Y_k|M_k^j(r), \mathcal{Z}^{k-1}] = \frac{1}{c}\mu_k^{j-}(r)\Lambda_k^j(r)$$

where c is such that $\sum_{j=1}^n \mu_k^j(r) = 1$.

Step 1.6. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_N$): The final state estimate update at time k is given by $\widehat{\widetilde{x}}_{k|k}(r) = \sum_{j=1}^n \widehat{\widetilde{x}}_{k|k}^j(r)\mu_k^j(r)$ and its covariance $\widetilde{P}_{k|k}(r)$ is given by

$$\sum_{j=1}^{n}\left\{\widetilde{P}_{k|k}^j(r) + [\widehat{\widetilde{x}}_{k|k}^j(r) - \widehat{\widetilde{x}}_{k|k}(r)][\widehat{\widetilde{x}}_{k|k}^j(r) - \widehat{\widetilde{x}}_{k|k}(r)]'\right\}\mu_k^j(r).$$

4.2.2. SECOND SCAN STEPS:

Here we update to scan $k+1$, given data up to time $k+1$, with a sliding scan window of size two. Compared to Sec. 4.2.1, here we have an additional conditioning on Θ_k.

Step 2.1. Interaction – mixing of the estimate from the previous time ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$):

$$\mu_k^j(r, \Theta_k) := P\{M_k^j(r)|\mathcal{Z}^k, \Theta_k\} = c\beta_k^j(r, \Theta_k)\mu_k^{j-}(r). \quad (16)$$

predicted mode probability:

$$\mu_{k+1}^{j-}(r, \Theta_k) := P\{M_{k+1}^j(r)|\mathcal{Z}^k, \Theta_k\} = \sum_{i=1}^n p_{ij}\mu_k^i(r, \Theta_k).$$

mixing prob. $\mu^{i|j}(r, \Theta_k) := P\{M_k^i(r)|M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\}$

$$= p_{ij}\mu_k^i(r, \Theta_k)/\mu_{k+1}^{j-}(r, \Theta_k). \quad (17)$$

mixed estimate $\widehat{\widetilde{x}}_{k|k}^{0j}(r, \Theta_k) := E\{\widetilde{x}_k(r)|M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\}$

$$= \sum_{i=1}^n \widehat{\widetilde{x}}_{k|k}^i(r, \Theta_k)\mu^{i|j}(r, \Theta_k). \quad (18)$$

covariance of the mixed estimate $\widetilde{P}_{k|k}^{0j}(r, \Theta_k) =$

$$\sum_{i=1}^n\{\widetilde{P}_{k|k}^i(r, \Theta_k) + [\widehat{\widetilde{x}}_{k|k}^i(r, \Theta_k) - \widehat{\widetilde{x}}_{k|k}^{0j}(r, \Theta_k)]$$
$$\times[\widehat{\widetilde{x}}_{k|k}^i(r, \Theta_k) - \widehat{\widetilde{x}}_{k|k}^{0j}(r, \Theta_k)]'\}\mu^{i|j}(r, \Theta_k). \quad (19)$$

Step 2.2. Predicted state ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$): This step is as Step 1.2 except for an additional conditioning on Θ_k. We compute $\widehat{\widetilde{x}}_{k+1|k}^j(r, \Theta_k) := E\{\widetilde{x}_{k+1}(r)|M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\}$ and the associated covariance $\widetilde{P}_{k+1|k}^j(r, \Theta_k)$. The mode-conditioned predicted measurement of target r, $\widehat{z}_{k+1}^j(r, \Theta_k)$, and the covariance of the residual $\nu_{k+1}^{j(i)}(r, \Theta_k) := z_{k+1}^{(i)} - \widehat{z}_{k+1}^j(r, \Theta_k)$ follow similarly.

Step 2.3. Measurement validation: Let

$$(j_r, \overline{\Theta}_k) := \arg\left\{\max_{j\in\mathcal{M}_n,\, \Theta_k} |S_{k+1}^j(r, \Theta_k)|\right\}. \quad (20)$$

Then measurement $z_{k+1}^{(i)}$ ($i = 1, 2, \cdots, m(k+1)$) is validated if and only if

$$[z_{k+1}^{(i)} - \widehat{z}_{k+1}^{j_r}(r, \overline{\Theta}_k)]'[S_{k+1}^{j_r}(r, \overline{\Theta}_k)]^{-1}[z_{k+1}^{(i)} - \widehat{z}_{k+1}^{j_r}(r, \overline{\Theta}_k)] < \gamma \quad (21)$$

where γ is an appropriate threshold. The volume of the validation region with the threshold γ is

$$V_{k+1}(r) := c_{n_z}\gamma^{n_z/2}|S_{k+1}^{j_r}(r, \overline{\Theta}_k)|^{1/2} \quad (22)$$

where n_z is the dimension of the measurement and c_{n_z} is the volume of the unit hypersphere of this dimension. The volume of validation region for the whole target set is approximated by $V_{k+1} = \sum_{r=1}^N V_{k+1}(r)$.

Step 2.4. State estimation with validated measurements ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$): This part is quite similar to Step 1.4 except for the additional conditioning on Θ_k. The variables $\widehat{\omega}_{ir}(\Theta_{k+1})$, $\delta_r(\Theta_{k+1})$, etc. have the same meaning as in Step 1.4. One can evaluate the likelihood that the target r is in model j_r as

$$\Lambda_{k+1}^{j_r}(r) := p[Y_{k+1}|M_{k+1}^{j_r}(r), \mathcal{Z}^k] = \sum_{\Theta_k}\sum_{\Theta_{k+1}}$$
$$p[Y_{k+1}|\Theta_k, \Theta_{k+1}, M_{k+1}^{j_r}(r), \mathcal{Z}^k]P\{\Theta_{k+1}\}P\{\Theta_k|M_{k+1}^{j_r}(r), \mathcal{Z}^k\}. \quad (23)$$

The first term in the last line of (23) can be written in a manner similar to (12), $P\{\Theta_{k+1}\}$ is computed as in (13) and the third term is given by

$$P\{\Theta_k|M_{k+1}^{j_r}(r), \mathcal{Z}^k\} = c\mu_{k+1}^{j_r-}(r, \Theta_k)P\{\Theta_k\}p[Y_k|\Theta_k, \mathcal{Z}^{k-1}], \quad (24)$$

$$p[Y_k|\Theta_k, \mathcal{Z}^{k-1}] = \sum_j p[Y_k|\Theta_k, M_k^j(r), \mathcal{Z}^{k-1}]\mu_k^{j-}(r). \quad (25)$$

The probability of the joint association events Θ_{k+1} and Θ_k given that model j is effective for target r from time k through $k+1$ is

$$P\{\Theta_{k+1}, \Theta_k|M_{k+1}^j(r), \mathcal{Z}^k, Y_{k+1}\}$$
$$= \frac{1}{c}p[Y_{k+1}|\Theta_{k+1}, \Theta_k, M_{k+1}^j(r), \mathcal{Z}^k]P\{\Theta_{k+1}\}$$
$$\times P\{\Theta_k|M_{k+1}^j(r), \mathcal{Z}^k\} =: \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) \quad (26)$$

where c is such that $\sum_{\Theta_{k+1}}\sum_{\Theta_k} \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) = 1$.

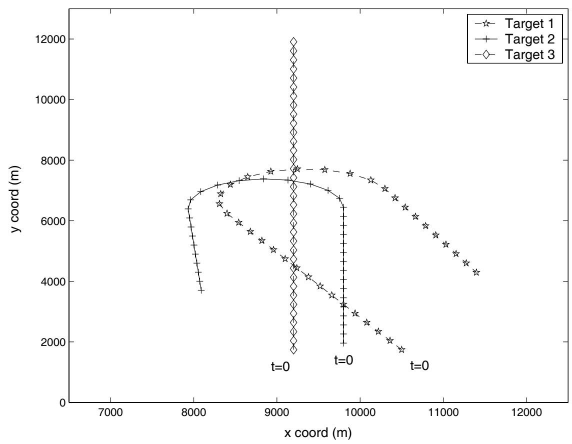

Figure 1. Trajectories (xy positions) of the three targets.

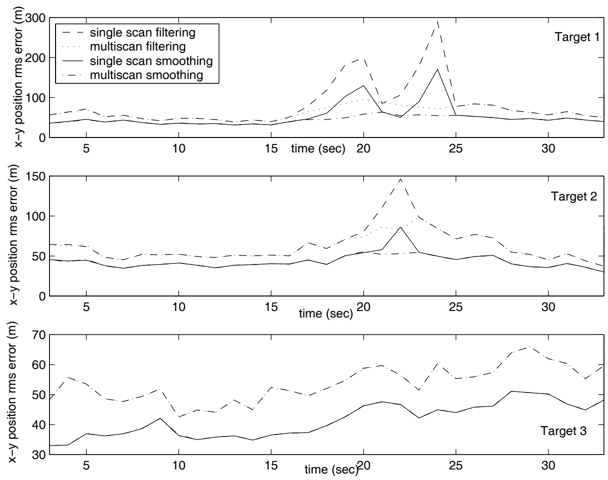

Figure 2. Root mean-square error (RMSE) in position using single scan IMM/JPDAF [5] and the proposed multiscan (window size 2 scans) approach; smoothing is with lag $d = 1$.

Using $\widetilde{\widetilde{x}}^j_{k+1|k}(r, \Theta_k)$ and its covariance $\widetilde{P}^j_{k+1|k}(r, \Theta_k)$, one computes the state update $\widehat{\widetilde{x}}^j_{k+1|k+1}(r)$ as

$$\widehat{\widetilde{x}}^j_{k+1|k+1}(r) := E\{\widetilde{x}_{k+1}(r) | M^j_{k+1}(r), \mathcal{Z}^k, Y_{k+1}\}$$

$$= \sum_{\Theta_{k+1}} \sum_{\Theta_k} \widehat{\widetilde{x}}^j_{k+1|k+1}(r, \Theta_{k+1}, \Theta_k) \beta^j_{k+1}(r, \Theta_{k+1}, \Theta_k) \quad (27)$$

where $\widehat{\widetilde{x}}^j_{k+1|k+1}(r, \Theta_{k+1}, \Theta_k)$ follows from a Kalman filter; we omit the details.

Step 2.5. Update of mode probabilities ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$):

$$\mu^j_{k+1}(r) := P[M^j_{k+1}(r) | \mathcal{Z}^{k+1}] = \frac{1}{c} \mu^{j-}_{k+1}(r) \Lambda^j_{k+1}(r) \quad (28)$$

where c is such that $\sum_{j=1}^n \mu^j_{k+1}(r) = 1$ and

$$\mu^{j-}_{k+1}(r) = \sum_{\Theta_k} \mu^{j-}_{k+1}(r, \Theta_k) P\{\Theta_k\} p[Y_k | \Theta_k, \mathcal{Z}^{k-1}]. \quad (29)$$

Step 2.6. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_N$): The final state estimate update at time $k+1$ is given by $\widehat{\widehat{x}}_{k+1|k+1}(r) = \sum_{j=1}^n \widehat{\widetilde{x}}^j_{k+1|k+1}(r) \mu^j_{k+1}(r)$ and its covariance is computed as in Step 1.6.

Finally we obtain the smoothed state estimate for the IMM/Mscan JPDA algorithm (in addition to filtered state estimate) as $\widehat{x}_{k+1-i|k+1}(r) = \widehat{\widehat{x}}^{(i)}_{k+1|k+1}(r)$ for $i = 0, ..., d.$, and similarly the associated state covariance.

5. SIMULATION EXAMPLE

We consider three targets whose true trajectories are shown in Fig. 1. The three motion models were selected for each target, exactly as in [5, Sec. 5]. The initial model probabilities and the mode switching probability matrix for three targets is also as in [5, Sec. 5]. A single sensor (radar) is used to obtain the measurements which are range, azimuth and elevation angles. The measurement noise w^j_k has covariance matrix $R = \text{diag}[400\text{m}^2,\ 49\text{mrad}^2,\ 4\text{mrad}^2]$. The sensor is assumed to be located at the origin of the coordinate system. The sampling interval was $T = 1$s and the probability of detection $P_D = 0.997$. The clutter was assumed to be Poisson distributed with expected number of $\lambda = 0.1/\text{m rad}^2$. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking. The simulation results were obtained from 50 Monte Carlo runs. Fig. 2 shows the RMSE (root mean-square error) for the filtered position estimates and smoothed state estimates for the three targets as a function of time. It is seen that the multiscan smoothing approach does provide a significant improvement over the multiscan filtering and the single scan smoothing approaches.

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