RADAR IRREGULAR SAMPLING

Radmila Pribić

Thales Naval Nederland, The Netherlands

ABSTRACT

Irregular Pulse Repetition Time (PRT) implies irregularly sampled signals in pulse radar. Irregular PRT is intentional because it improves the radar anti-jamming performance. This radar specific case involves not only standard analysis from irregular samples but also demands additional research. The frequencies above the sampling frequency should be sought, and unwanted signals (so-called clutter) should be suppressed. The state of art, especially applicability of discrete Fourier Transform (DFT) and frame decomposition, is investigated. Solving the desired frequencies and filtering the clutter are treated as most serious of all radar specific problems.

1. INTRODUCTION

Radar is an electronic system used for detection, location and classification of objects. It operates by transmitting a particular type of waveform, and detects the nature of the echo signal. In this introduction, only pulse radar principles sufficient for understanding the sampling problem are summarized. This includes the existing regular sampling, and an introduction to the irregular sampling. For further details about numerous theoretical and practical aspects of radar principles, the interested reader is referred to the excellent radar texts such as e.g. [7] and [5].

Pulse radar transmits pulses and receives the pulses reflected from an object (Fig.1). The received pulses are delayed in time by $T_R = 2R/c$, where c and R are the speed of light and distance of the object from the radar, respectively. Accordingly, the pulses change also in phase by $\phi_R = \omega_c T_R$, where ω_c is the radar carrier frequency. This implies also frequency shift, if R changes in time, i.e. if $dR/dt = v_D \neq 0$, where v_D is radial (or Doppler) velocity of the object.

In a typical radar application, the object motion is assumed to be uniform, i.e. $R = v_D\, t + R_0$, where R_0 is an initial distance. The unknown parameters to be estimated are R_0 and v_D from the time delay T_R and the phase change ϕ_R, respectively. Since radar echoes (in the base band) are discrete signals, samples for R_0 and v_D are taken during each PRT and during a so-called coherent processing interval (CPI), respectively (Fig.1). For purpose of the typical parameter estimation, the distance R can be assumed to change negligibly, $R \approx R_0$, because the constant velocity v_D is low, $v_D \ll c$. Thus, for the Doppler estimation, samples can be taken each PRT, at the same discrete time delay belonging to R_0. In general, radar echoes from a simple (point) target, can be written at discrete times t_m, as follows:

$$x(t_m) = \gamma \cdot g(t_m) \cdot \exp(j2\pi f_D t_m) \qquad , \qquad (1)$$

where γ, $g(t)$ and f_D indicate the radar cross section (RCS), the antenna gain, and the Doppler frequency, respectively. Each m, t_m increases by (ir)regular PRT, during a CPI of M pulses, $m=1,M$. If γ and $g(t)$ can be assumed constant, e.g. $\gamma g(t)=1$, $x(t)$ is periodic with f_D.

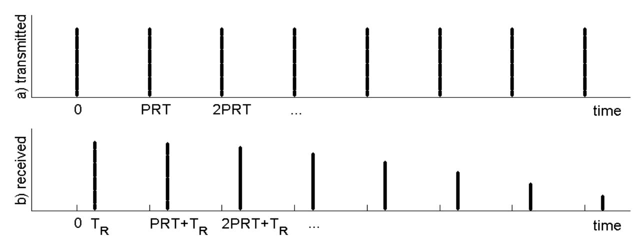

Fig.1 Signal in pulse radar: a) transmitted, and b) received at regular PRT. Received pulses (being reflected from an object) are delayed and Doppler modulated.

When PRT is regular, Doppler processing utilizes DFT over a CPI. A CPI contains usually 8 to 64 pulses. In pulse radar, Pulse Repetition Frequency (PRF), PRF=1/PRT, equals a sampling frequency f_s. Doppler frequency f_D, $2\pi f_D = d\phi_R/dt = 2\omega_c v_D/c$, can be higher than f_s because a broad range of v_D should be detected by radar. This obviously ruins the Nyquist criterion. Frequencies above f_s are usually obtained from two CPIs with different PRTs (Fig.2). This solution for the Doppler ambiguity problem is based on the periodicity of DFT with PRF. A Doppler frequency f_D higher than f_s, can be written as

(Fig.2): $f_D = f_{D1} + n_1 f_{s1} = f_{D2} + n_2 f_{s2}$, where f_{s1} and f_{s2} are the two known PRFs, f_{D1} and f_{D2} are estimated by using DFT, and finally, the integers n_1 and n_2 are numerically solved. The maximum Doppler frequency that can be solved is the least common multiple of f_{s1} and f_{s2}.

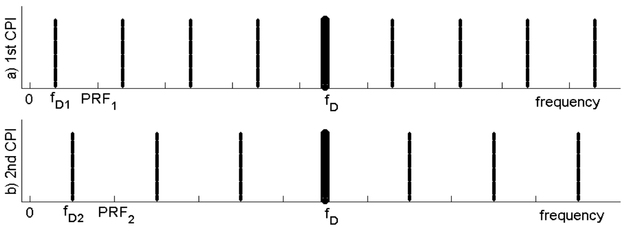

Fig.2 Obtaining Doppler frequency f_D that is higher than sampling frequency (PRF) by using two different PRFs in two adjacent CPIs, indicated in a-b.

This spectral analysis of regular radar echoes has been satisfactory for many decennia. However, irregular PRT is also used because it makes radar perform better against the deceptive jamming (e.g. [11]). In particular, irregular PRT is more difficult to be mimicked by such a jammer. Since this implies irregular sampling (imagine irregular PRTs instead of constant PRT in Fig.1), PRF agile radar requires completely new solutions for the spectral analysis. There are no such solutions yet. When irregular PRT is being used for the anti-jamming purposes, no spectral analysis (i.e. Doppler processing) is being tried in pulse radar.

Applicability of current spectral analysis from irregular samples, and the basic radar problems of Doppler ambiguity and clutter suppression, are most relevant in this study of spectral analysis on irregular PRT.

Irregular samples are well-understood in theory (e.g. [3]), but their algorithms are usually too complicated (e.g. [1]). Current work on irregular sampling is mostly motivated by image processing (e.g. [12]). Algorithms based on frame decomposition, have been developed by the NUmerical Harmonic Analysis Group (*NUHAG*), University of Vienna ([2]). Time-frequency techniques are also efficiently applied, such as e.g. wavelet transform (WT) to irregular samples obtained by decimation or expansion of regular samples (e.g. [4] and [13]). Since typical radar echoes are periodic and stationary signals (eq.(1)), these techniques can hardly be applicable. Thus, such results are not presented in this study because they would be too poor to be interesting.

In radar signal processing, irregular PRT has also been studied, but with emphasis on usage of the Fourier analysis (e.g. [14]), rather than on solving the irregular sampling problem. In the recent work done at the U.S. National Severe Storms Laboratory (*NSSL*) ([9]), the familiar DFT is used legitimately.

In the following, the applicability of the DFT and frame algorithms to PRF agile pulse radar is investigated, especially regarding their potential in solving Doppler ambiguity and clutter filtering.

2. APPLICABILITY OF IRREGULAR SAMPLING

The following techniques illustrate the potential of the state of art in the spectral analysis from radar irregular samples.

The Lomb-Scargle periodogram (LSP) is the classical DFT-based periodogram corrected by the statistical behaviour and time-translation invariance ([6] and [10]). In the further text, it is abbreviated by irDFT (irregular DFT).

In the *NUHAG* algorithm in [2] (further abbreviated by NUHAG), a complex band-limited signal $x(t)$ given by M irregular samples, is first rewritten as trigonometric polynomials $p(t)$ of period 1 and degree K, $K < M/2$.

The solution for the Fourier coefficients a_k, $|k| \le K$, $x(\tau) = \sum_k a_k \cdot \exp(j2\pi f_k \cdot \tau)$, $|f_k| \le B$, is based on properties of the frame operator: $\sum_m p(t_m) \cdot D_K(t - t_m)$, where $D_K(t) = \sum_k \exp(j2\pi k \cdot t)$, represents the frame.

The *NSSL* algorithm in [9] (further abbreviated by NSSL), is used for the radar Doppler processing of an interlaced sampling scheme.

In general, an NSSL sampling set $\{t_m\}$, $m = 1,M$, is multirate with rate K, and the mean interval T_s. An NSSL time interval $(t_{m+1} - t_m)$, as well as the whole sequence KT_s, are integer multiples of the largest common time interval T_ε, $KT_s = LT_\varepsilon$, so that the smallest regular set $\{iT_\varepsilon\}$, $i = 1,N$, can contain $\{t_m\}$, $\{t_m\} \subset \{iT_\varepsilon\}$.

Irregular samples $x(t_m)$ can be zero-padded to regular samples $r(iT_\varepsilon)$ being a product of the sampling scheme c_i, $c_i = \delta(iT_\varepsilon - t_m)$, and the regular samples $x(iT_\varepsilon)$. Based on this relation (in the vector form): $\boldsymbol{r} = diag(\boldsymbol{c}) \cdot \boldsymbol{x}$, i.e. $dft(\boldsymbol{r}) = dft(\boldsymbol{c}) * dft(\boldsymbol{x}) = \boldsymbol{C} \cdot dft(\boldsymbol{x})$, the spectrum of $\boldsymbol{x}$, can be derived as follows:

$$|dft(\boldsymbol{x})| = |\boldsymbol{C}|^{-1} \cdot |dft(\boldsymbol{r})| \qquad , \qquad (2)$$

where $\boldsymbol{C}$ is a Toeplitz matrix whose row vectors are cyclically shifted $dft(\boldsymbol{c})$. Since $\boldsymbol{C}$ is singular (its rank is M, $M<N$), and, thus not invertible, the NSSL idea is to use the magnitudes in order to be able to benefit from the full rank N (with particular sampling schemes only). Furthermore, there are no complex additions in the product $\boldsymbol{C} \cdot dft(\boldsymbol{x})$. This implies limitations on the bandwidth of $\boldsymbol{x}$, but it is no constraint for most radar applications.

Thus, the spectrum $|dft(\boldsymbol{r})|$ contains L weighted replicas of the spectrum $|dft(\boldsymbol{x})|$. The deconvolution gives the strongest replica, i.e. the signal spectrum $|dft(\boldsymbol{x})|$.

3. RADAR SPECIFIC SPECTRAL ANALYSIS

Typical radar echoes simulated as normalized complex noise-free exponentials from eq.(1), are input signals in this radar irregular sampling study. The sampling is regular, and irregular deterministic and random. The deterministic irregular sampling involves repeating periodically the same sequence of K irregular intervals. (Interlaced usually means $K=2$, while multirate or bunch stands for an arbitrary K.) The random sampling involves uniformly distributed jitter added to regular sampling times. The frequencies are also normalized by the average sampling frequency, $f_s = 1$.

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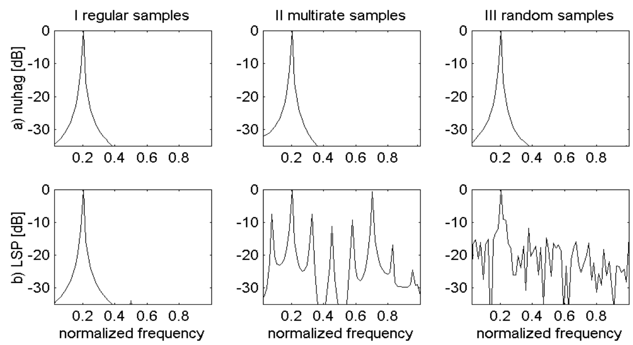

Fig.3 Spectra obtained by known techniques: a) NUHAG and b) LSP (irDFT), from I) regular and II-III) irregular samples of a complex noise-free exponential at frequency *0.2*. Number of (regular or irregular) samples equals *64*. Multirate samples are of rate $K=8$.

In Fig.3, results with the frequencies lower than the sampling frequency are presented first.

If a signal is periodic and regularly sampled, irDFT (in LSP) suffices (Fig.3bI). If the samples are irregular, irDFT on multirate and random samples reveals the sampling pattern: deterministic repetitions (Fig.3bII) and (pseudo) random noise (Fig.3bIII), respectively.

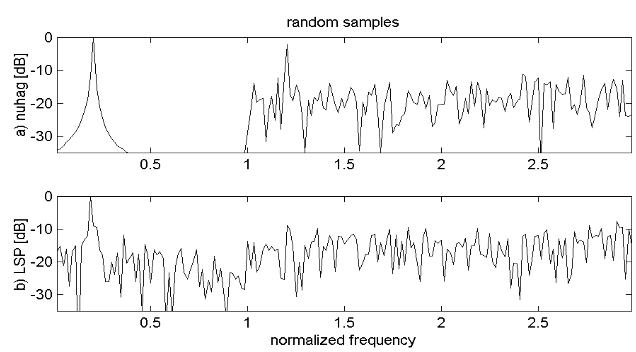

Fig.4 Spectra obtained by: a) NUHAG and b) LSP, from random samples (Fig.3III), at frequencies beyond the sampling frequency.

NUHAG is superior, because it supports any sampling, and moreover, provides accurate spectral components (Fig.3a), but only within the sampling frequency band. In other frequency bands (not defined formally in [2]), an increased noise level and aliasing would appear (Fig.4a).

The irDFT from random samples, provides spectra beyond f_s (without aliasing) but it adds significant noise coming from the random sampling itself (Fig.4b). Unfortunately, this can hardly be afforded in a radar application because the signal-to-noise ratio (SNR) is already critical. Although SNR increases linearly with a number of samples, longer processing times should also better be avoided.

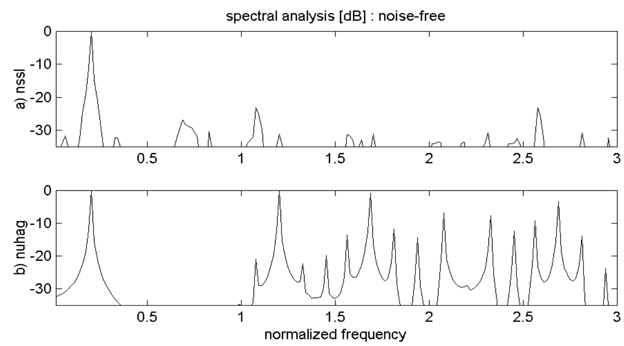

Fig.5 Spectra obtained by: a) NSSL, and b) NUHAG, from multirate samples (Fig.3II), at frequencies beyond the sampling frequency.

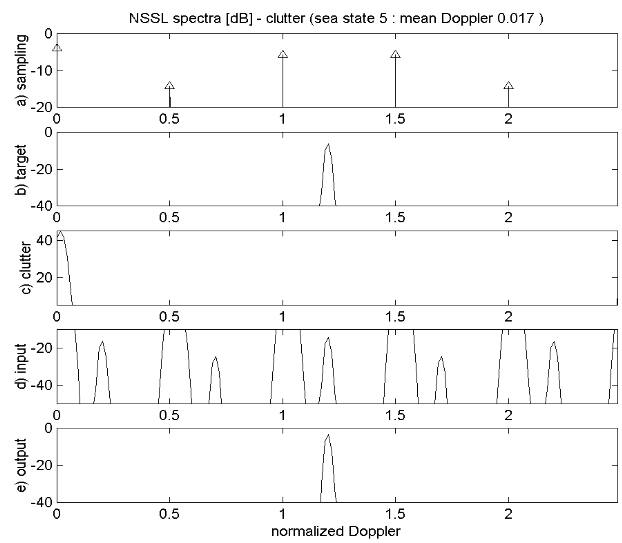

Fig.6 Extended NSSL algorithm illustrated by spectra of: a) irregular sampling, b) point-target, c) sea, d) input, and e) output. Sampling is multirate with $K=2$, $L=5$ (i.e $5T_\varepsilon = 2T_s$), and $M=64$, as in [9]. Point-target echo is a noise-free complex exponential at frequency 1.2. Clutter corresponds to sea state 5.

NLLS is comparable with NUHAG for multirate samples (Fig.5a-b). Furthermore, NLLS enables solving the radar problems of Doppler ambiguity and clutter filtering. NLLS supports frequencies above the Nyquist frequencies, namely up to *L/K* times the sampling frequency (e.g. Fig.6). If the clutter bandwidth is known, the complex clutter signal can be estimated, and filtered from $dft(\boldsymbol{r})$ (eq.(2)), before the deconvolution. This filter, originally meant for known land clutter in [9], is extended for sea clutter in [8].

In Fig.6, the extended NSSL algorithm is illustrated when the multirate sampling contains five non-zero spectral components (Fig.6a) which modulate the input signal (Fig.6d) containing a noise-free target echo (Fig.6b) and nonrandom sea clutter (Fig.6c). The power spectral density (psd) of the sea clutter is assumed to be Gaussian shaped. The clutter filtering and deconvolution result in the frequency contents of the target echo (Fig.6e).

The range of unambiguous frequencies, may expand limitlessly, but clutter filtering becomes more involved with increasing complexity of the NLLS sampling. For example, imagine how crowded Fig.6d could become with a larger *L*, i.e. if a basic irregular sequence (Fig.6a) would be longer and, thus, more complicated.

4. CONCLUSIONS

Applicability of current spectral analysis, together with Doppler ambiguity and clutter suppression, are most relevant when studying radar irregular sampling.

Current spectral analysis from irregular samples, provides no satisfactory solution for the radar specific problem. Decreasing the sampling noise level in irDFT from random samples, and also in NUHAG at frequencies beyond the sampling frequency, should further be studied.

An illustration of the radar problems of the Doppler ambiguity and clutter suppression is given by using results of NSSL that has been adopted. The extended NSSL tested on simulated sea clutter, performs satisfactorily. It shall also be tested with real measurements. Other irregular sequences shall also be investigated that are tractable by this algorithm, but also effective against the deceptive jammers, and optimal for the unambiguous Doppler extension as well as for the clutter filtering.

5. ACKNOWLEDGMENT

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6. REFERENCES

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