ON TURBO-COMPRESSION AND MODELING OF BLACK AND WHITE IMAGES[1]

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ABSTRACT

This paper considers lossless compression of black and white images using recently developed distributed turbo compression techniques. An empirical study of image pixel statistics is initially conducted in order to apply turbo compression algorithms to distributed compression of image rows. In particular, it is noted that while the entropy of the pixels throughout the image does not vary much, the distribution and conditional distributions of pixel values are non-stationary. Corresponding compression bounds are noted, appropriate coding schemes based on turbo-codes are developed and simulation results are reported.

1. INTRODUCTION

Turbo codes and LDPC codes have been shown to perform well in the compression of independently and identically distributed data [1], as well as in the more general case of distributed compression, including the Slepian-Wolf problem [2-5]. This research is based on the premise that given the above, turbo-codes may be a promising technique in the problem of image compression. Existing image compression [10] techniques present certain characteristics that can be disadvantageous in specific applications, and turbo-codes could potentially avoid those. We thus seek to use turbo-codes in order to develop a compression method which will be lossless, unlike habitual JPEG compression, noise robust and based on 2-dimensional correlation models, unlike run-length coding, as well as low-complexity and distributed.

It was first realized that in order to develop a good turbo compression scheme, it was crucial to obtain an adequate statistical model of the image data. The first part of this paper is thus devoted to the development of a suitable image model based on the study of empirical image data. Among other characteristics, we seek to model the non-stationarity observed in image data [6]. Our findings are summarized in Section 2.

Our final contribution is to incorporate this image model into the design of appropriate turbo-coding schemes. Section 3 describes the development of these schemes, and we report on the results obtained through simulations in Section 4.

2. STATISTICAL MODELING OF BLACK AND WHITE IMAGES IN SLEPIAN-WOLF CONTEXT

2.1. Image Modeling

In the process of lossless image turbo-compression, we will attempt to utilize local dependencies among pixels of the image. When compressing a pixel in the context of a two-dimensional image, we must first choose effectively the neighboring pixels aiding our compression. Since the conditional entropy of the pixel given its neighbors gives a limit on the achievable (lossless) compression rate, we would like to choose the neighboring pixels in such a way that a worthwhile compression can be achieved while reasonable modeling and implementation simplicity are maintained.

Figure 1 illustrates some possible choices used for our calculations and simulations. One particularly natural and simple formulation may be to use information from a pixel in the previous image row (2-row model in Figure 1). We found that the second and third row pixels are significantly correlated with the original pixel; hence the 3-row model is also included. Nonetheless, subsequent rows yielded little additional information. We also considered the alternate T model, which conditions a pixel not only on its predecessor but also on the neighbors of the latter. Finally, we considered a square model.

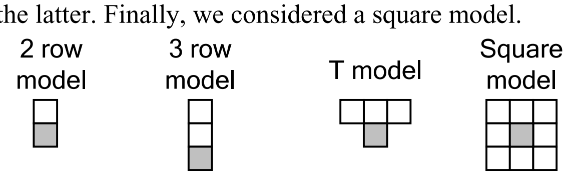

Figure 1: Entropy model definitions: m1 pixels in grey, m2 to m3 pixels in white.

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For a given image and selected neighboring pixel model, our approach was to calculate empirically the joint probability distribution of the pixels and then to evaluate the ensuing empirical conditional (pixel) entropy. Table 1 reports on the empirically calculated conditional pixel entropies for several standard grayscale images converted to black and white through thresholding, for various neighbor models from Figure 1. In each case, two values are given: one is the conditional empirical entropy calculated over the whole image, while the second is the average of the conditional empirical entropies calculated for each individual row. Since entropy is a concave function in the input probability distribution, the latter approach yields lower conditional entropies and thus a potential for better achievable compression. Nonetheless, as the differences between the two quantities are not appreciable, an adaptive rate coding method (within an image) was not found to be critical. Nonetheless, an adaptation to these statistical variations was found to be useful in the pixel recovery and will be discussed in the coding section.

Image	2 row		3 row		T model		Square model	
Camera	.42	.67	.37	.56	.23	.28	.12	.16
Text	.80	.85	.34	.43	.34	.44	.16	.16
Goldhill	.63	.77	.58	.70	.40	.45	.18	.30
Bridge	.69	.76	.63	.70	.40	.44	.19	.30
Baboon	.85	.89	.77	.81	.53	.56	.30	.43

Table 1: Empirical pixel entropies of various images under different conditioning models, first subcolumn as stationary processes, second subcolumn as non-stationary processes.

Finally, we illustrate through Figure 2 the non-stationarity of the joint statistics for a particular image.

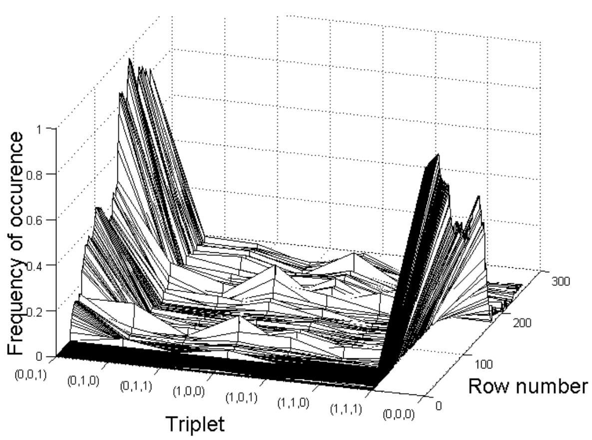

Figure 2: Illustration of non-stationarity properties of images: distribution of triplets (three consecutive pixels) as it changes through the image 'camera'.

2.2 Slepian-Wolf Modeling

The Slepian-Wolf problem concerns the separate encoding of correlated messages, and their joint recovery, as illustrated in Figure 3. In this example, two correlated messages, m_1 and m_2, are generated and must be encoded by two encoders that do not communicate with each other, at rates denoted by R_1 and R_2. The two encoded messages are then transmitted over a channel, and are received by a decoder which attempts to recover m_1 and m_2 jointly.

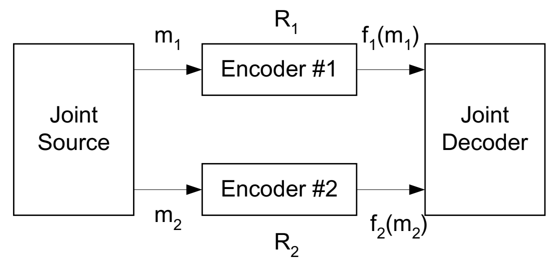

Figure 3: The Slepian-Wolf Problem

Evidently, were m_1 and m_2 encoded jointly, the optimal encoding rate would be their joint entropy, denoted by $R_1 + R_2 = H(m_1, m_2) = H(m_1) + H(m_1 \mid m_2)$. Perhaps surprisingly, Slepian and Wolf [7] have proven that in the case of separate encoding, the compression rate can also asymptotically attain this joint entropy. More precisely, as made clearer by the region of attainability in Figure 4, the zero-error objective can be obtained by compressing the m_1 and m_2 to the rates of $R_1 \geq H(m_1)$ and $R_2 \geq H(m_2)$ respectively, while by allowing an arbitrarily small error rate we can reach any compression rate pair respecting the following constraints:

$$R_1 \geq H(m_1 \mid m_2)$$
$$R_2 \geq H(m_2 \mid m_1)$$
$$R_1 + R_2 \geq H(m_1, m_2)$$

Subsequently, we can generalize the Slepian-Wolf Problem to include as many sources as desired.

Previous work on coding for this problem can be found in [2-5] and many others.

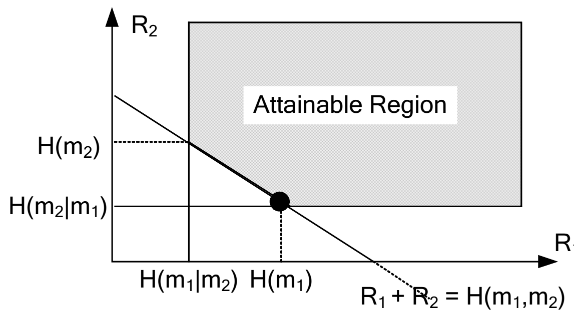

Figure 4: The Slepian-Wolf attainable region

In the case of image compression considered here, we make use of these results by modeling the source of each pixel as a separate source of the Slepian-Wolf problem. Thus, we can attempt to encode each row of the image without the referring to the joint realizations of successive pixels without entailing any measurable loss in rate.

Furthermore, we fix our interest upon a subset of the attainable region, notably the region satisfying $R_1 \geq H(m_1)$. By convexity and time-sharing arguments, this can be shown to a sufficient goal for any Slepian-Wolf coding scheme, since any other point can be attained through the use of a time-sharing scheme between the encoders.

3. FIXED RATE TURBO CODING OF IMAGES

We have previously developed a fixed rate 3:2 serial turbo coding scheme, and applied it to the compression of Bernouilli data as well as correlated messages of large block sizes (about 5000 bits) [5]. Based on the statistical modeling described in section 2, we first apply this coding scheme, unmodified, to image data. We then develop a modification of the scheme having aim of adapting it to the non-stationary nature of image data. Results for both the unmodified and improved coding schemes are then reported on in section 4.

The serial concatenated codes used in the original scheme consist in a repetition-like FSM encoder with a random output interleaver followed by a Latin Square encoder of rate 3:1 or 5:2. These embodied the coding principles described in [8,], and can be found in [5]. The overall rate was 3:2, depending on the Latin Square encoder chosen.

The decoding was based on the BCJR algorithm [1], but it should be noted that the complexity of the decoding could have been reduced by employing the simplifications entailed by the use of the repetition encoder [9].

The block diagrams of the encoder and decoder are given in Figures 5 and 6, while more details on the performance and design of this coding scheme can be found in [5].

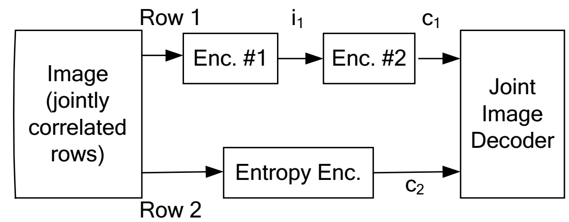

Figure 5: Serial turbo-coding for Slepian-Wolf problem

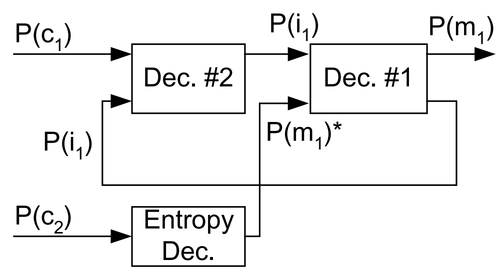

Figure 6: Serial turbo decoding for Slepian-Wolf problem

Based on the statistical investigation reported in Section 2, it is known that the image data is highly non-stationary. Thus, the conditional entropy varied greatly from one row to another, a fact leading to difficulties in the recovery of individual rows. To counteract this fact without immediately modifying the coding scheme, as well as take advantage of the improved performance of turbo-codes for larger packet sizes, immediately before decoding we concatenated each nth row of the image together, thus producing what we called “superrows”. The result was essentially to produce a shorter but wider image, thus obtaining the larger packet sizes improving the performance of turbo-codes.

This encoding scheme was applied to the compression of the images by compressing the first superrow to its entropy, and then compressing all other superrows using the serially concatenated codes as depicted in Figure 5. The recovery of each superrow was then accomplished by using the fully recovered previous supperrows as side information, as depicted in Figure 6.

Finally, we modified the coding scheme in order to take into account the non-stationary pixel distributions. Our approach was to calculate empirically the joint probability distribution $\tilde{p}\left(m_1^i m_2^i \ldots m_N^i\right)$ for each row based on the previous N rows. This information was then transmitted along with the actual data. At the recovery side, this information was used to bias $p\left(M_1^i = m_1^i\right)^* = \tilde{p}\left(m_1^i m_2^i ... m_N^i \mid m_2^i ... m_N^i\right)$, as opposed to previously, where $p\left(M_1^i = m_1^i\right)^*$ was constant for all rows based on the average over the whole image, which implicitly assumed a stationary model. As will be detailed in the next section, the addition of these local statistics improved the coding performance.

4. EXPERIMENTAL RESULTS

In this section, we report on results implementing the compression methods described in section 3 and their performance for various test images of size 256 by 256 pixels. We applied the described coding schemes to each image, basing the conditional distribution on the side information on the 'T', 2-row and 3 row models. In each case, we compared the recovery using average and localized statistical information, i.e. under the stationary and non-stationary assumptions described in section 3. The entries of Table 2 list the number of rows that we needed to concatenate in order to obtain lossless recovery, or in the case of a '---' entry, the failure to recover the image with maximum concatenation. We observe an improvement as additional rows are considered, and an interesting performance in the case of the one-row 'T' model. Furthermore, we observe that the inclusion of local statistics improves the performance, as observed by the errorless recovery for smaller interleaver sizes (implied by the smaller number of row concatenations needed for errorless recovery) as well as the recovery of some images which could not be decompressed based on average statistics only. For example, the image `baboon` could not be decompressed in any case because of its high entropy.

Image	T model		2 row		3 row	
Camera	128	128	128	128	128	128
Text	128	64	---	128	128	128
Goldhill	32	64	4	4	4	16
Bridge	32	32	4	4	4	16
Baboon	---	---	---	---	---	---

Table 2: Contents indicate largest value of 'n' enabling errorless recovery (every n^{th} row being concatenated). First subcolumn using stationary assumption, second subcolumn using non-stationary assumptions. Failure to recover image for any 'n' is indicated by '---'.

5. CONCLUSION

To summarize, the first contribution of this research work was to initiate the investigation of image modeling appropriate for coding design, particularly in the Slepian-Wolf context. Several models were suggested, and the achievable compression was calculated through a study of the empirical entropies yielded under these modeling assumptions. Turbo-coding compression methods based on these models were then implemented, and results in noiseless compression were reported.

It is hoped that this work should motivate future efforts in the modeling of images in a way motivating the development of compression codes. Furthermore, there remains much to be done from both the modeling and coding point of view for grayscale or colour images as well as sequences of images (such as movies) through possible extension of the methods presented here.

6. REFERENCES

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