LINEAR SYSTEM ANALYSIS OF MOTION COMPENSATED DE-INTERLACING

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ABSTRACT

Majority of the work on deinterlacing techniques are ad hoc in nature and lack theoretical justification [2]-[6]. The designs are heuristic and the results are in most cases experimental. It is very difficult to quantify the overall performance of a de-interlacing system based on its performance over a few video sequences. In this paper,we present a linear system analysis for motion-compensated video upconversion system, analyze the effect of motion vector accuracy in system performance and investigate switching algorithm between fixed and motion-compensated de-interlacing algorithm.

1. INTRODUCTION AND BACKGROUND REVIEW

The choice of interlaced and progressive scanning formats often have been the centre of intense discussions due to their respective pros and cons, especially in the context of making a choice about the future of digital television. Choosing one or the other format reverts to the problematic choice between an ingenious bandwidth reduction and an improved visual quality at the display. From signal processing point of view, halving the information rate, as in the case of interlaced format, reduces the quality of the displayed image and generates a variety of objectionable visual artefacts.

Fig.1 summarizes the deinterlacing process where a progressive frame (twice number of rows) are created using two interlaced fields. The interlaced video is captured by a camera with odd and even fields at different time instants. The model is based on two key aspects, i.e. image sampling in time and spatial sampling. The following assumptions are made:

1)The video scene $I(x, y, t)$ is defined as the 2D projection of the 3-D world on the camera sensor. Uniform translation motion (d_x, d_y) is assumed for simplicity. Thus the video scene can be represented at any time instant t, by $I(x, y, t) = I(x - d_x, y - d_y, 0)$.

2)The video scene $I(x, y, t)$ is spatially sampled using the raster scan. The sampling grid is represented by $\delta(x - mX_0, y - nY_0)$, where X_0 and Y_0 are sampling spacing. We assume $X_0 = Y_0 = 1$.

3)The video scene is assumed to be spatially band-limited such that for stationary case it can be perfectly reconstructed from the sample scene.

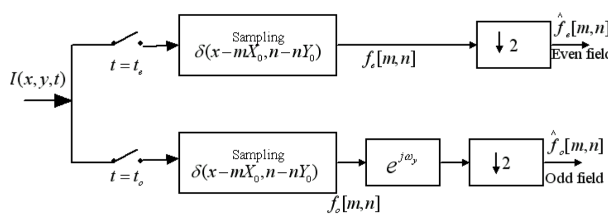

Fig. 1. Linear system model of the interlaced system

The authors in [1] present a linear system analysis for fixed interpolation where no motion vector is used. We summarize their result here and extend it for motion-compensated deinterlacing. Fig. 1 illustrates the model for interlaced field capture. The even and odd frame are captured at time instant $t = t_e$ and $t = t_o$, respectively. The spatial signals are then sampled using the raster $\delta(x - m, y - n)$. The even field consists of the even lines of the digitized frame captured at $t = t_e$ and odd field consists of lines formed by shifting the even vertical lines of the frame by one sample. The frame $f_e[m, n]$ is defined as in eq.(1) and its Fourier Transform is given by

$$f_e[m, n] = I(m, n, t_e) = I(m - d_x t_e, n - d_y t_o, 0) \quad (1)$$

$$F_e(e^{-j\omega_x}, e^{-j\omega_y}) = e^{-jt_e(d_x\omega_x + d_y\omega_y)} I(e^{-j\omega_x}, e^{-j\omega_y}, 0) \quad (2)$$

From here on we will be using $F(\omega_x, \omega_y)$ to represent $F(e^{-j\omega_x}, e^{-j\omega_y})$ and $I(\omega_x, \omega_y)$ to represent $I(\omega_x, \omega_y, 0)$. Using the property of sub-sampling in frequency domain for the odd and even fields and the assumption that $t_e = -t_o = -T$ and $d_xT = S_x, d_yT = S_y$, where S_x and S_y are the displacement respectively, we obtain

This work is supported in part by National Semiconductor Inc and matching fund from UC DiMi

$$\hat{F}_e(\omega_x,\omega_y) = \tfrac{1}{2}\,[F_e(\omega_x,\frac{\omega_y}{2}) + F_e(\omega_x,\frac{\omega_y}{2}+\pi)] \quad (3)$$

$$\hat{F}_e(\omega_x,\omega_y) = \tfrac{1}{2}\,e^{j(S_x\omega_x+\frac{S_y\omega_y}{2})}\,\{I(\omega_x,\frac{\omega_y}{2}) + e^{jS_y\pi}I(\omega_x,\frac{\omega_y}{2}+\pi)\} \quad (4)$$

$$\hat{F}_o(\omega_x,\omega_y) = \tfrac{1}{2}\,e^{j\omega_y/2}\,[F_o(\omega_x,\frac{\omega_y}{2}) - F_o(\omega_x,\frac{\omega_y}{2}+\pi)] \quad (5)$$

$$\hat{F}_o(\omega_x,\omega_y) = \tfrac{1}{2}\,e^{j\omega_y/2}\,e^{-j(S_x\omega_x+\frac{S_y\omega_y}{2})}\,\{I(\omega_x,\frac{\omega_y}{2}) - e^{-jS_y\pi}I(\omega_x,\frac{\omega_y}{2}+\pi)\} \quad (6)$$

Eqs (4),(6) show that each field consists of a base band spectrum $I(\omega_x,\omega_y)$ and an alias spectrum $I(\omega_x,\omega_y+\pi)$. The base band spectra is same for both fields, except for a phase due to vertical shift of one line. The alias spectra in both fields also have the same magnitude but opposite signs. Given two fields, deinterlacing techniques combine them to obtain a progressive frame. The different methods to convert interlaced to progressive format can be categorized as follows:

- Intraframe de-interlacing
- Intrafield de-interlacing
- Motion adaptive de-interlacing
- Motion compensated de-interlacing

As one can expect, the first two methods do not use motion information and consequently are inferior but simpler to the other two methods. The motion adaptive de-interlacing algorithm switches between intraframe and intrafield deinterlacing depending on the presence or absence of motion. Since it does not utilize the motion vector, its performance is inferior to the motion compensated de-interlacing. Detailed analysis of the non-motion compensated methods are not presented due to the length constraint of the paper.

2. MOTION ANALYSIS OF INTERLACED VIDEO

In motion compensated de-interlacing, two consecutive fields are pair wise combined to reconstruct the frame with one of the field undergoing motion compensation. The motion compensation can be done pair wise for unequal parity fields or equal parity fields, where unequal and equal parity fields are used for the case where the motion compensating field pair is opposite and with same parity, respectively. In the discussion below, the performance of both cases are evaluated and the results are used to compare the performance of the motion compensated de-interlacing algorithms.

Equal parity motion compensation: In this case we compensate between two even fields as shown in Fig. 3. The analysis is also valid for odd fields. The motion compensated error signal can be represented as ($t_{e1}=0, t_{o1}=T, t_{e2}=2T$);

$$\hat{F}_{e1}(\omega_x,\omega_y) - \Delta\,\hat{F}_{e2}(\omega_x,\omega_y) = \tfrac{1}{2}I(\omega_x,\frac{\omega_y}{2})[1-e^{-j\{(2S_x+N_x)\omega_x+(S_y+N_y)\omega_y\}}] + \frac{1}{2}I(\omega_x,\pi+\frac{\omega_y}{2})[1-e^{-2jS_y\pi}e^{-j\{(2S_x+N_x)\omega_x+(S_y+N_y)\omega_y\}}] \quad (7)$$

where $\Delta = e^{-j(N_x\omega_x+N_y\omega_y)}$. The base-band signal can be minimized by choosing $(N_x,N_y)=(-2S_x,-S_y)$. The resulting alias term

$$\tfrac{1}{2}I(\omega_x,\pi+\frac{\omega_y}{2})[1-e^{-2jS_y\pi}]$$

is zero in case of integer vertical motion.

Unequal parity motion compensation: The other case of motion compensation between unequal fields will be interesting to analyze as in Fig. 3. The motion compensated error signal is

$$\hat{F}_{e1}(\omega_x,\omega_y) - \Delta\,\hat{F}_{o1}(\omega_x,\omega_y) = \tfrac{1}{2}I(\omega_x,\frac{\omega_y}{2})[1-e^{-j\{(S_x+N_x)\omega_x+(\frac{S_y}{2}+N_y)\omega_y\}}e^{j\frac{\omega_y}{2}}] + \frac{1}{2}I(\omega_x,\pi+\frac{\omega_y}{2})[1+e^{-jS_y\pi}e^{j\frac{\omega_y}{2}}e^{-j\{(S_x+N_x)\omega_x+(\frac{S_y}{2}+N_y)\omega_y\}}] \quad (8)$$

The difference signal can be minimized by choosing $(N_x,N_y)=(-S_x,-\frac{S_y}{2})$. However, we will still have a base-band component and an alias component, i.e., the base-band spectrum will not vanish due to the half line phase shift. The resulting difference signal is

$$\tfrac{1}{2}I(\omega_x,\frac{\omega_y}{2})[1-e^{j\frac{\omega_y}{2}}] + \frac{1}{2}I(\omega_x,\pi+\frac{\omega_y}{2})[1+e^{-jS_y\pi}e^{j\frac{\omega_y}{2}}]$$

The signal contains a vertically high-passed base-band signal and a non-zero alias term. The two terms will never be zero simultaneously.

If we compare the results of motion compensation between equal parity fields and unequal parity fields, it can be seen that the former one has advantage. The only disadvantage of the process is the temporal separation between the fields. The motion to be measured can be too large and the complexity of the motion estimator can be overwhelming.

3. MOTION COMPENSATED DE-INTERLACING

We will use the insight gained in the previous section to analyse the results for motion compensated de-interlacing. We will analyse the interpolation scheme for two cases. The first will be unidirectional compensation and the other will be bi-directional interpolation with three field storage. From the discussion of the last section we have seen that the motion compensation between equal parity fields performs better than the unequal parity fields. We will evaluate both cases where the motion compensated field is generated from the equal parity fields and the unequal parity fields.

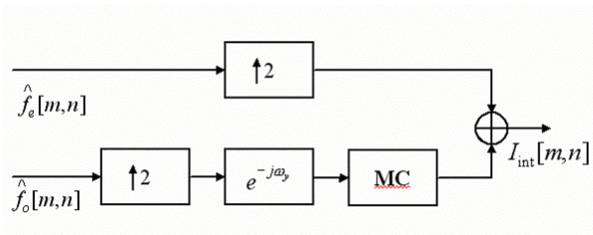

Fig. 2. Motion compensated de-interlacing of interlaced fields

3.1. Unidirectional motion compensation:

In Fig.2 the interpolated frame $I_{int}(m, n)$ is obtained by pair wise combining the even field $\hat{f}_e[m, n]$ and the motion compensated odd field $\hat{f}_o[m, n]$. It is assumed that the motion compensation applies a global shift of (N_x, N_y) pixels. The interpolated frame $I_{int}(m, n)$ can now be expressed as:

$$I_{int}(\omega_x, \omega_y) = \hat{F}_e(\omega_x, 2\omega_y) + e^{-j\omega_y} e^{-j(N_x\omega_x+N_y\omega_y)} * \hat{F}_o(\omega_x, 2\omega_y) \tag{9}$$

Using the expression for odd and even field that was derived

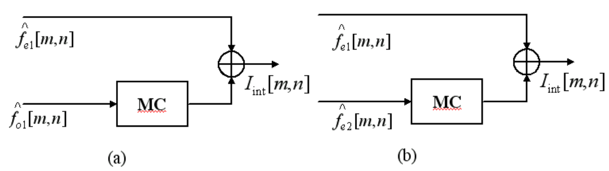

Fig. 3. (a) Odd parity motion Compensation (b) Even parity motion compensation

previously in Eqn.(4) and Eqn.(6) and $t_e = 0$ and $t_o = T$

$$I_{int}(\omega_x, \omega_y) = \frac{1}{2}[(1 + \gamma\Delta)I(\omega_x, \omega_y) + \gamma(1 - \gamma\Delta e^{-jS_y\pi}) * I(\omega_x, \pi + \omega_y)] \tag{10}$$

where $\gamma = e^{-j(\omega_x S_x+\omega_y S_y)}$ and $\Delta = e^{-j(N_x\omega_x+N_y\omega_y)}$ respectively. The resulting de-interlaced frame consists of distorted base-band and alias spectrum. The distorting function for the base-band spectrum is given by $(1+\gamma\Delta)$ and for the alias spectrum is given by $(1-\gamma\Delta e^{-jS_y\pi})$ respectively. The interpolation error can be minimized for both parts separately. The performance of the de-interlacer is analysed under ideal and non-ideal motion compensation cases.

Ideal motion compensation : If one can estimate the shift between the two fields perfectly i.e, $(N_x, N_y) = (S_x, S_y)$, the interpolated frame becomes

$$\begin{aligned} I_{int}(\omega_x, \omega_y) &= [I(\omega_x, \omega_y) \\ &+ e^{-j\frac{S_y\pi}{2}} j\sin(\frac{S_y\pi}{2}) I(\omega_x, \pi + \omega_y)] \end{aligned} \tag{11}$$

where,$\gamma\Delta = 1$ The first term is $F_e[m, n]$ and the second term is the alias spectrum distorted by the sine function. The term is zero for even integer values of S_y. This is an interesting observation. It implies that the motion compensated interpolation will work only for even integer vertical motion. For odd integer vertical motion it is impossible to perfectly reconstruct the original frame.The odd integer vertical motion is thus called " critical velocity".

Non-ideal motion compensation: If the motion estimation process is not perfect i.e, $(N_x, N_y) = (2S_x + \delta_x, 2S_y + \delta_y)$, where δ_x and δ_y are the error between the true motion and the measured motion. In presence of the imperfection in motion estimation process the de-interlaced frame is given by :

$$I_{int}(\omega_x, \omega_y) = \frac{1}{2}[(1 + \delta)I(\omega_x, \omega_y) + \gamma e^{jS_y\pi}(1 - e^{-jS_y\pi}\delta) * I(\omega_x, \pi + \omega_y)] \tag{12}$$

where, $\delta = e^{-j(\delta_x\omega_x+\delta_y\omega_y)}$. The magnitude of the base-band distortion and the alias distortion is given by,

$$\left|1 + e^{-j(\delta_x\omega_x+\delta_y\omega_y)}\right| = 2\cos(\frac{\delta_x\omega_x + \delta_y\omega_y}{2}) \tag{13}$$

$$\left|1 - e^{-j(S_y\pi+\delta_x\omega_x+\delta_y\omega_y)}\right| = 2\sin(\frac{\frac{S_y\pi}{2} + \delta_x\omega_x + \delta_y\omega_y}{2}) \tag{14}$$

respectively. It can be observed that for small error in motion estimation it approaches the performance as in case 1. The base-band term vanishes for $\delta_x\omega_x + \delta_y\omega_y = (2n + 1)\pi, \forall n \in \mathcal{Z}$ and the alias term vanishes for $S_y\pi = -2(\delta_x\omega_x+ \delta_y\omega_y)$. As it can be seen that the "zero lines" of the sine and cosine functions do not overlap. The effect of this can be seen in the form of staircase artefact in the de-interlaced frame in Fig. 6 due to motion vector estimation error.

3.2. Three field motion compensated de-interlacing

In the previous section we observed that equal parity motion compensation results in a better performance. We use that insight here to interpolate the even field $\hat{f}_{eint}[m, n]$ that is temporally at same position as $\hat{f}_{o1}[m, n]$. The interpolated frame is generated by merging the odd field with the motion compensated even field. The motion compensated even field is generated by bi-directional interpolation of the previous and current even field.

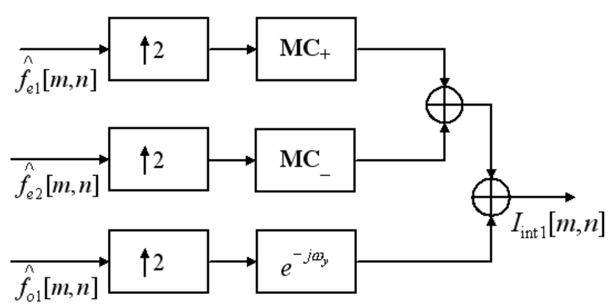

Fig. 4. Motion compensated de-interlacing using three fields

Proceeding as before, and using $t_{e1} = -T, t_{e2} = T$ and

$t_{o1} = 0$ for simplifying the result.

$$\hat{f}_{eint}(\omega_x, \omega_y) = \frac{1}{2}\cos(A) I(\omega_x, \omega_y) + \frac{1}{2}\cos(A + S_y \pi) * I(\omega_x, \omega_y + \pi) \quad (15)$$

$$where, \quad A = (S_x - N_x)\omega_x + (S_y - N_y)\omega_y$$

$$\hat{F}_{int}(\omega_x, \omega_y) = \hat{f}_{eint}(\omega_x, \omega_y) + \hat{f}_{o1}(\omega_x, 2\omega_y) \quad (16)$$

$$\hat{F}_{int}(\omega_x, \omega_y) = \frac{1}{2}[\cos(A) + 1] I(\omega_x, \omega_y) + \frac{1}{2}[\cos(A + S_y\pi) - 1] I(\omega_x, \omega_y + \pi) \quad (17)$$

We can analyse the performance of the de-interlacer for ideal and non-ideal motion compensation. For ideal motion compensation $A = (S_x - N_x)\omega_x + (S_y - N_y)\omega_y = 0$, eqn[18] reduces to

$$\hat{F}_{int}(\omega_x, \omega_y) = I(\omega_x, \omega_y) - \sin(\frac{S_y \pi}{2})^2 \, I(\omega_x, \omega_y + \pi) \quad (18)$$

For this type of de-interlacer , distortion function for the alias term converges to zero at twice the rate that of unequal parity de-interlacer. Similarily for non-ideal motion compensation, $\hat{F}_{int}(\omega_x, \omega_y)$ is similar to eq.[18] where $A = \delta_x \omega_x + \delta_y \omega_y$ is the error in motion vector estimation. The salient points for the motion compensated interpolation are
1) Equal parity motion compensation results in a better performance than unequal parity motion compensation.
2) Motion compensated interpolation results in prefect reconstruction if the motion informations are accurate and for even integer vertical motion.
3) It is impossible to perfectly reconstruct the frame using motion compensated de-interlacing for odd integer vertical motion.
4) 3-field motion compensated de-interlacing performs better than simple motion compensated de-interlacing involving 2 fields.

Fig. 5. Original Progressive frame

Fig. 6. Result of motion compensated de-interlacing,with errors in motion vectors

4. CONCLUSION AND FUTURE WORK

In this paper the performance of the motion compensated de-interlacing was analysed. The reason for the artefacts were analysed. For the analysis we assumed a very simple linear system model. In the subsequent work we will analyse the performance of the motion compensated de-interlacing taking into account the motion vector error distribution. The interpolation filters that will reduce the aliasing artefacts will be presented. We will start with time invariant 2D interpolation filter to attenuate the aliasing component. In second step, motion adaptive interpolation filter will be developed to compensate the varying aliasing component of the input video signal.

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