A NEW NON-LINEAR EXPONENTIAL 2-D ADAPTIVE FILTER AND ITS APPLICATION IN TEXTURE CHARACTERIZATION

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ABSTRACT

We propose in this paper a new non-linear exponential adaptive bi-dimensional (2-D) filter for image modeling. The filter coefficients are updated with the Least Mean Square (LMS) algorithm. Furthermore, the proposed non-linear model is used for texture modeling with a 2-D Auto-Regressive (AR) adaptive model. The characterization efficiency of the proposed exponential model is compared with the 2-D linear AR model updated with the LMS algorithm. The comparison criteria is based on the computation of a characterization rate using the ratio of "between-class" variances with respect to "within-class" variances of the estimated coefficients. Extensive experiments show that the exponential model coefficients give better results in texture discrimination than those of the linear model, even in a noisy context.

1.INTRODUCTION

In many 2-D applications such as image enhancement, image identification and compression, linear adaptive filtering does not give good performances because of the non-linear and non-stationary general nature of images. Looking for better results, many researches had turned their attention to non-linear adaptive filtering. In [6] adaptive 2-D Volterra filters have been applied in non-linear channel distorted image restoration. Since these filters use a high number of parameters to represent the nonlinear character, research on model which provides a low computation cost is a serious problem [4][5].

In the present paper, we propose a new non-linear model for 2-D signals which takes in consideration high order non-linearity without increasing the number of filter coefficients. We propose the use of the exponential of the 2-D support matrix instead of the support matrix itself. Thus any non-linearity order can be considered using the same number of coefficients of the linear case. The proposed non-linear model is used for texture modeling with a 2-D Auto-Regressive adaptive model.

In fact, texture analysis plays an important role in several image processing and pattern recognition applications such as remote sensing, cartography, robot vision, military surveillance and medical imaging. It has long been the topic of intense research [3][9]. Texture can be found in the background of natural scenes as well, as filling elements of surface images, thus the textural features are an important pattern elements in image interpretation. Various methods for texture features extracting for texture characterisation have been proposed during the last two decades. One such method characterizes texture by discrete Wavelet representation [8]. Fractal based features have been also used as features for texture characterization [3]. This features depend mostly on textural characteristics than on intensity information. Several authors have made a comparison of the performance of various features for texture characterisation purpose. In [9], Ojala *et al* have compared four texture features: gray level differences, Laws texture features, center-symetric covariance features, and local binary patterns.

One of the most promising methods for texture features extraction is the parametric modelling, where the coefficients of the 2-D AR parametric model are used for texture characterisation and synthesis. Various adaptive parametric linear filters have been proposed for texture characterization [10][11]. The main contribution of the present work is the use of a non-linear exponential parametric 2-D filter for image modeling. We show how much this model can improve texture characterization in comparison with the linear model.

2. RECALL OF THE 2-D LINEAR MODEL

An image can be represented by a 2-D transversal AR parametric linear model with a support of order ($p\times p$) (p has an odd value) (Figure 1). The value of a pixel at position (n,r) is represented by the following relationship:

$$y(n,r)=\sum_{i=-\frac{p-1}{2}}^{\frac{p-1}{2}}\ \sum_{j=-\frac{p-1}{2},\ (i,j)\neq(0,0)}^{\frac{p-1}{2}} W_{i,j}\, y(n+i,r+j). \qquad (1)$$

n and r are in the interval $[1..L]$ ($L\times L$: the image size), $W_{i,j}$ are the 2-D AR transversal coefficients. In the stationary case, these coefficients do not depend on the position of the pixel (n,r).

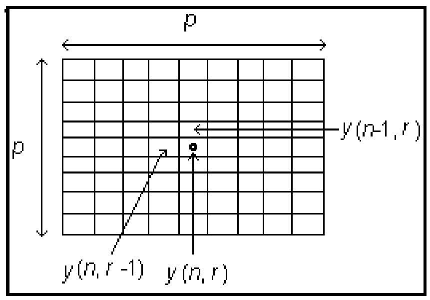

Figure 1: Bi-dimensional filter support

For the filter coefficients adaptation, the Least Mean Square algorithm is the most widely used algorithm due to its implementation simplicity. It is based on the minimization of the mean square error between the filter output and the desired output. This algorithm is extended to the 2-D case in [2]. Considering the 2-D linear AR model (1), a simplified version of the 2-D LMS algorithm can be given by:

For n and r from 1 to L:

- Calculation of the adaptive filter output using (1).
- Coefficients adaptation:

For i and j from $-\frac{p-1}{2}$ to $\frac{p-1}{2}$

$$W_{i,j}=W_{i,j}+\mu\left(d(n,r)-y(n,r)\right)y(n-i,r-j)$$

$d(n,r)$ represents the desired output. In texture modeling, it is the gray level value of the texture image pixel. μ is the step size of the algorithm; the initial values of the coefficients $W_{i,j}$ are set to zero.

3. THE PROPOSED EXPONENTIAL MODEL

In the proposed non-linear filter, the filter output $y(n,r)$ is calculated using the exponential of the 2-D support pixel values.

For this, we define a matrix $Y(n,r)$ as a filter window of size $(p\times p)$ containing the values of the 2-D support pixels:

$$Y(n,r)=\begin{pmatrix} y\left(n-\frac{p-1}{2},r-\frac{q-1}{2}\right) & \cdots & & y\left(n-\frac{p-1}{2},r+\frac{q-1}{2}\right) \\ y\left(n-\frac{p-1}{2}+1,r-\frac{q-1}{2}\right) & \cdots & & \\ \vdots & \vdots & \ddots & \vdots \\ y\left(n+\frac{p-1}{2},r-\frac{q-1}{2}\right) & \cdots & & y\left(n+\frac{p-1}{2},r+\frac{q-1}{2}\right) \end{pmatrix}$$

We also define a matrix $Ey(n,r)$ as the matrix:

$$Ey(n,r)=\mathrm{Exp}\left(Y(n,y)\right)-I_{p\times p} \quad .$$

We note Exp of a matrix A of size $p\times p$ the quantity:

$$Exp(A)=I_{p\times p}+A+\frac{A^2}{2!}+\cdots+\frac{A^k}{k!}+\ldots$$

Thus, the non-linear exponential filter output is given by:

$$y(n,r)=Ey(n,r).{*}W \qquad (2)$$

Where W denotes the coefficient matrix defined as:

$$W=\begin{pmatrix} W_{-\frac{p-1}{2},-\frac{q-1}{2}} & W_{-\frac{p-1}{2},-\frac{q-1}{2}+1} & \cdots & W_{-\frac{p-1}{2},\frac{q-1}{2}} \\ W_{-\frac{p-1}{2}+1,-\frac{q-1}{2}} & & \cdots & \\ \vdots & \vdots & \ddots & \vdots \\ W_{\frac{p-1}{2},-\frac{q-1}{2}} & & \cdots & W_{\frac{p-1}{2},\frac{q-1}{2}} \end{pmatrix}$$

and '.*' denotes the sum of the dot product of $Ey(n,r)$ and W.

Note that the model (2) can be also written as:

$$y(n,r)=\sum_{i=-\frac{p-1}{2}}^{\frac{p-1}{2}}\sum_{j=-\frac{p-1}{2}}^{\frac{p-1}{2}}{}_{(i,j)\neq(0,0)} W_{i,j}Ey_{i,j}(n,r)$$

4. ADAPTATION OF THE NON-LINEAR FILTER WITH THE 2-D LMS ALGORITHM

The 2-D LMS algorithm can be applied for the adaptation of the exponential filter by minimizing the square error value between the filter output and the desired output

$$J=\xi^2(n,r)=\left(d(n,r)-y(n,r)\right)^2$$

The gradient of J with respect to the matrix coefficients is

$$\nabla J=\frac{\partial J}{\partial W}=2\,\xi(n,r)\frac{\partial\xi(n,r)}{\partial W}=-2\,\xi(n,r)\frac{\partial y(n,r)}{\partial W}$$
$$\approx-2\,\xi(n,r)\,Ey(n,r)$$

Hence the adaptation equation can be written as:

$$W=W-\frac{1}{2}\mu\frac{\partial J}{\partial W}=W+\mu\xi(n,r)\,Ey(n,r)$$

Then the 2-D LMS algorithm for the proposed exponential filter can be written as follows:

For n and r from 1 to L:

- Calculation of the adaptive filter output using (2).

-Coefficients adaptation: For i and j from $-\frac{p-1}{2}$ to $\frac{p-1}{2}$

$$W=W+\mu\left(d(n,r)-W(n,r)\right)Ey(n,r)\,.$$

5. TEXTURE CHARACTERIZATION WITH THE NON-LINEAR EXPONENTIAL FILTER

Consider a set of ten different gray-scale textures of (256×256) pixels (Figure 2) extracted from the Brodatz album [1]. In order to check whether the use of the non-linear exponential filter improves the texture characterization in comparison with the linear filter, we propose to evaluate the characterization efficiency of the coefficient estimated by each filter.

Fifty images of (64×64) pixels are randomly chosen from each texture class of Figure 2. We calculate the texture coefficients using both adaptive filters for the 500 images. Note $x_{k,n}$ the n^{th} estimated coefficient vector for the k^{th} texture class. The mean of the k^{th} texture class coefficient vectors is $\mu_k = \frac{1}{50}\sum_{n=1}^{50} x_{k,n}$ and $\mu_c = \frac{1}{10}\sum_{k=1}^{10} \mu_k$ is the mean of all the coefficient vectors.

We define a characterization rate J as $J = trace\left(S_{\text{int}ra}^{-1}.S_{\text{int}er}\right)$.

The matrix $S_{\text{int}ra}$ is the mean of the within-class (intra-class) dispersion matrix given by :

$$S_{\text{int}ra} = \frac{1}{500}\sum_{k=1}^{10}\sum_{n=1}^{50}\left(x_{k,n} - \mu_k\right)\left(x_{k,n} - \mu_k\right)^t .$$

The matrix $S_{\text{int}er}$ is the mean of the between-class (inter-class) dispersion matrix calculated by

$$S_{\text{int}er} = \frac{1}{10}\sum_{k=1}^{10}\left(\mu_k - \mu_c\right).\left(\mu_k - \mu_c\right)^t .$$

The greater the characterization rate is, the more robust the classification process is. The comparison of the capability of the filter coefficients will be presented through the next experiments.

Experiment 1: Influence of the filter order

The aim of the following experiment is to compare the capability of the non-linear exponential filter and 2-D linear filter in texture classification for different 2-D filter order. Hence, as stated above, we use the characterization rate J based on the dispersion ratio as a comparison criteria. This rate is calculated for different filter orders ranging from 3×3 to 7×7 (odd values) without any additive noise. Figure 3 depicts the characterization rate with respect to the 2-D filter orders. The step size is equal to 0.4 for both models. The size of the image is 64×64. It should be noted that for any order, the characterization rate provided by the exponential adaptive filter is greater than the one provided by the linear adaptive filter. Furthermore, the coefficients seem to give better characterization rate for a high filter order.

Experiment 2: Influence of and additive noise

The 2-D filter order is fixed to 5×5. We plot the characterization rate for both adaptive with respect to the Signal to Noise (SNR) value in the case of additive gaussian noise (Figure 4). Clearly, The characterization rate of the exponential adaptive filter is greater than that of the 2-D linear filter. For both filters, the increase of additive noise variance causes attenuation in the characterization rate. The additive noise perturbs the classification process. Therefore, the non-linear exponential filter based coefficients are better texture discriminators than those of the linear filter ones.

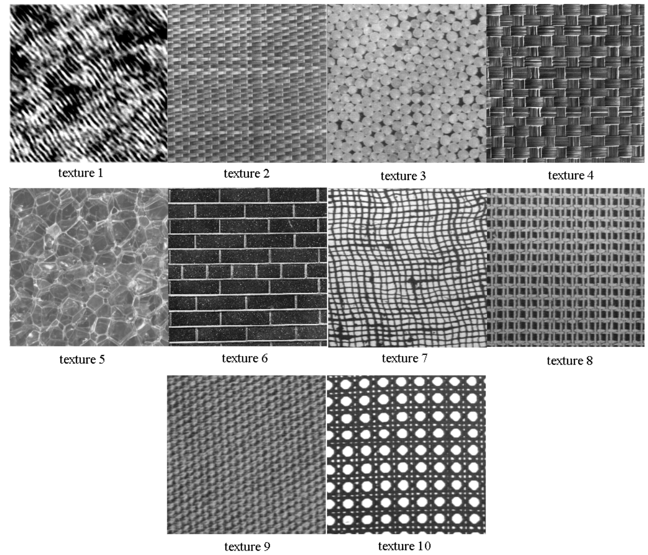

Figure 2: Ten texture images used in the study

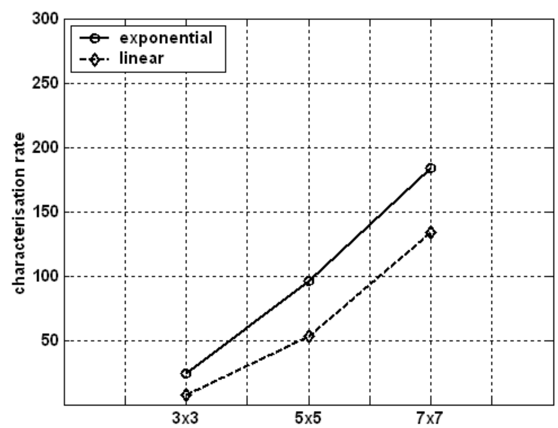

Figure 3: Characterization rate of the 2-D exponential filter and the 2-D linear filter with respect to the filter orders.

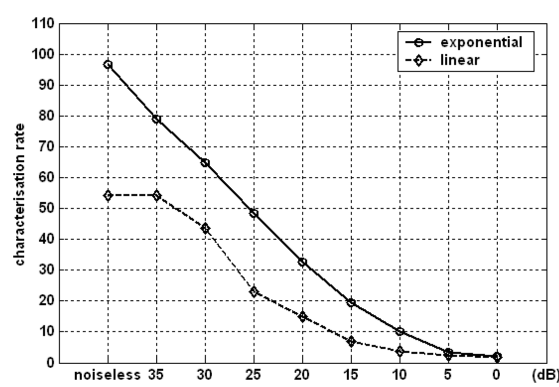

Figure 4: Characterization rate of the 2-D exponential filter and the 2-D linear filter for various SNR values.

Experiment 3: Influence of the image size

The characterization rate is now calculated for different image sizes ranging from 16×16 to 128×128 pixel, without any additive noise and for a filter order of 5×5.

Figure 5 depicts the characterization rate with respect to the image sizes. The characterization rate provided by the exponential adaptive filter is superior than the one provided by the linear adaptive filter. For a large image size, the performance of the classification becomes high.

Experiment 4: Texture classification with a neural network

In this last experiment, our purpose is to test the ability of the proposed non-linear model to classify images issued from the 10 textures of Figure 2 with a multi-layer neural network trained with data provided from both linear and exponential filters. The 2-D coefficients estimated from 500 images of 64×64 pixels (50 images randomly chosen from each texture) are used as input vectors to the multilayer neural network. The network is trained using the gradient descent back- propagation algorithm [7] with 50% of the available texture images (250 images of 64×64 pixels) and tested with the other part. The network weights were updated on each presentation of a feature vector. The set of training examples is changed at each iteration and their order is randomly chosen. For each texture, we define the classification sensitivity as the ratio of the number of positive tests to the total number of tests. In order to determine the optimum neural network to achieve a maximal classification sensitivity for each algorithm and each SNR value, we carried out several experiments using various architectures, that is: various training coefficients and various numbers of neurons in each layer. We used two hidden layers and three binary coded outputs. The momentum was 0.9 and the initial random values of the weights were set between –1 and 1. The threshold value of the network sigmoid was 0.2. In Table 1, we present respectively the results of classification sensitivity obtained with the coefficients of the 2-D exponential and the 2-D linear models.

Model	Noise-less	SNR 20 dB	SNR 10 dB	SNR 0 dB
Linear	100 %	100 %	94 %	85 %
Exponential	100 %	100 %	99.2 %	86 %

Table 1: Classification sensitivity for both 2-D filters

These results show that, in comparison with the coefficients of the linear model, the coefficients of the exponential model provide excellent performances for texture classification, even for low SNR. These results confirm the conclusion of the last experiments and can be interpreted by some non-linear nature inside the texture image.

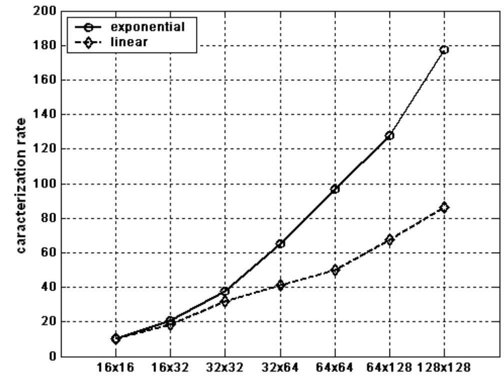

Figure 5: Characterization rate of the 2-D exponential filter and the 2-D linear filter versus the texture image size

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