LOGARITHMIC QUANTISATION OF WAVELET COEFFICIENTS FOR IMPROVED TEXTURE CLASSIFICATION PERFORMANCE

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ABSTRACT

The coefficients of the wavelet transform have been widely used for texture analysis tasks, including segmentation, classification and synthesis. Second order statistics of such values have been shown to give excellent performance in these applications, and are typically calculated using *co-occurrence matrices*, which require quantisation of the coefficients. In this paper, we propose a non-linear quantisation function which is experimentally shown to better characterise textured images, and use this to formulate a new set of texture features, the *wavelet log co-occurrence* signatures.

1. INTRODUCTION

The wavelet transform has emerged over the last two decades as a powerful new theoretical framework for the analysis and decomposition of signals and images at multiple resolutions [1]. One of the most common forms of the transform used for image analysis applications is the separable two dimensional wavelet transform, defined as

$$A_j = [H_x * [H_y * A_{j-1}]_{\downarrow 2,1}]_{\downarrow 1,2} \quad (1)$$

$$D_{j1} = [G_x * [H_y * A_{j-1}]_{\downarrow 2,1}]_{\downarrow 1,2} \quad (2)$$

$$D_{j2} = [H_x * [G_y * A_{j-1}]_{\downarrow 2,1}]_{\downarrow 1,2} \quad (3)$$

$$D_{j3} = [G_x * [G_y * A_{j-1}]_{\downarrow 2,1}]_{\downarrow 1,2} \quad (4)$$

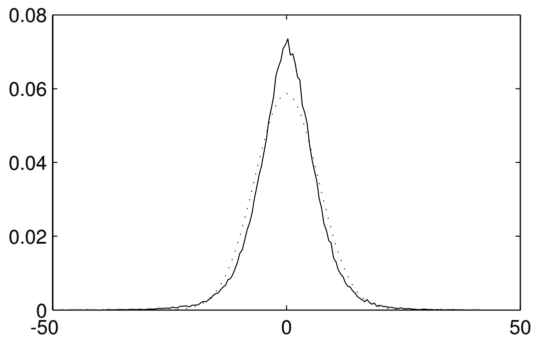
where G and H are the high and low-pass filters along the subscripted axis, $*$ is the convolution operator, j is the resolution level, and $\downarrow a, b$ represents downsampling along the x and y axes by factors of a and b respectively. The resulting images A_j and $D_{ji},\ i \in \{1, 2, 3\}$ are known as the approximation and detail coefficients respectively.

Initial research into the use of the wavelet transform for the purpose of texture analysis focussed on using the energy of each wavelet band as a feature for texture characterisation [2, 3]. More recently it has been shown that using the second order statistics of the wavelet transform coefficients can provide a better representation of texture, with significantly reduced error rates in classification experiments [4]. Using this technique, second order statistical features of the wavelet coefficients are extracted by means of co-occurrence matrix features. Such a matrix is formed by using uniform quantisation to map the continuous valued coefficients resulting from an overcomplete wavelet frame representation to the discrete indices of the co-occurrence matrix. In order to reduce the total number of features, the co-occurrence matrices are averaged over the four directions $\{0°, 45°, 90°, 135°\}$, and restricted to a distance of one pixel. Thus, only one co-occurrence matrix from each detail subband is created. Extracting common co-occurrence features from each such matrix for the first four levels of decomposition gives a total of 96 features, which the authors refer to as *co-occurrence signatures* [4].

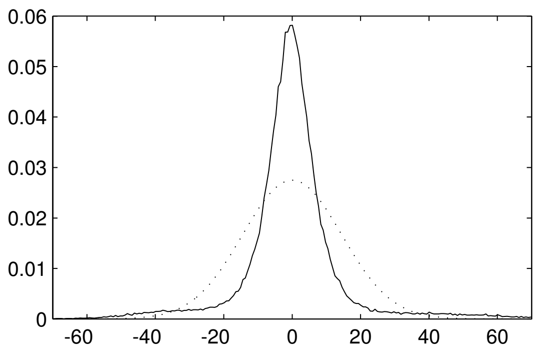
In this paper we propose a method of quantising the wavelet coefficients based on the logarithm function which leads to an improved representation for many textures. The validity of using such a function is experimentally proven by showing that image reconstruction can be performed with lower distortion when compared to uniform quantisation of the wavelet coefficients. Three separate feature sets which represent second order statistics of the coefficients are then proposed and experimentally evaluated, with the results showing improved classification accuracy compared to existing texture features.

2. LOGARITHMIC QUANTISATION OF WAVELET COEFFICIENTS

It has been observed by a number of authors that the wavelet coefficients of natural and other textured images are non-Gaussian in nature, and often contain large peaks at the origin and long tails [5, 6, 7]. In spite of this, the vast majority of texture features extracted for classification of such images, such as the energy or mean deviation of each band, assume a Gaussian-like distribution of these coefficients. Examples of the distribution of wavelet coefficients of textured images and the corresponding Gaussian distributions are shown in figure 1. These examples show the errors which are introduced using such a model. Using a generalised Gaussian function to model the distribution of coefficients, as proposed by Van de Wouwer *et. al.* [4] can



(a)



(b)

Fig. 1. Histograms of wavelet coefficients for (a) well matched, and (b) mismatched images. The dotted line shows the Gaussian distribution of equal variance commonly used to model such distributions.

in many cases overcome such limitations, however in general is still inadequate for representing texture.

When calculating many second order statistics, it is often necessary to perform quantisation of the continuous wavelet coefficients, and such a process of typically performed in a linear manner, described by

$$q(x) = \text{round}\left(I\frac{x}{a_s}\right) \tag{5}$$

where x is the value being quantised, A_s is the saturation level of the quantiser, I is the desired number of quantisation levels, and round$(\cdot)$ represents rounding to the nearest integer value. Such a function, when used with a sufficient value of I, does not substantially alter the shape of the histogram, and thus still suffers from the poor modelling previously described for most natural textures. Other quantisation techniques, such as Lloyd-max optimal quantisation, overcome this problem by using non-uniform decision regions to ensure the mean-squared error is minimised [8]. Although this is of great benefit in compression applications, it cannot be used in a classification task as the quantiser must be recalculated for each individual image.

By modifying the quantisation function $q(x)$, it is also possible to significantly alter the shape of the resulting histogram. In order to remove the long tails of the histogram that make modelling of the coefficients difficult, a function is required that compacts the histogram at the high limit. One example of such a function is the logarithm, and this shall be the focus of further investigation. One implementation of logarithmic quantisation of a continuous variable x can be realised by transforming x such that uniform quantisation of the resulting variable can be performed. An ideal case of this transform would map each threshold value a_i to the linear center of the new quantisation cell, ie $\frac{2i-1}{2}$, giving

$$q(x) = \text{round}\left[\kappa \log\left(\frac{x}{a_s\delta} + 1\right) + 1/2\right] \tag{6}$$

where

$$\kappa = \frac{I-1}{\log\left(1/\delta + 1\right)} \tag{7}$$

Thus, to design a logarithmic quantiser having I levels, knowing the desired saturation point a_s, it is required to choose only one parameter $\delta > 0$, the non-linearity factor. Lower values of δ indicate a high level of non-linearity in the transform, and as $\delta \to \infty$, the quantiser becomes a uniform quantiser. By removing the addition of $\frac{1}{2}$ from (6), it is possible to make the quantiser symmetric about the origin.

The non-linear transform described in (6) is in some ways similar to that proposed by Unser and Eden, who suggested that a logarithmic function applied to wavelet coefficients yields a more stable representation and better class separation in the context of texture segmentation [9]. In their work, a second non-linearity was also applied to the coefficients before the logarithm function in order to remove the sign of the coefficients. Since the wavelet detail coefficients are zero-mean, and their histograms generally near-symmetric about the origin (see figure 1) [4], the sign of each coefficient is relatively unimportant and such an operation results in little loss of useful information. In [9], Unser and Eden show that taking the square of the coefficients leads to the best characterisation of the textures. Modifying (6) to include this operator gives

$$q_1(x) = \text{round}\left[\kappa \log\left(\frac{x^2}{a_s^2\delta} + 1\right)\right] \tag{8}$$

Another possible choice for this rectifying function is the magnitude operator $|\cdot|$, which gives

$$q_2(x) = \text{round}\left[\kappa \log\left(\frac{|x|}{a_s\delta} + 1\right)\right] \tag{9}$$

Using the proposed quantisation function on the wavelet coefficients of textured images has the effect of shortening the tails of the resulting histograms, while decreasing the large peak typically found at the origin. Such histograms are

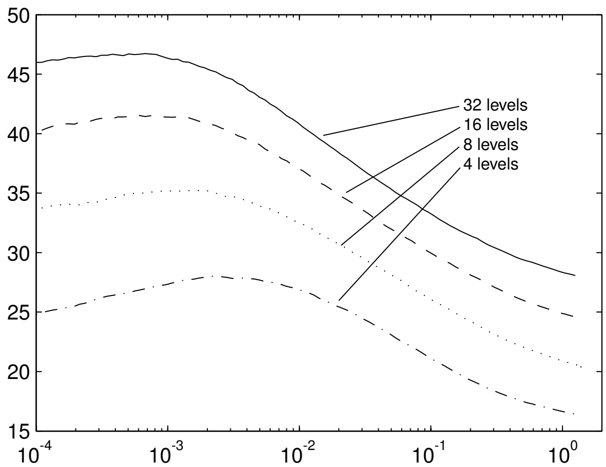


Fig. 2. PSNR(dB) vs δ for 4, 8, 16 and 32 quantisation levels.

then able to be better modelled by a Gaussian distribution, leading to improved characterisation of the texture by both first and second order statistics.

2.1. Image Distortion

The application of the logarithmic quantisation function described above to the wavelet coefficients of a textured image necessarily causes some distortion of this image if reconstruction is performed. Measurement of this distortion is possible using the peak signal to noise ratio (PSNR) which compares the original image to the distorted version. The PSNR of a modified image is defined as

$$PSNR = 20 \log_{10}(r/\varepsilon) \tag{10}$$

where ε is the RMS error between the reconstructed and original images, and r is the dynamic range of the image. A number of texture samples were quantised using the proposed technique, and the images reconstructed from the resulting coefficients. Since the sign of the coefficients is lost during the quantisation process, this information is reinstated before calculating the inverse DWT. The averaged PSNR over these images for a number of quantisation levels and values of δ is shown in figure 2, with the PSNR for uniform quantisation being 41.71dB, 35.82dB, 29.82db and 23.46dB for 32, 16, 8 and 4 levels respectively. From these results it can be seen that the logarithmic quantisation technique provides a significantly better representation of the textures, with the PSNR peaking at approximately $\delta = 0.001$ for 32 levels. It must be noted that PSNR is not always an accurate measure of perceived image quality, and that this value of δ may not be optimal for use in all texture classification tasks.

2.2. Wavelet Log Co-occurrence Signatures

The wavelet co-occurrence signatures proposed in [4] are extracted from a co-occurrence matrix constructed using uniform quantisation of the original wavelet coefficients. In this section, a new feature set is proposed utilising the logarithmic quantisation function developed previously. Three variations on this set are proposed for evaluation, with a different non-linearity applied before quantisation in each case. Thus, a *wavelet log co-occurrence matrix* $P_{j,l,d,\theta}(i,j)$ can be defined as the probability of two pixels from the wavelet detail image D_{jl} separated by distance d and angle θ having quantised values of i and j respectively, using one of three quantisation functions

$$q_1(x) = \begin{cases} \text{round}\left[\kappa \log\left(\frac{x}{a_s\delta}+1\right)\right], & x >= 0 \\ -\text{round}\left[\kappa \log\left(\frac{|x|}{a_s\delta}+1\right)\right], & x < 0 \end{cases} \tag{11}$$

$$q_2(x) = \text{round}\left[\kappa \log\left(\frac{|x|}{a_s\delta}+1\right)\right] \tag{12}$$

$$q_3(x) = \text{round}\left[\kappa \log\left(\frac{x^2}{a_s^2\delta}+1\right)\right] \tag{13}$$

$q_1(x)$ quantises the wavelet coefficients with no rectifying function, and thus requires twice as many quantisation levels to allow for negative values. $q_2(x)$ and $q_3(x)$ use the magnitude and squaring operators respectively to rectify the coefficients before quantisation.

From such matrices, the following co-occurrence features are extracted: energy, entropy, inertia, local homogeneity, maximum probability, cluster shade, cluster prominence and information measure of correlation. A mathematical description of these features can be found in [4]. In order to avoid an excessively large number of features, the value of d is restricted to 1, and the matrices for the four directions $\{0°, 45°, 90°, 135°\}$ are averaged to form a single matrix for each detail image D_{jl}. As correlations over small distances have been shown to provide the majority of information, this process does not result in a significant loss of classification accuracy. A total of 24 features are therefore extracted at each resolution level of the wavelet transform. For four decomposition levels, this leads to a total of 96 features, which we call the *wavelet log co-occurrence* (WLC$_k$) signatures, with $k \in \{1,2,3\}$ indicating the quantisation function used.

3. EXPERIMENTAL RESULTS

The performance of the wavelet log co-occurrence features was evaluated experimentally using a selection of 25 texture images from the Brodatz database [10]. The original plate number of these images are D1, D3, D4, D5, D9, D11, D16, D18, D19, D21, D24, D29, D37, D52, D53, D55, D68, D76,

Wavelet Energy	**Wavelet Co-oc.**	**WLC$_1$** $\delta = 10^{-3}$	**WLC$_2$** $\delta = 10^{-3}$	**WLC$_3$** $\delta = 10^{-4}$
6.7%	2.7%	1.4%	2.0%	1.8%

Table 1. Classification errors for wavelet log co-occurrence signatures compared to wavelet energy features and wavelet co-occurrence signatures extracted with uniform quantisation. In all cases, the value of k giving the lowest error rate is used.

D77, D80, D82, D84, D93, and D112, and were chosen on the basis of being relatively uniform in appearance and either non-directional or unidirectional in nature. Additionally, the classification performance using the simple wavelet energy signatures on this set of images is poor enough to allow for a meaningful comparison between the techniques. Each of the images was divided into non-overlapping regions of size 64x64, providing a total of 100 sample images for each texture class.

The proposed wavelet log co-occurrence signatures were extracted from these samples using each of the three quantisation functions, with 32 quantisation levels used in all cases. For comparison, the wavelet co-occurrence signatures proposed in [4] using uniform quantisation are also extracted with the same number of levels, as well as the wavelet energy features which have been widely used as a benchmark in the literature. Using a k-nn classifier, each sample image was classified, with the results shown in table 1. The *leave-one-out* technique was used to generate these results, as it has been previously shown to give an upper bound of the Bayes' error, and is thus a conservative estimate of actual classifier performance.

From the results of table 1, it can be seen that all of the WLC signatures outperform the uniformly quantised wavelet co-occurrence signatures, with the WLC$_1$ features showing error rates reduced by approximately 50% overall. The WLC$_2$ and WLC$_3$ signatures, in which the coefficients were rectified using the magnitude and squaring operators respectively, did not perform as well as the non-rectified WLC$_1$ signatures, from which it can be concluded that the sign of the coefficients is of some importance when calculating second-order statistics. Examination of the class error distributions shows that the WLC$_1$ features provide a lower error rate than the uniformly quantised wavelet co-occurrence features for all but a single class, with particular improvement noted for the D9 image, which was poorly classified by the standard wavelet co-occurrence features.

4. CONCLUSIONS AND FUTURE WORK

In this paper we have presented a new approach to the quantisation of wavelet coefficients, and shown that using such an approach can lead to a better characterisation of textured images. Using this approach, three new texture feature sets, the *wavelet log co-occurrence* signatures have been proposed. Experimental results have shown that these features outperform similar features extracted from co-occurrence matrices using uniform quantisation when applied to a range of natural textures from the widely used Brodatz album.

5. REFERENCES

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