TWO-DIMENSIONAL FREQUENCY ESTIMATION WITH MULTIPLICATIVE NOISE USING NON-CAUSAL MINIMUM VARIANCE REPRESENTATION

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ABSTRACT

In this paper, the problem of two-dimensional (2-D) frequency estimation of a complex sinusoid embedded in a white Gaussian additive noise and a multiplicative noise is addressed. For this purpose, we derive a non-causal Minimum Variance Representation, the coefficients of which are described according to the frequencies to be estimated. Therefore, estimates are given without a complete computation of the Power Spectral Density over the 2-D frequency plane, but directly from the coefficients.

Accuracy and robustness of this new 2-D frequency estimator are statistically assessed by Monte Carlo simulations. The results obtained show that a good local frequency estimation can be directly achieved with the proposed model, even for signal embedded in multiplicative noise.

1. INTRODUCTION

Two-dimensional (2-D) frequency estimation has been widely studied. Among its classical applications, we could mention the use of spectral properties for image segmentation or classification (e.g., [1]). In [2], authors proposed to decompose a texture into a sum of an indeterministic and a deterministic field, which can be characterized by 2-D resonant frequencies. Two-dimensional frequency estimation is also of interest in fields such as sonar and radar. This problem can be achieved using 2-D Fourier transform based methods. In [3], Kay and Nekovei described the use of the Fourier transform as the optimal maximum likelihood estimator for a single sinusoid in white Gaussian noise. Nevertheless, the use of this method requires a large data set and the stationarity over it. Actually, when the number of data available is small, those methods suffer from the resolution limit called the Rayleigh limit. These assumptions, which are very restrictive for real life images, reduce the use of such methods, particularly in the presence of additive and multiplicative noises. Therefore, short term frequency estimators have to be developed. Among the main methods proposed, we could mentioned 2-D autoregressive (AR) methods, 2-D maximum entropy methods or Minimum Variance Representations (MVR's) (e.g., [4], [5], [6]). These methods are commonly said to be of high resolution [7], [8] and can give better estimates than classical methods, especially when the number of data available is small.

The problem of 2-D frequency estimation is dealt with in this work using non-causal MVR's in presence of additive and multiplicative noises. The use of these methods corresponds to a modelling of the data as the output signal of an all-pole infinite impulse response (IIR) filter driven by a 2-D Gaussian random process. The parameters of this filter can be determined with classical spectral analysis techniques such as the Yule-Walker method. When the model is well chosen and after a numerical evaluation of the parameters, the power spectral density (PSD) of the output signal of the model is close to the true PSD of the experimental signal. The poles of such a model, which correspond to the roots of the transfer function denominator, are used in this study to estimate the 2-D frequency. The main problem in 2-D autoregressive modelling is that the roots of 2-D polynomials are generally continuous contours in a complex space, unlike isolated points for 1-D polynomials [8]. Therefore, although MVR's and autoregressive models have already been used for 2-D applications such as texture synthesis [9], or texture classification (e.g., [10]), no measurement can be performed directly from the autoregressive parameters, because poles are difficult to assess. For this purpose, we present in this paper a particular model of 2-D MVR's that ensures that the roots are isolated points. Coefficients of this model are directly estimated according to the frequencies to be estimated.

The paper is organized as follows. In section 2, the model of the complex signal with additive and multiplicative noises is presented. The non-causal MVR and frequency estimates are given in section 3. Performances are statistically assessed by Monte Carlo simulations in section 4. Finally, in section 5, we present the main conclusions.

2. TWO-DIMENSIONAL COMPLEX SIGNAL MODEL

In this work, we assume that observed process $s(k,l)$ is a complex sinusoid with varying amplitude embedded in complex white Gaussian additive noise given by

$$s(k,l) = A(k,l)\exp\left(j2\pi(f_1 k + f_2 l + \theta)\right) + n(k,l), \quad (1)$$

where :

- $k \in [0..K-1]$ and $l \in [0..L-1]$,
- f_1 and f_2 are the frequencies along the horizontal and the vertical directions respectively,
- θ is the initial phase, uniformly distributed in $[0..2\pi]$,
- $n(k,l)$ is the zero-mean complex white gaussian additive noise, with variance σ_n^2. In this case, variances of real and imaginary parts of $n(k,l)$ are $\sigma_n^2/2$.
- $A(k,l)$ is a real low-pass random process. The variance of this process is denoted σ_A^2.
- f_A corresponds to the normalized cut-off frequency of the varying amplitude $A(k,l)$.

The signal to noise ratio (SNR) is defined here as $SNR = \sigma_A^2/\sigma_n^2$. Figure 1 represents an example of the spectrum of $s(k,l)$ given by eq. (1), with $f_1 = f_2 = 0.15$, $K = L = 32$.

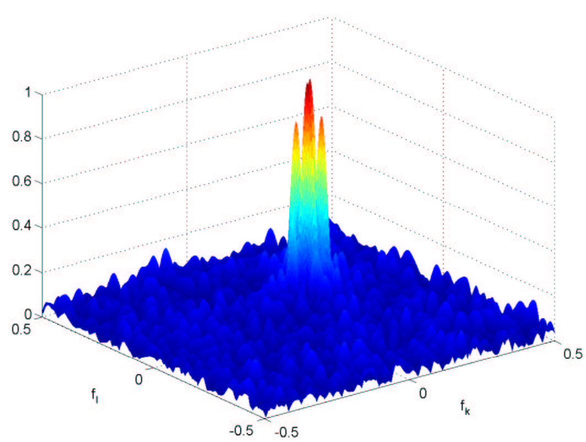

Fig. 1. Spectrum of a 32 × 32 2-D complex signal with additive and multiplicative noises.

3. ALGORITHM

As already mentioned in introduction, we propose in this work a new estimator for f_1 and f_2 based on the construction of a non-causal MVR. These methods consist in finding the coefficient set $\{a(m,n)\}$ that minimizes the variance of the prediction error defined in the 2-D general case by

$$\epsilon(k,l) = I(k,l) - \hat{I}(k,l), \quad (2)$$

where $\hat{I}(k,l)$, also called a (M,N)th-order minimum variance model for $I(k,l)$ using the prediction support V, is defined in [6] by

$$\hat{I}(k,l) = \sum_{(m,n)\in V} a(m,n) I(k-m, l-n). \quad (3)$$

In the general case, the support V of a noncausal model of order (M,N) is

$$V = \begin{array}{l} (m,n) : \\ \{-M \le m \le M, -N \le n \le N, (m,n) \ne (0,0)\}. \end{array}$$

The associated orthogonality condition, $E\left[\hat{I}(k,l)\epsilon(k,l)\right] = 0$, implies the normal equations [6] :

$$\sigma_\epsilon^2 d(k,l) = \phi_I(k,l) - \sum_{(m,n)\in V} a(m,n)\phi_I(k-m, l-n), \quad (4)$$

for all (k,l). σ_ϵ^2 represents the variance of the random process $\epsilon(k,l)$, $\phi_I(k,l)$ is the autocorrelation of $I(k,l)$, and $d(k,l)$ is the 2-D Kronecker delta function :

$$d(k,l) = \begin{cases} 1 & \text{for } k = l = 0, \\ 0 & \text{everywhere else.} \end{cases} \quad (5)$$

$I(k,l)$ given by the linear time invariant equation (2) can be written as the output signal of a 2-D infinite impulse response (IIR) filter defined by the z-domain transfer function

$$H(z_1, z_2) = \frac{1}{1 - \sum_{(m,n)\in V} a_{m,n} z_1^{-m} z_2^{-n}}, \quad (6)$$

where z_1^{-1} and z_2^{-1} are complex variables corresponding to unit delays in the k and l directions respectively :

$$\begin{array}{c} z_1^{-1} I(k,l) = I(k-1,l) \\ z_2^{-1} I(k,l) = I(k,l-1). \end{array}$$

Finally, if the order of this model (i.e. M and N) and the coefficients are well estimated, the true PSD of the analyzed image $I(k,l)$ can be estimated using eq. (6) in noting that $z_{1,2} = \exp(j2\pi f_{k,l})$. The presence of the denominator in the transfer function makes the model resonant for a set of frequencies, therefore IIR filters are generally used for estimating resonant frequencies corresponding to peaks in the

PSD, which is of our interest in this study. We are here interested in estimating frequencies f_1 and f_2 without evaluating the whole 2-D frequency function.

The main drawback of the use of non-causal models in 2-D is that, unlike in one-dimension, no factorization method of the polynomial denominator exists, and poles are not necessarily isolated points. Therefore, in our case, a natural relationship cannot be determined between the coefficients and resonant frequencies. In order to ensure the existence of poles and to estimate the peak frequencies without searching the maximum of the entire spectrum calculated from equation (6), we propose to use a model with an imposed and already factorized structure. The desired poles of the transfer function, according to figure 1 are

$$\begin{cases} z_1 = r_1 \\ z_2 = r_2 \end{cases} \tag{7}$$

where $r_1 = R_1 \exp(j2\pi f_1)$ and $r_2 = R_2 \exp(j2\pi f_2)$ are two complex numbers. We finally propose a transfer function corresponding to a resonant function when $z_1 = r_1$ and $z_2 = r_2$. These conditions are expressed in terms of distance in the following transfer function :

$$H(z_1, z_2) = \frac{1}{\left|(1 - r_1 z_1^{-1})\right|^2 + \left|(1 - r_2 z_2^{-1})\right|^2}. \tag{8}$$

The development of eq. (8) gives the following formula of the 2-D z-transform :

$$\begin{aligned} H(z_1, z_2) = 1/ \quad & [2 + R_1^2 + R_2^2 - R_1 \exp(+j2\pi f_1) z_1^{-1} \\ & -R_1 \exp(-j2\pi f_1) z_1 \\ & -R_2 \exp(+j2\pi f_2) z_2^{-1} \\ & -R_2 \exp(-j2\pi f_2) z_2]. \end{aligned} \tag{9}$$

We finally obtain a 2-D non-causal MVR model of the order $(1, 1)$. In figure 2, the spectrum of the proposed MVR is represented for the following parameters :

- $R_1 = R_2 = 0.95$,
- $f_1 = 0.1$ and $f_2 = 0.05$.

Modulus R_1 and R_2 of the roots of the autoregressive model represent the frequency response selectivity of the filter defined by eq. (8), according to the frequencies f_k and f_l respectively. In this work, there is no need to have two different values, therefore, we fix $R_1 = R_2 = R$. This yields a circular sharp response around frequencies (f_1, f_2), which is more appropriate for accurate estimations in the presence of multiplicative noise. Moreover, the use of a transfer function defined in terms of distance on eq. (8) involves the non-causal nature of the model, and allows quasi isotropic shapes of the peak in the estimated PSD.

We are now interested in estimating the coefficients in their particular expression given by eq. (9). To be more precise, the set of three parameters f_1, f_2 and R has to be estimated. Moreover, this model ensure the existence of one pole and therefore the direct calculation of the frequencies.

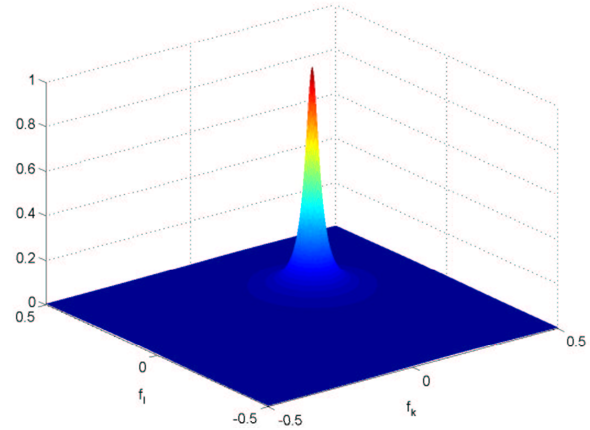

Fig. 2. Frequency response of 2-D MVR with $R_1 = R_2 = 0.9$ and $f_1 = f_2 = 0.15$.

The normal equation (4) becomes for $k = l = 0$:

$$\begin{aligned} \sigma_\epsilon^2 = \quad & (2 + 2R^2)\phi_I(0,0) - R\exp(j2\pi f_1)\phi_I(-1,0) \\ & -R\exp(-j2\pi f_1)\phi_I(1,0) \\ & -R\exp(-j2\pi f_2)\phi_I(0,-1) \\ & -R\exp(-j2\pi f_2)\phi_I(0,1). \end{aligned} \tag{10}$$

The set of coefficients $X = [R, f_1, f_2]^T$ is finally obtained by minimizing the prediction error variance from eq. (10) :

$$\begin{aligned} X^{OPT} = \underset{X}{\text{argmin}} \quad & |(2 + 2R^2)\phi_I(0,0) \\ & -R\exp(+j2\pi f_1)\phi_I(-1,0) \\ & -R\exp(-j2\pi f_1)\phi_I(1,0) \\ & -R\exp(+j2\pi f_2)\phi_I(0,-1) \\ & -R\exp(-j2\pi f_2)\phi_I(0,1)|, \end{aligned} \tag{11}$$

with $R \in]0..1[$ and $f_{1,2} \in [-0.5..0.5]$.

4. PERFORMANCES

In order to study performances of this 2-D frequency estimator, Monte Carlo simulations have been performed. Estimations given by our estimator have been compared with the classical frequency estimator based on the use of the Fourier transform estimated on 1024×1024 points, for the following parameters :

- θ uniformly distributed in $[0...2\pi]$,
- $K = L = 32$,
- $f_1 = f_2 = 0.15$,

- $f_A = 0.03$.

For this purpose, the modulus R of the MVR pole is set constant to 0.9, allowing resonant frequency estimates. Preliminary results show that the two methods are unbiased. Moreover, when the additive noise is low (this value depends on the PSD of the varying amplitude), the frequency estimation variance for the Fourier-based method depends only on the size of the analyzed image and the cut-off frequency f_A. Beyond this threshold, the variance of the proposed algorithm ($2.8\ 10^{-5}$) is much more lower than the variance of the Fourier-based method ($2.2\ 10^{-4}$), as shown in figure 3.

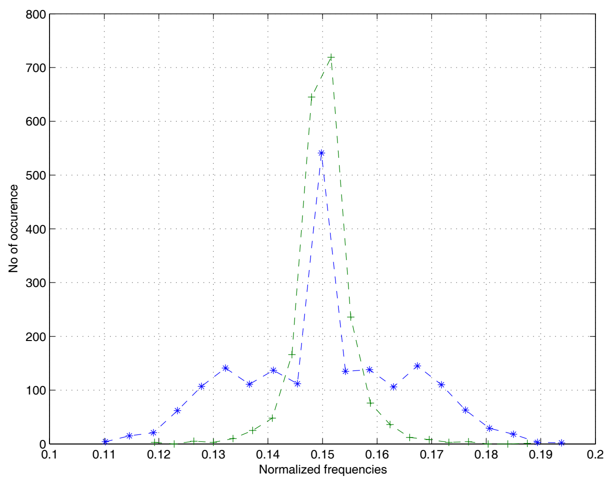

Fig. 3. Histogram plots of the frequency estimation for the Fourier-based method ($- * -$) and the proposed estimator ($- + -$).

Histogram plots in figure 3 show that the presence of the multiplicative noise increases the estimation variance of the Fourier method. It should be noted that this variance has to be linked not only to the varying amplitude bandwidth but also to the PSD estimation variance of the image using short windows. Our findings indicate that the proposed algorithm gives better estimates because of the use of a single frequency resonant MVR. Thus, this resonant frequency appears in the center of the estimated 2-D bandwidth.

5. CONCLUSION

This paper is concerned with the problem of 2-D frequency estimation in presence of additive and multiplicative noises. An original point of this work is the use of a non-causal Minimum Variance Representation. In the 2-D high resolution AR-based methods, causal supports have been merely used. This is due to the difficulty of estimating the parameters of the model in a maximum likelihood sense. In this work, we propose a new model ensuring the existence of a pole in the 2-D frequency plane, the coefficients of which are analytically defined according to the pole parameters and, in particular, according to the frequencies to be estimated.

Performances of this algorithm have been studied for synthetic images with additive and multiplicative noises. Results are then compared with those obtained with the classical frequency estimator based on the use of the Fourier transform. Our findings reveal the satisfactory behavior of the proposed model for signal embedded in multiplicative noise.

6. REFERENCES

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