EMPIRICAL CHOICE OF SMOOTHING PARAMETERS IN ROBUST OPTICAL FLOW ESTIMATION

Mingren Shi, Victor Solo

School of Electrical Engineering
University of New South Wales
Sydney, NSW 2052, AUSTRALIA
email: vsolo@syscon.ee.unsw.edu.au

ABSTRACT

Optical flow estimation algorithms such as the Lukas-Kanade method and Horn and Schunk method require selection of a tuning parameter. In the former case a neighbourhood size, in the latter, a penalty parameter. Selection of these tuning parameters is difficult in general but has a profound effect on the results. So automatic methods of selection are of great interest. In previous work we have developed selection methods for the algorithms above. Here we develop a selection procedure for a robust version of the Lukas-Kanade method. This is a non-trivial task since the robust algorithm is nonlinear.

1. INTRODUCTION

The problem of estimating motion (i.e. velocities) from a sequence of image intensities $I = I(t, x_1, x_2)$ is a much studied one in computer vision e.g. [1],[2],[3],[4]. Most approaches to this problem start with the brightness constraint equation (BCE) which assumes brightness does not change with time so

$$\begin{aligned} \frac{dI}{dt} &= 0 = \frac{\partial I}{\partial t} + u\frac{\partial I}{\partial x_1} + v\frac{\partial I}{\partial x_2} \\ &= I_t + uI_{x_1} + vI_{x_2} \end{aligned}$$

where I_t, I_{x_1}, I_{x_2} are image intensity gradients; u, v are the x_1, x_2 components of optical flow. By temporal and spatial differencing the image gradients can be estimated (as $\hat{I}_t, \hat{I}_{x_1}, \hat{I}_{x_2}$) from image intensities. The BCE provides one constraint on the two velocities u, v. But further information is needed and this is supplied by the assumption of spatial continuity for u, v.

Such continuity can be supplied either as in [5](HS) by Tikhonov regularization or as in [6](LK) by local estimation. More recent approaches to optical flow estimation have emphasized the necessity to take account of outliers due e.g. to occlusion; thus the above methods are modified [4],[2]. Our aim here is to develop a method for automatic selection of the neighbourhood size for such a robust [6] algorithm. Our approach can handle the HS approach but will be pursued elsewhere.

This work was partly supported by the Australian Research Council

For ill-conditioned inverse problems [7] (of which Optical flow estimation is an example) the problem of estimating tuning parameters has a large literature e.g.[8]. There are two approaches: deterministic, most notably cross-validation based on minimising an estimator of a mean squared error quality measure e.g.[9]; stochastic based on a(n) (empirical) Bayesian approach e.g[10],[11].

Unfortunately, as discussed in [12], the Bayesian approach is usually computationally very demanding and approximations are necessary. We pursue the deterministic approach here; although cross-validation is also computationally demanding for nonlinear problems our approach [12] has computational demands that are usually modest by comparison. The Bayesian approach has been mentioned in the computer vision literature [13] but not applied to optical flow.

While there has been some work on tuning parameter selection in image processing e.g.[14] there has been only a little on optical flow [15],[16],[17] and none of it applies to robust optical flow estimation.

In section 2 we formulate the robust BCE . In section 3 we develop our new selection criterion. Section 4 contains results and conclusions in section 5.

In the sequel $P = (x_1, x_2)$ is a point on the plane.

2. NOISY BCE

We formulate the velocity estimation problem as a regression problem in a standard way as follows

$$\begin{aligned} y &= \mu + \epsilon \\ y &= -I_t \end{aligned}$$

$$\begin{aligned} \mu &= uI_{x_1} + vI_{x_2} = g^T f \\ f, g &= (u, v)^T, (I_{x_1}, I_{x_2})^T \end{aligned}$$

Here y represents a lexicograhically ordered image of $M \times M$ pixels as do $u, v, I_{x_1}, I_{x_2}, \mu$ (so e.g. y is an $M^2 \times 1$ vector). The noise ϵ , treated as white noise of variance σ^2, soaks up violations of the BCE due e.g. variations in ambient illumination, occlusions etc.

A number of approaches have been taken to robust estimation of optical flow [2],[4],[18]. Since we are concentrating on automatic tuning parameter selection we use a straightforward approach.

At point P the 'overlapped' velocity estimate $\hat{f}_P$ is obtained by solving the robust regression problem [19].

$$\begin{aligned} \hat{f}_P &= arg.min_f J_P(f) \\ J_P(f) &= \Sigma_Q \rho(y_{P-Q} - g^T_{P-Q} f) w_{P-Q} \end{aligned}$$

where $\rho(\cdot)$ is a potential function e.g. we use

$\rho(\epsilon) = \sqrt{\epsilon^2 + \gamma^2} - \gamma$ where γ is a small fixed parameter; w_Q is a hump shaped kernel (e.g. a product of triangles or a Gaussian) whose region of support defines the neighbourhood of pixels used to calculate $\hat{f}_P$. As long as the kernel is continuous its use eliminates Gibbs ringing and 'zigzagging' in the criterion below.

Carrying out the optimisation leads to the estimator (with $e_{P-Q} = y_{P-Q} - g^T_{P-Q} f$)

$$\begin{aligned} \hat{f}_P &= M_P^{-1} N_P \\ M_P &= \Sigma g_{P-Q} g^T_{P-Q} w_{P-Q} \psi_{P-Q} \\ N_P &= \Sigma \psi(e_{P-Q}) w_{P-Q} g_{P-Q} y_{P-Q} \end{aligned}$$

where $\psi(e) = \frac{\rho'(e)}{e} = \frac{1}{\sqrt{e^2+\gamma^2}}$ is the influence function. This equation has to be iterated at each P. It usually converges in a few steps. It is initialized with the overlapped LK estimator using $\psi(e) = 1$.

3. AUTOMATIC NEIGHBOURHOOD SELECTION

To measure the quality of the optical flow estimator. we use intensity based mean squared error (mse). (A criterion using velocity base mse is much harder to develop and will be pursued elsewhere). We also note that methods such as AIC (Akaike Information Criterion),MDL (Minimum Description Length) are inapplicable since they require that the problem be specified in terms of a model dimension which is not the case here.

So our quality measure or statistical 'risk' is

$$\begin{aligned} R &= E \parallel \hat{\mu} - \mu \parallel^2 \\ &= E \parallel g^T(\hat{f} - f) \parallel^2 \end{aligned}$$

where $\parallel z \parallel^2 = z^T z$ and $\hat{f}$ is the optical flow estimator from the algorithm of interest. Ideally we would choose the neighbourhood size to minimize R. Now R cannot be calculated since f is unknown so the idea is to find an empirically computable surrogate for R and minimize that instead. It can be shown (modifying [12]) that an unbiassed estimator of R (known as Stein's unbiassed risk estimator SURE) is (with $e = y - \hat{\mu}$ = residual)

$$\hat{R} = \parallel e \parallel^2 - 2\sigma^2 trace(\frac{\partial e^T}{\partial y}) + M^2 \sigma^2$$

Dropping terms not dependent on the neighbourhood size leaves

$$\hat{R} = -2\hat{\mu}^T y + \parallel \hat{\mu} \parallel^2 + 2\sigma^2 \Sigma_P \frac{\partial \hat{\mu}_P}{\partial y_P}$$

Following [12] it can be shown that

$$\frac{\partial \hat{\mu}_P}{\partial y_P} = -J^{-1}_{ffP} J_{fyP}$$

where e.g. $J_{fyP} = \frac{\partial^2 J}{\partial f \partial y_P}$. For our robust estimator we find

$$\begin{aligned} J_{ffP} &= \Sigma_Q \rho''(e_{P-Q}) g_{P-Q} g^T_{P-Q} w_{P-Q} \\ J_{fyP} &= -\rho''(e_P) g_P w_0 \end{aligned}$$

In our case $\rho''(e) = \frac{\gamma^2}{(e^2+\gamma^2)^{3/2}}$.

This leads to the criterion (with $\hat{\mu}_P = g_P^T \hat{f}_P$)

$$\hat{R} = \Sigma(-2\hat{\mu}_P y_P + \hat{\mu}_P^2 + 2\sigma^2 g_P^T J^{-1}_{ffP} g_P w_0 \rho''(e_P))$$

We call this the robust SURE criterion. It applies to any choice of potential. For a squared error potential function it collapses to that of [16].

4. RESULTS

We illustrate our new results with a well known example; the rotating Rubik cube sequence used in [20]. In Fig.1 is a plot of the robust SURE criterion. It exhibits a well defined minimum (at 11×11) although in practice one will want to look at estimates constructed based on neighbourhood sizes in the vicinity of the minimum [9] e.g. 9×9. In Fig.2 is shown the optical flow estimates for the regular LK method (with an optimal neighbourhood size chosen using the appropriately simplified criterion). Fig.3 shows the robust optical flow estimates based on the optimal neighbourhood size. The regular estimator shows some erratic behaviour in several places. This fluctuation is absent from the robust estimator. The estimates are based on $\gamma = .01$ and 10 iterations at each point. Changing γ does not change the qualitative nature of the results but does have some small impact on the details. In Fig.4,Fig.5 we show SURE and estimated flow with 20 iterations. We see the qualitative nature of the result is unchanged. However the minimum has

Fig. 1. Robust SURE for Rubik Cube Sequence.

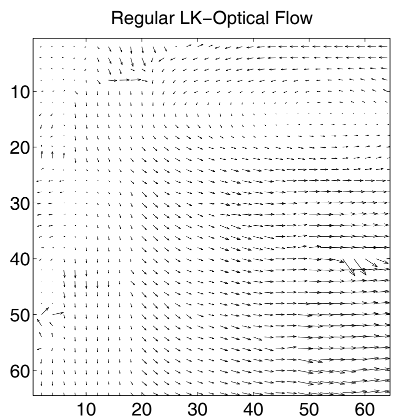

Fig. 2. Regular LK Optical Flow for Rubik Cube Sequence

shifted to 9×9. The difference is not great and as indicated before we would now look at 11×11, as well perhaps at 7×7. The estimated flow with a 9×9 neighbourhood size is a little more noisy than the 11×11 estimate and we would recommend the latter in this case.

5. CONCLUSION

We have exhibited for the first time an automatic procedure for selection of neighbourhood size for robust Lukas-Kanade type optical flow estimation. The method has been illustrated successfully on a standard image sequence and shows a well defined optimal neighbourhood size choice. There is a small influence of iteration count on the results. In practice however one may wish to view flow reconstructions using neighbourhood sizes in the vicinity of the minimum and so these small variations will not be a problem.

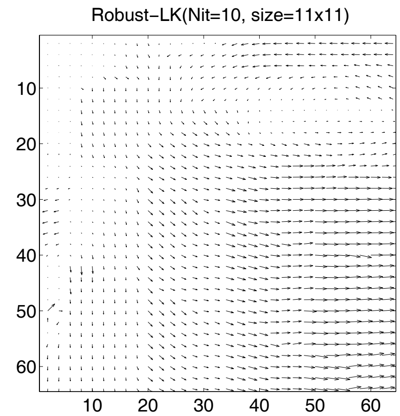

Fig. 3. Robust LK Optical Flow for Rubik Cube Sequence

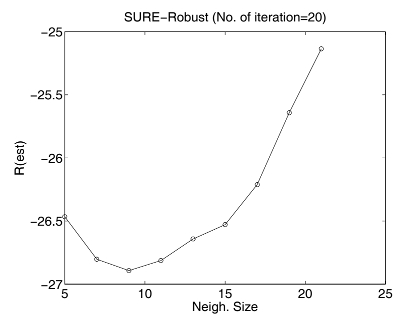

Fig. 4. Robust SURE for Rubik Cube Sequence (20 iterations).

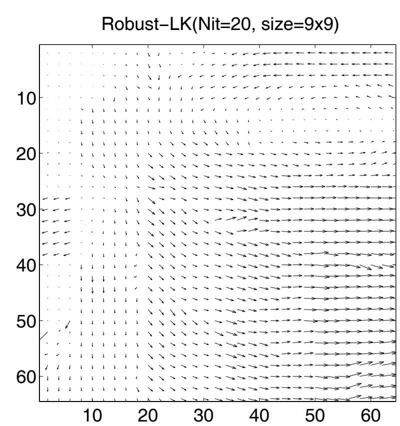

Fig. 5. Robust LK Optical Flow for Rubik Cube Sequence (20 iterations).

6. REFERENCES

[1] B.K.P. Horn, *Robot Vision*, Massachusetts, MIT Press, Cambridge, 1986.

[2] M J Black and P Anandan, "The robust estimation of multiple motions :parametric and piecewise-smooth flow fields," *Comp Vis Image Understanding*, vol. 63, pp. 75–104, 1996.

[3] R.M. Haralick and L.G. Shapiro, *Computer and Robot Vision, Vol II*, Addison-Wesley, New York, 1993.

[4] J. Weber and J. Malik, "Robust computation of optical flow in a multi-scale differential framework," *Int Jl Comp Vision*, vol. 14, pp. 67–81, 1995.

[5] B.K.P. Horn and B.G. Schunck, "Determining optical flow," *Artificial Intelligence*, vol. 17, pp. 185–203, 1981.

[6] B. Lucas and T. Kanade, "An iterative image registration technique with an application to stereo vision," in *DARPA Image Understanding Workshop*. DARPA, 1981, pp. 121–130.

[7] A. Tikhonov and V. Arsenin, *Solutions of Ill-Posed Problems*, Wiley, New York, 1977.

[8] V.A. Morozov, *Regularization Methods for Ill-posed Problems*, CRC Press, Boca Raton, 1993.

[9] M.P. Wand and M.C. Jones, *Kernel Smoothing*, Chapman and Hall, London, 1995.

[10] Z Zhou, R Leahy, and J Qi, "Approximate maximum likelihood hyperparameter estimation for gibbs priors," vol. 6, pp. 844–861, 1997.

[11] AM Thompson and J Kay, "On some Bayesian choices of regularisation parameter in image restoration," *Inverse problems*, vol. 9, pp. 749–761, 1993.

[12] V. Solo, "A SURE-fired way to choose smoothing parameters in ill-conditioned inverse problems," in *Proc. IEEE ICIP96*. IEEE, 1996, IEEE Press.

[13] R Szeliski, *Bayesian Modeling of Uncertainty in Low-Level Vision*, Kluwer Academic Publishers, Amsterdam, 1989.

[14] M.G. Kang and A.K. Katsaggelos, "General choice of the regularization functional in regularised image restoration," *IEEE Trans. Im.Proc.*, vol. 4, pp. 594–602, 1995.

[15] V V Estrela and N P Galatsanos, "Spatially-adaptive regularized pel-recursive motion estimation based on cross-validation," in *Proc IEEE ICIP98*. IEEE, 1998, pp. –.

[16] L. Ng and V. Solo, "A data-driven method for choosing smoothing parameters in optical flow problems," in *Proc. IEEE ICIP97*. IEEE Press, 1997, pp. III:360–363.

[17] L Ng and V Solo, "A data driven smoothing parameter selection technique for optical flow estimation," in *Proc IEEE ICIP2000, Seattle*. IEEE, 2000.

[18] A.B. Hadiashar and D. Suter, "Robust optic flow computation," *Int Jl Comp Vision*, vol. 29, pp. 59–77, 1998.

[19] R G Staudte and S J Sheather, *Robust Estimation and Testing*, J Wiley, New York, 1990.

[20] J.L. Barron, D.J. Fleet, and S.S. Beauchemin, "Systems and experiment: Performance of optical flow techniques," *Int Jl Comp Vision*, vol. 12, pp. 43–74, 1994.