RINGING ARTIFACT REDUCTION IN THE WAVELET-BASED DENOISING

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ABSTRACT

Wavelet is a well-known noise reduction tool for its several nice properties. Since it is a kind of the transform domain, thresholding in the wavelet domain can cause problems at the neighborhood of discontinuous points, seen as ringing artifact in 2-D image. In the low-bit rate network and the heavy error-prone environment, it is a serious problem for an end-user. In this paper, we propose an efficient ringing reduction algorithm based on the standard wavelet decomposition. Since its structure is the reverse of the wavelet denoising method, our algorithm has advantages about computation load and memory efficiency over other ringing reduction filters. Experimental results show that the proposed algorithm reduces ringing along the edge dramatically.

1. INTRODUCTION

Denoising has been used for visually pleasing purpose in the post-processing step. Nowadays, noise removal gathers attention from the wide field because the fact that it gives great efficacy to the subsequent compression algorithm if it is used as pre-processing. In addition to improving visual quality, denoising helps detect motion vectors and scene changes correctly and reduces, literally, noise which acts like high frequency contents and prevents the efficiency of run length encoding. As a rule of thumb, by using denoising as pre-processing, one can get about two times more compression ability than compression standard algorithm alone does.

Wavelet is a main tool for the denoising with its property of good localization in both spatial and spectrum domains, superior separation of noise and signal contents. It also gives flexibility to make use of several scales, or resolutions with inter or intra dependencies, which combines joint use of scale and space consistency[1]. Also its basis functions can be chosen optimally according to the applications. Therefore, wavelet is the most powerful basis for seeing the forest and the trees simultaneously[2].

In spite of excellent properties of wavelet, it has severe artifact known as ringing which manifests itself at the strong edge and in the low-bit rate network. This ringing artifact is a common problem for the transform domain processing because of its finite number of basis functions[3]. In the standard wavelet decomposition, down-sampling process broadens the region of ringing effect. Therefore, ringing artifact is more complex problem for wavelet than for other transforms.

The simplest method to reduce ringing is not to use down-sampling process which leads to the notion of, known as *á trous* algorithm, time-invariant wavelet unlike standard wavelet. However, because of the limitation of memory space, it can not have enough decomposition levels to denoise. The efforts to reduce ringing in the standard wavelet have been made by [4][5]. They used the periodic time-invariant property of wavelet and proposed $SpinCycle$ algorithm. However, because of its heavy computational complexity, it has problem extending to 2-D image. Another authors proposed to apply a vector thresholding, but the computation and the recording of all footprints are pretty heavy[6].

This paper is organized as follows. Section 2 introduces theoretical background about wavelet thresholding and ringing artifacts and Section 3 proposes our reduction filter for ringing artifact. Experimental results are explained in Section 4. Section 5 concludes the paper.

2. THEORETICAL BASIS

2.1. Wavelet Thresholding

In the wavelet domain, the problem can be formulated as

$$y = x + n \tag{1}$$

where y is a wavelet noisy coefficient, x is an original wavelet coefficient and n is additive white Gaussian noise. If *Maximum a Posteriori*(MAP) estimation is used, estimate of x can be obtained by

$$\begin{aligned} \hat{x} &= \arg\max[p_{x|y}(x|y)] && (2)\\ &= \arg\max[p_{y|x}(y|x)p_x(x)] && (3)\\ &= \arg\max[\log\{p_{y|x}(y|x)\} + \log\{p_x(x)\}]. && (4) \end{aligned}$$

The subscripts on p are used to denote that $p_{x|y}$, $p_{y|x}$, and p_x are different functions. Log function is monotonically increasing, so it does not affect the estimate of the original. Various types of prior knowledge can be incorporated into the form of $p_x(x)$[7]. If zero-mean Gaussian assumption is used, that is,

$$p_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{x^2}{2\sigma^2}) \tag{5}$$

then, the solution is

$$\hat{x} = \frac{\sigma^2}{\sigma^2+\sigma_n^2}y. \tag{6}$$

The above form is the classical Wiener filter, often called LMMSE filter[8]. On the other hand, if the Laplacian assumption is used as

$$p_x(x) = \frac{1}{\sqrt{2\sigma^2}}\exp(-\frac{\sqrt{2}|x|}{\sigma}), \tag{7}$$

then, the estimate can be obtained as

$$\hat{x} = \text{sign}(y)(|y| - \frac{\sqrt{2}\sigma_n^2}{\sigma})_+. \tag{8}$$

Here, $(x)_+$ is defined as

$$(x)_+ = \begin{cases} 0, & if\ x < 0 \\ x, & otherwise. \end{cases} \tag{9}$$

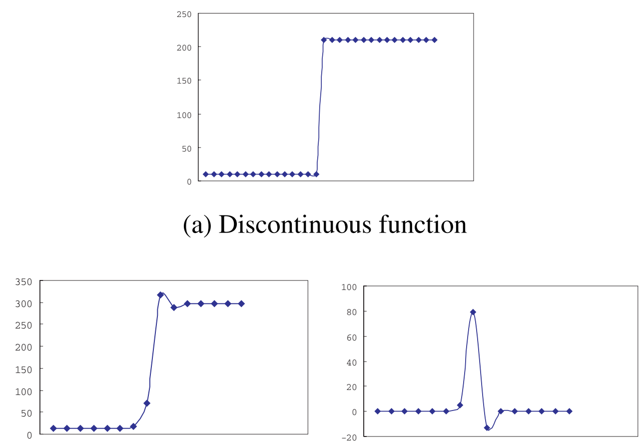
Eq. (8) is often called soft thresholding which is first devised by Donoho[9].

The σ in Eq. (6) and (8) can be calculated globally or locally, but local estimation of σ improves visual quality by using local adaptivity[10][11].

2.2. Ringing Artifacts

Ringing artifacts are a common problem for the transform domain processing. Because of the irrelevance between data in the DCT and the absence of time-invariance in the wavelet, thresholding is a typical technique for denoising and compression algorithm instead of convolution.

Discontinuous function in the spatial domain is described by the function of infinite duration with decays to the zero value in the transform domain. In this case, the truncation of the small values is a rational choice for memory and computational load. However, this can cause an artifact at the neighborhood of discontinuous point in the spatial domain, which is known as Gibbs phenomenon. This is also the case for the wavelet domain. In the wavelet domain, due to the spatial and spectrum localization property, it produces overshoot with more local and smaller amplitude at the neighborhood of discontinuous point. They are called pseudo-Gibbs phenomena[12].



(a) Discontinuous function

(b) Low-frequency part (c) High-frequency part

Fig. 1. Ringing artifacts in the wavelet domain.

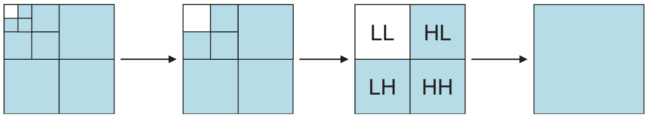
Figure 1a shows a discontinuous 1-D signal and Fig. 1b and Fig. 1c present wavelet decomposition results. As can be seen in the figures, Gibbs phenomenon manifests itself as an overshoot at the discontinuous point in the low-frequency part. If the low band is further decomposed into second low and high bands with downsampling, then the region of ringing in the image is broadened.

3. THE PROPOSED ALGORITHM

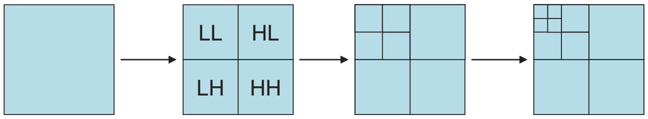
Unlike the traditional wavelet denoising algorithm, in the proposed denoising framework, LL band is also the target for the denoising. After all bands(including LL band) are processed the denoised LL band is further decomposed. This is just the reverse process of the conventional wavelet denoising method. In the conventional algorithm, it begins to manipulate from the finest bands and merge into the coarser bands, while in our proposed algorithm we start to manipulate from the coarsest bands and split into the finer bands. If the conventional algorithm is denoted as process / merge / process / merge / $\cdots$, the proposed algorithm can be represented as process / split / process / split / $\cdots$. These structure can be described as Fig. 2. The shaded region is the target for the denoising algorithm. In the conventional algorithm, LL bands from the finest scale is not an interest for denosing while in the proposed algorithm LL band is more interesting band than any other bandpass bands.

To reduce ringing artifacts, LL band is processed by the weighted local Wiener filter. Generally, LL band does not have zero-mean, so slight modifications must be made to Eq. (6) as follows

$$\hat{x} = m_L + \frac{\sigma_L^2}{\sigma_L^2+\sigma_n^2}(y - m_L). \tag{10}$$



(a) The conventional algorithm



(b) The proposed algorithm

Fig. 2. Block diagram of the algorithms.

Here, m_L and σ_L denote local statistics. In the local mask, only the pixels that satisfy $|X(i,j) - X(i-s, j-t)| < T$ where $X(i,j)$ is center pixel and $-M/2 \leq s, t \leq M/2$, are used in calculating mean and variance. Here, T is the threshold value and M is the size of the spatial mask. As T gets smaller, the resultant image gets noisier. On the other hand, as T gets higher, the result would be smoother. In the other bandpass region, that is, LH, HL, and HH, the local soft thresholding algorithm is applied as in [10].

Noise variance in Eq. (6) and (8) can be obtained by

$$\sigma_n = \frac{med\{|Y(i,j)|\}}{0.6745} \tag{11}$$

where $med\{\cdot\}$ is a median function which returns the middle value in the ordered sequence and $Y(i,j)$ are coefficients of the coarsest HH band[7]. The median function can give robustness to the noise estimator and Eq. (11) works well to various standard images. The proposed algorithm can be summarized as follows.

Step 1 Estimate the noise variance σ_n^2 in the noisy image using Eq. (11).

Step 2 Decompose the image into low and bandpass bands, that is, LL , LH, HL, and HH bands.

Step 3 Apply the local soft thresholding to the LH, HL, HH bands and apply the weighted local Wiener filter to the LL band.

Step 4 Go to *Step 2* with setting LL band to image if the decomposition level requirement is not met, or go to *Step 5*.

Step 5 Merge all decomposed bands into the final denoised result.

4. EXPERIMENTAL RESULTS

In our experiment, we adopted CDF(Cohen, Daubechies, Feauveau) 9/7 transform as the wavelet decomposition[13]. CDF 9/7 transform has (4,4) vanishing moments. The lifting equation of the transform is as follows[14]

$$d_1(i) = d_0(i) - \frac{203}{128}\{s_0(i+1) + s_0(i)\} \tag{12}$$

$$s_1(i) = s_0(i) - \frac{217}{4096}\{d_1(i) + d_1(i-1)\} \tag{13}$$

$$d_2(i) = d_1(i) + \frac{113}{128}\{s_1(i+1) + s_1(i)\} \tag{14}$$

$$s_2(i) = s_1(i) + \frac{1817}{4096}\{d_2(i) + d_2(i-1)\} \tag{15}$$

$$d(i) = d_2(i)/1.149604398 \tag{16}$$

$$s(i) = s_2(i) \times 1.149604398. \tag{17}$$

Here, $s_0(i)$ and $d_0(i)$ denote even and odd pixels in the spatial image and $s(i)$ and $d(i)$ represent low-pass and high-pass decomposed signals. Although Eq. (12)–(17) are expressed as 1-D form, they can be easily extended to 2-D image using row-by-row and column-by-column processing.

Figure 3a and 3b present original and its corrupted version, respectively. The noisy image is 10 dB of SNR, which amounts to $\sigma_n = 19.7142$. SNR is calculated as

$$\text{SNR} = 10\log\frac{||\vec{x}||^2}{||\vec{n}||^2}. \tag{18}$$

(a) Original image

(b) Noisy image

(c) The conventional algorithm

(d) The proposed algorithm

Fig. 3. Results of the conventional and proposed algorithm.

Cameraman image is good for ringing test because of its definite difference between background and the coat the man is wearing. Figure 3c is the result of local soft thresholding with 3 decomposition levels and Fig. 3d is the result of proposed algorithm with 2 decomposition levels. Both the algorithms used parameter values with $M = 5$ and Fig. 3d is the result of the proposed algorithm with $T = 90$. The proposed algorithm eliminates ringing along the edge efficiently while it maintains the characteristics in the other flat area. Undoubtedly, there is no question about the superiority of the result of Fig. 3d over that of Fig. 3c.

5. CONCLUSION

In this paper we proposed an efficient ringing artifact reduction algorithm in the wavelet-based denoising. Although wavelet has efficient noise reduction ability, ringing artifact which is seen in the heavy noise-prone network or low-bit rate environment is a main obstacle for high visual quality. Proposed algorithm is just the reverse process of the conventional wavelet denoising method and its computational load is almost the same as the local soft thresholding method. Therefore our ringing reduction algorithm requires reasonable memory and computation relative to *SpinCycle*[4] and vector thresholding[6].

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