A MULTIRESOLUTION DIRECTIONAL FILTER BANK FOR IMAGE APPLICATIONS

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ABSTRACT

In this paper, we proposed a new formulation of directional filter banks (DFBs). By using a non-uniform and non-separable filter bank, a critically sampled multiresolution directional image representation can be obtained efficiently. The resulting DFB yields non-uniform frequency division which composes of one lowpass channel with a decimation factor of one-fourth and six highpass directional channels with a decimation factor of one-eighth. It overcomes the limited directional selectivity of the separable wavelets and limited resolution of the conventional DFB. The lowpass channel can be used to obtain multiresolution representation by simply re-iterating the same DFB decomposition. On the other hand, the directional subbands can be further refined by simply applying a two-channel DFB at each highpass channel. A simple design method yielding near orthogonal uniform and non-uniform multidimensional filter banks is discussed, and, finally, a numerical experiment is presented to demonstrate potential of the new image basis.

1. INTRODUCTION

Wavelets and filter banks have been used effectively in many signal processing applications [1]. An advantage of using wavelet bases instead of Fourier bases is due to approximation power of wavelet series in signal with singularity since it would take a larger number of Fourier coefficients than wavelet coefficients to represent a signal with discontinuities. Typically, one constructs a two-dimensional (2D) wavelet by taking the tensor product of one-dimensional (1D) wavelets. This 2D wavelet is still effective at approximating point singularity (e.g. points in an image), but it is not very efficient for line singularity (e.g. edges in an image). This fact was notified by many researchers [2, 3], and therefore finding a more effective basis for images is currently a very active research area.

The directional filter bank (DFB) originally introduced in [4] has been proven to be effective in processing images with directional information [5, 6]. The DFB decomposition shares two important properties with the traditional discrete wavelet transform (DWT), namely maximally decimated and perfect reconstruction (PR). Although the DFB in [4] can extract directional information into 2^n subbands (see Figure 1 (b)), the decomposition does not have lowpass or highpass subbands, and DC energy is spread among all the directional subbands. Usually, the DC energy is not divided evenly due to practically design limitation.

In [7], Do et al. propose a pyramidal DFB in order to implement the contourlet transform, a discrete version of the curvelet transform [8]. The proposed structure is a combination of the Laplacian pyramid and the DFB, and is oversampled by a factor of $4/3$. Recently, a maximally decimated version is proposed in [9], which yields more directional subbands at higher frequency. However, the problem of dividing the DC component is still not resolved because there are four lowpass directional subbands at DC. In addition, the proposed filter bank is implemented by a binary tree structure with very large support filters, which can lead to very large support of the overall filters' impulse responses and directional error in the reconstructed image.

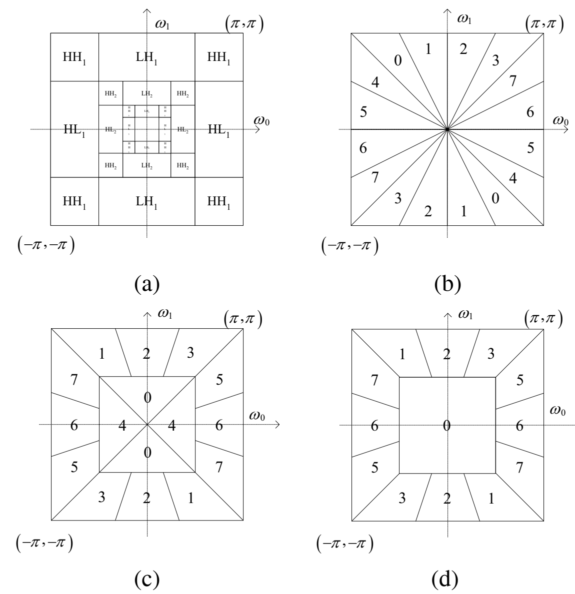


Figure 1: Frequency partition of the filter banks: (a) DWT, (b)conventional DFB, (c) new uniform DFB and (d) nonuniform DFB.

The paper is organized as follows. In the next section, the new DFB recently introduced in [10] and its properties are briefly presented. Limitations of the binary tree structure used in the construction of the maximally decimated DFBs in [9, 10] are discussed. An extension to nonuniform decomposition which allows for multiresolution decomposition is presented in Section 3. In Section 4, a simple filter design algorithm adapted from [11] for both uniform and non-uniform cases is discussed. A numerical experiment is presented in Section 5 to demonstrate the potential of the new DFB, and Section 6 concludes the paper.

*N*otations: For notation of 2D multirate signal processing, we refer to [12]. The followings are some special matrices that are used to decimate subband images in the paper:

$$\mathbf{Q} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \; \mathbf{Q}_2 = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}, \; \mathbf{D}_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

2. THE NEW DIRECTIONAL FILTER BANKS

In [10], we proposed a new DFB (uDFB) with frequency partition depicted in Figure 1(c) as an alternative to the conventional DFB in [4] (Figure 1(b)). The uDFB yields a uniform eight-channel filter bank with corresponding decimation matrix $\mathbf{Q}_2$. The signal is decomposed into two lowpass channels (both connected to DC) and six other highpass directional channels. According to Figure 1(c), the lowpass subbands 0 and 4, once appropriately combined as discussed in the next section, can be used to obtain multiresolution representation, similar to that of the separable DWT (Figure 1(a)), and the highpass directional subbands 1, 2, 3, 5, 6 and 7 can be used to obtain narrower directional selectivity, while maintaining critically sampled property.

The uDFB is constructed using a binary tree with three levels as shown in Figure 2(a). According to the passband supports shown in the tree structure, it can be shown that the filter shapes satisfy the admissibility and permissibility conditions [13] with respect to the decimation matrix $\mathbf{Q}_2$. In the conventional DFB [4], the number of frequency wedges is fixed, and it is difficult to achieve a multiresolution decomposition since there is no specific lowpass subband and/or further divide for finer directions while maintaining the critical sampling. The uDFB, however, can be cascaded with two-channel filter banks with simple passband shape to obtain arbitrary (2^n) number of directional subbands at different resolutions.

One design problem with the binary tree is the effect of frequency 'scrambling' [cite Do], which means that the aliased high frequency component will be next to the lowpass component after decimated. Such a problem occurs in the DFB in [4], and is corrected by using a resampling block in [14]. A similar problem appears in the implementation of the tree-structured DFB in [9, 10]. In order to illustrate this problem, let us consider the top branch in the second level of the tree structure in Fig. 2(a) as in Fig. 3. Using a noble identity [12], it is clear that the equivalent decimation filter is given by:

$$\begin{aligned} H(\mathbf{z}) &= H_0^{(1)}(\mathbf{z}) H_1^{(2)}(\mathbf{z}^{\mathbf{Q}}) \\ H(\omega_0, \omega_1) &= H_0^{(1)}(\omega_0, \omega_1) H_1^{(2)}(\omega_0 - \omega_1, \omega_0 + \omega_1). \quad (1) \end{aligned}$$

If this branch is a highpass filter with one zero at DC (one vanishing moment), it is required that $H_1^{(2)}(0,0) = 0$ since $H_0^{(1)}(0,0)$ is in the transition band of $H_0^{(1)}(\omega_0, \omega_1)$. This implies that $H(\pm\pi, \pm\pi) = 0$, which will severely affect the passband shape of the highpass filter. Furthermore, some high frequency components will leak into the lowpass channel after decomposition.

In order to avoid above problem, the uDFB is proposed to be optimized directly as a uniform eight-channel filter bank as shown in Figure 2(b). This framework has a number of advantages: (i) it exploits the permissibility of passband supports to design filter with good frequency characteristics and regularity requirement (see Section 4) is possible, and (ii) the size of the impulse response can be more compact than that from the binary tree structure since the entire eight channel filters can be implemented directly.

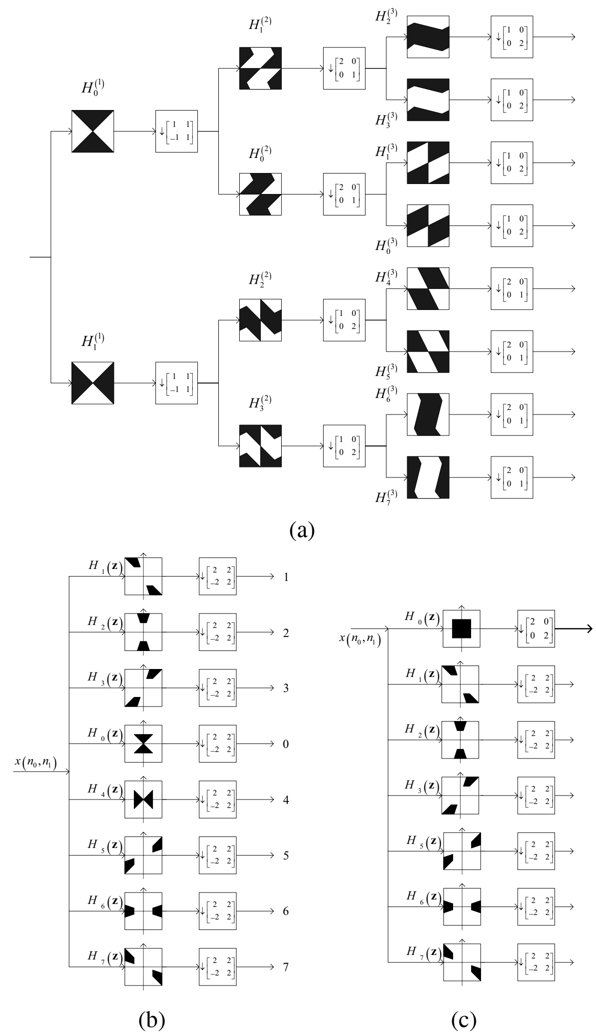


Figure 2: (a) Uniform eight channels DFB and (b) Non-uniform seven channels DFB.

3. MULTIRESOLUTION DIRECTIONAL DECOMPOSITION USING NONUNIFORM DFB

A natural extension to a multiresolution image decomposition using a filter bank is to reiterate the same filter bank on the lowpass coefficients. For 1D octave-band multiresolution, the passband support (bandwidth) of the lowpass filter is one-half of the entire frequency space, and hence is one-fourth for the 2D case. The above uDFB has two lowpass subbands (0 and 4), which should be combined to obtain a decimation factor of one-fourth. One simple remedy is to use a two-channel PR filter bank (transmultiplexer) to combine the low pass signals. This approach increases computational complexity of the design and implementation since additional filters are used. Moreover, the size of the overall support of the filter's impulse response will be less compact. A better approach is to use a nonuniform filter bank with one lowpass component with a decimation matrix $\mathbf{D}_2$ of which the decimation factor is one-fourth as shown in Figure 2(c). The other six directional subbands remain the same. Since the uDFB satisfies both admissible and permissible conditions, so does the new nonuniform DFB (nuDFB). The frequency partitioning of the nuDFB is presented in

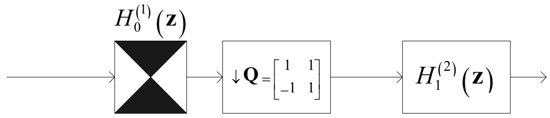


Figure 3: One branch of the tree structure to implement DFB.

Figure 1(d) which is similar to that of the DWT in Figure 1(a) in that they have the same lowpass frequency support $(-\pi/2, \pi/2)^2$. Instead of decomposing the image details into horizontal, vertical and diagonal subbands, the nonuniform DFB uses six directional subbands. This new uniform DFB provides more flexibility in the decomposition: (i) in order to obtain a (directional) multiresolution of an image, one can reiterate the entire filter bank on the lowpass channel (coarse approximation), and (ii) in order to double the directional resolution, one can cascade a two-channel filter bank with fan-shaped passband at the output of each of the highpass channels.

4. A DESIGN METHOD OF THE UDFB AND NUDFB

In this section we present a simple design method adapted from [11]. The method allows one to design a near PR orthogonal or biorthogonal filter bank only by solving linear equations iteratively. For convenient, let $\mathcal{S} = \{1, 2, 3, 5, 6, 7\}$ be the set of the indices of the directional subbands. The design problem can be formulated as an optimization problem with an objective function:

$$\begin{aligned}\Phi &= \sum_{j=0}^{M-1}\left(\sum_{\mathbf{n}} f_j(\mathbf{n})h_j(\mathbf{n}) - 1\right)^2 \\ &\quad + \sum_{j=0}^{M-1}\sum_{\mathbf{m}\neq\mathbf{0}}\left(\sum_{\mathbf{n}} f_j(\mathbf{n}-\mathbf{m}\mathbf{Q_2})h_j(\mathbf{n})\right)^2 \\ &\quad + \sum_{j,k=0, j\neq k}^{M-1}\sum_{\mathbf{m}}\left(\sum_{\mathbf{n}} f_j(\mathbf{n}-\mathbf{m}\mathbf{Q_2})h_k(\mathbf{n})\right)^2, \text{ or} \end{aligned} \quad (2)$$

$$\begin{aligned}\Phi &= \sum_{j\in\mathcal{S}}\left(\sum_{\mathbf{n}} f_j(\mathbf{n})h_j(\mathbf{n}) - 1\right)^2 \\ &\quad + \sum_{j\in\mathcal{S}}\sum_{\mathbf{m}\neq\mathbf{0}}\left(\sum_{\mathbf{n}} f_j(\mathbf{n}-\mathbf{m}\mathbf{Q_2})h_j(\mathbf{n})\right)^2 \\ &\quad + \sum_{j,k\in\mathcal{S}, j\neq k}\sum_{\mathbf{m}}\left(\sum_{\mathbf{n}} f_j(\mathbf{n}-\mathbf{m}\mathbf{Q_2})h_k(\mathbf{n})\right)^2 \\ &\quad + \sum_{j\in\mathcal{S}}\sum_{\mathbf{m}}\left(\sum_{\mathbf{n}} f_j(\mathbf{n}-\mathbf{m}\mathbf{D_2})h_0(\mathbf{n})\right)^2, \end{aligned} \quad (3)$$

where (2) and (3) are, respectively, for the cases of uDFB and nuDFB. The objective function mainly consists of two parts constituting the PR property. The first term corresponds to the 'no-distortion' condition while the rest correspond to the 'aliasing-cancellation' conditions [12]. It is clear from (2) and (3) that when h_j (f_j) are fixed, Phi is a quadratic function of f_j (h_j), which can be minimized by solving a linear equation

$$\frac{\partial\Phi}{\partial f_j} = 0 \quad \left(\frac{\partial\Phi}{\partial h_j} = 0\right). \quad (4)$$

In addition, for orthogonal filter banks, regularity of degree R, which is the number of zeros at DC of the highpass filters (vanishing moments), can be imposed with the following linear constraint:

$$\begin{aligned}&\frac{\partial^{r_1+r_2} f_j(z_1,z_2)}{\partial {z_1}^{r_1}\partial {z_2}^{r_2}}\Big|_{z_1,z_2=0} = 0 \\ &\left(\frac{\partial^{r_1+r_2} h_j(z_1,z_2)}{\partial {z_1}^{r_1}\partial {z_2}^{r_2}}\Big|_{z_1,z_2=0} = 0\right). \end{aligned} \quad (5)$$

where $r_1,\ r_2$ be non-negative integers and $r_1+r_2 < R$. The above conditions in (5) serve as extra constraints in the optimization, and can be simultaneously solved with (4) using Lagrange multiplier. The design procedure can be carried out by iteratively solving (5) and (4) while h_j (f_j) are fixed. The obtained h_j (f_j) are then fixed and used to optimize for f_j (h_j).

During the iteration, orthogonality of the filter bank can be obtained by imposing that each synthesis filter is the time reverse version of the corresponding analysis filter, i.e. $f_j(\mathbf{n}) = h_j(-\mathbf{n})$. In order to converge to an orthogonal solution, the averages of the analysis and synthesis filters is used to update the calculated filter coefficients, i.e.

$$\begin{aligned}&f_j^n(\mathbf{n}) = \frac{h_j^{n-1}(\mathbf{n}) + f_j^{n-1}(\mathbf{n})}{2} \\ &\left(h_j^n(\mathbf{n}) = \frac{f_j^{n-1}(\mathbf{n}) + h_j^{n-1}(\mathbf{n})}{2}\right). \end{aligned} \quad (6)$$

The design procedure can be summarized as follows:

1. Initialize the analysis filters h_j^0 with appropriate passband supports.
2. Solve for the synthesis filters $\hat{f}_j^0$ by solving the above linear equations (5) and (4). Update f_j^0 by setting $f_j^0(\mathbf{n}) = \frac{h_j^0(-\mathbf{n})+\hat{f}_j^0(\mathbf{n})}{2}$.
3. Fix f_j^0 and solve for $\hat{h}_j^1$ using the linear equations (5) and (4). Update h_j^1 by setting $h_j^1(\mathbf{n}) = \frac{\hat{h}_j^1(\mathbf{n})+f_j^0(-\mathbf{n})}{2}$.
4. Repeat the same above process for f_j^i and h_j^i until the cost function is lower than a defined level.

Remarks: Although the design method is simple and straightforward, it has some limitations. First, the method is not robust and the convergent rate is strongly dependent on the initialization filters. Second, the cost functions do not impose any frequency response characteristics (except from the regularity conditions) and hence the resulting filters will also be dependent on the initialization. Third, the algorithm is not guaranteed to converge to a PR solution at the optimality, i.e. $\Phi = 0$.

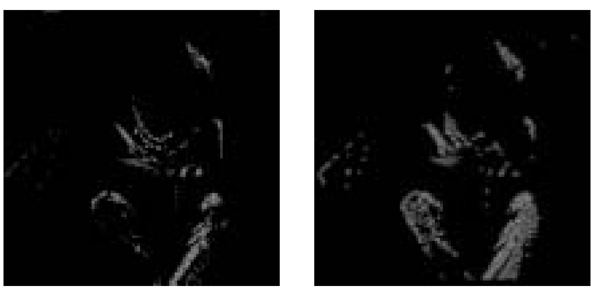
5. NUMERICAL EXPERIMENT

In this experiment, approximation performances of the DWT and the nuDFB are compared. The two filter banks are both orthogonal and have identical lowpass filter h_0, which is the Daubechies filter

Table 1: SNR of the reconstruction images in the high frequency subbands after a number of coefficients have been retained for the cases of DWT and nuDFB.

# coefficients	DWT	nuDFB
32	0.058 dB	0.077 dB
64	0.104 dB	0.132 dB
128	0.177 dB	0.223 dB
256	0.314 dB	0.368 dB
512	0.551 dB	0.612 dB
1024	0.947 dB	0.988 dB

of length 4. Hence the total energy in the highpass subbands is the same for both cases. An image is decomposed at one level using the filter banks and, for each decomposition, a number of subband coefficients are retained. These coefficients are determined by their magnitudes in order to minimize the reconstruction error. Table 1 tabulates signal-to-noise ratio (SNR), which compares the reconstruction error after a number of highpass coefficients are retained to the reconstruction when all highpass coefficients are used. Note that the lowpass coefficients are not used in the calculation of SNR. It is evident that, with a same small number of coefficients, the nuDFB coefficients provide a better approximation of the image than the DWT coefficients. Figs. 4 (a) and (b) show the reconstructions of edges of the 'Barbara' image when 1024 highpass coefficients are retained. It is clear that the DWT represents edges in the image by many points appearing continuously, while the nuDFB approximates by very few coefficients at the subbands perpendicular to the directions of the edges.



(a) (b)

Figure 4: Details of the Barbara image reconstructed from 1024 coefficients at high frequency subbands using (a) DWT and (b) nuDFB.

6. CONCLUSION

This paper presents a maximally decimated, multiresolution and multidirectional filter bank. It is shown that when the 2D uniform non-separable filter bank is constructed using a binary tree, additional constraints on the passband supports of the filters must be satisfied. Hence by directly generating all the filters, the filter bank can be obtained with better characteristics such as regularity and vanishing moments. The proposed filter bank is further extended to a non-uniform subbands by combining the two lowpass filters. This extension is suitable for multiresolution image decomposition, and can be used to obtain a pyramidal directional representation. A simple design method is discussed where orthogonality and regularity can be easily imposed. A numerical simulation is presented and shows that the proposed DFB yields better performance in signal compression than the traditional DWT.

7. REFERENCES

[1] Gilbert Strang and Truong Q. Nguyen. *Wavelets and Filter Banks*. Wellesley Cambridge, 1996.

[2] Martin Vetterli. Wavelets, approximation and compression. *IEEE Signal Processing Magazine*, 18(5):59–73, 2001.

[3] Minh N. Do. *Directional Multiresolution Image Representations*. PhD thesis, Swiss Federal Institute of Technology, Lausanne, November 2001.

[4] Roberto H. Bamberger and Mark J. T. Smith. A filter bank for the directional decomposition of images: theory and design. *IEEE Transactions on Signal Processing*, 40(7):882 –893, April 1992.

[5] J.G.Rosiles and Mark J.T. Smith. Texture classification with a biorthogonal directional filter bank. In *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pages 1549 –1552, 2001.

[6] Sang-Il Park, Mark J.T. Smith, and Jun Jae Lee. Fingerprint enhancement based on the directional filter bank park. In *Proceedings. 2000 International Conference on Image Processing*, pages 793–796, Nagoya, Japan, January 2000.

[7] Minh N. Do and Martin Vetterli. Contourlets. In J. Stoeckler and G. V. Welland, editors, *Beyond Wavelets*, chapter to appear. Academic Press, Amsterdam, The Netherlands, 2003.

[8] Emmanuel J. Candès and David L. Donoho. Curvelets - a surprisingly effective nonadaptive representation for objects with edges. In L. L. Schumaker et al., editor, *Curves and Surfaces*. Vanderbilt University Press, Nashville, TN, 1999.

[9] Yue Lu and Minh N. Do. Crisp-contourlets: a critically sampled directional multiresolution image representation. In *Proc. of SPIE conference on Wavelet Applications in Signal and Image Processing X*, San Diego, USA, August 2003.

[10] Truong Trong Nguyen and Soontorn Oraintara. A directional decomposition: theory, design and implementation. *Submitted to the International Symposium on Circuits and Systems,ISCAS 2004*, 2004.

[11] M.Ikehara and Truong Q. Nguyen. On 2d perfect reconstruction linear phase filter banks. In *Thirty-First Asilomar Conference on Signals, Systems and Computers*, pages 721–725, 1997.

[12] P.P. Vaidyanathan. *Multirate Systems and Filter Banks*. Prentice-Hall,Englewood Cliffs, NJ, 1993.

[13] Soontorn Oraintara and Truong Q. Nguyen. Multidimensional 2-channel filter banks. *Thirty-first Asilomar Conference on Signals, Systems and Computers*, pages 1269–1273, 1997.

[14] Sang Il Park, Mark J.T. Smith, and Russel M. Mersereau. A new directional filter bank for image analysis and classification. In *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pages 1417 –1420, March 2000.