SHAPE RECONSTRUCTION FROM UNORGANIZED POINTS WITH A DATA-DRIVEN LEVEL SET METHOD

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ABSTRACT

We propose a new method for shape reconstruction from noisy and unorganized point data. We represent a shape through its signed distance function and formulate shape reconstruction as a constrained energy minimization problem directly based on the observed point set. The associated energy function includes both the likelihood of the observed data points and a smoothness prior on the reconstructed shape. To solve this optimization problem, an efficient data-driven level set method is developed. Our method is robust to local minima, clutter, and noise. It is also applicable to situations where the data are sparse. The topological nature of the underlying shape is handled automatically through the level set formalism.

1. INTRODUCTION

Reconstruction of a shape from a set of noisy and unorganized point samples of its surface is a challenging inverse problem arising in various areas. Point observations of a surface can arise from noisy range images, slices of segmented medical images, and computer vision techniques, such as correlation-based range estimation, voxel carving, etc. In this paper, we propose a new method for the reconstruction of a smooth underlying shape given such point measurements based on a data-driven level set method.

For the problem of shape reconstruction from point clouds, solutions based on implicit shape representations and level set methods have been proposed in the past, because of their ability to handle topological uncertainties in the scene. Hoppe et al. proposed an algorithm for constructing a signed distance function from unorganized points in [1]. Their method can resolve the topology automatically, but is not robust to noise. To overcome this difficulty, Taubin[2]

represents the shape implicitly as the level set of a parameterized polynomial and then fits this function to the data set by minimizing the average distance between the data and the implicit surface. But the order of the polynomial has to be selected by trial and error, which limits the ability of this method to automatically represent complicated shapes. Instead of using a global polynomial function, Bajaj et al. proposed a method using patches of polynomials to fit a complex shape to a data set in [3]. They used the α-shape technique to construct a signed distance function of the data points based on the duality between the Voronoi diagram and the Delaunay triangulation, but the parameter α has to be selected interactively.

Recently, partial differential equation (PDE) based approaches to solve level-set problems were introduced by Osher and Sethian [4] as a robust technique to track dynamic interfaces. Surface fitting methods using this technique start from an initial surface and deform the surface toward the data set by evolving a PDE until some criterion is met. In [5], Whitaker proposed a PDE-based level set approach for 3D shape reconstruction from range data, but the method is designed only for dense range data and independent Gaussian noise. More recently, in [6] Zhao et al. developed a method which reconstructs a shape as the minimal surface corresponding to a distance transform derived from the data set. But this approach is designed for use with clean data, thus data uncertainty is not taken into account.

In this paper we propose a data-driven, PDE-based, level-set method to reconstruct a shape surface from observations of a set of noisy and unorganized points. We represent the shape as the zero level set of a signed distance function and formulate the problem of shape reconstruction as a constrained energy minimization problem. Our energy function explicitly accounts for noise in the data and the smoothness of the shape is controllable through the inclusion of a prior shape term. A direct, data-driven level set method is then derived to efficiently solve this optimization problem. The resulting method is applicable to sparse data sets and is robust to local minima and clutter in contrast to previous

methods.

The rest of the paper is organized as follows. In Section 2, we formulate the constrained energy minimization problem based on the signed distance function representation. A data-driven level set method is then derived in Section 3 to solve this optimization problem. Experimental results on 3D shape reconstruction are presented in Section 4. Finally, conclusions are made in Section 5.

2. PROBLEM FORMULATION

Let the set of surface/shape data points in Euclidean space be denoted as $\mathbf{X} = \{\underline{x}_1, \cdots, \underline{x}_N\}$. The shape to be reconstructed is represented as the zero level set of a signed distance function ϕ. Using the property of a signed distance function, the signed distance from each data point to the shape, denoted as $g(\underline{x}_i, \phi)$, can be expressed as:

$$g(\underline{x}_i, \phi) = \phi(\underline{x}_i) = \int \phi(\underline{x})\delta(\underline{x} - \underline{x}_i)d\underline{x}. \tag{1}$$

where $\delta(\underline{x} - \underline{x}_i)$ is the delta function. The set of distances between the data points $\mathbf{X}$ and the shape ϕ is collected as a vector and denoted by $\mathbf{g}(\mathbf{X}, \phi)$. The likelihood of the data $\mathbf{X}$ given the shape ϕ is denoted as $p(\mathbf{X} \mid \phi)$ and we assume it to be a function of $\mathbf{g}(\mathbf{X}, \phi)$, so that the likelihood is $p(\mathbf{g}(\mathbf{X}, \phi) \mid \phi)$.

We formulate the shape reconstruction problem as the following constrained energy minimization problem:

$$\phi = \arg \min_{\substack{\phi \\ |\nabla\phi|=1}} E(\phi) \tag{2}$$

where the energy function is defined as follows

$$E(\phi) = \underbrace{-\log p(\mathbf{g}(\mathbf{X}, \phi) \mid \phi)}_{E_d} + \underbrace{\mu \int \delta(\phi)|\nabla\phi|d\underline{x}}_{E_s} \tag{3}$$

This energy function E is composed of two parts: a data fidelity term E_d and a smoothing term E_s. The constraint $|\nabla\phi| = 1$ is necessary to ensure that ϕ is a signed distance function.

For the data fidelity term E_d we use the negative log-likelihood of the data, so variability in the point data is directly incorporated into the formulation. This term enables our method to robustly reconstruct shapes from noisy and unorganized point data sets, unlike current methods based on level set techniques. To date in our experiments, we have assumed that $p(\mathbf{g}(\mathbf{X}, \phi) \mid \phi)$ is a Gaussian distribution:

$$p(\mathbf{g}(\mathbf{X}, \phi) \mid \phi) \propto e^{-(\mathbf{g}-\mathbf{u})^T \mathbf{W}^{-1}(\mathbf{g}-\mathbf{u})/2} \tag{4}$$

where $\mathbf{u}$ is the mean distance and $\mathbf{W}$ is the covariance matrix. This model corresponds to assuming that the closest distance of a point to the shape is a Gaussian random variable.

For the smoothing term E_s we use the length of the curve in 2D and area of the surface in 3D. By including E_s, we enforce smoothness of the resulting shape [7]. By adjusting μ, we can control the degree of smoothness of the reconstructed shape.

3. LEVEL-SET-BASED SOLUTION METHOD

Solution of the constrained optimization problem in (2) is challenging because of the high dimensionality, presence of the gradient constraint, and possible numerical instabilities. In this section, we propose a method to evolve the zero level set of ϕ in the gradient descent direction of the energy function E while approximately enforcing the unit magnitude gradient constraint required of a signed distance function. At each iteration, first we find the gradient descent direction of ϕ with respect to the data fidelity term E_d and the smoothing term E_s, and then they are extended to the whole domain using the technique in [8] to construct an evolution speed such that the singed distance property of ϕ is maintained approximately.

The first variation of E_d with respect to ϕ is obtained as:

$$\frac{dE_d}{d\phi} = \frac{dE_d}{d\mathbf{g}}^T \frac{d\mathbf{g}}{d\phi} \tag{5}$$

From Eq.(3), we have

$$\frac{dE_d}{d\mathbf{g}} = -\frac{1}{p}\frac{dp}{d\mathbf{g}}$$

Now

$$\frac{d\mathbf{g}}{d\phi} = [\delta(\underline{x} - \underline{x}_1) \cdots \delta(\underline{x} - \underline{x}_N)]^T$$

since $dg(\underline{x}_i, \phi)/d\phi = \delta(\underline{x} - \underline{x}_i)$. We approximate $\delta(x)$ in a manner similar to [9]:

$$\delta_\alpha(\underline{x}) = \begin{cases} 0, & |\underline{x}| > \alpha \\ \frac{1}{2\alpha}\left[1 + \cos\left(\frac{\pi|\underline{x}|}{\alpha}\right)\right], & |\underline{x}| \leq \alpha \end{cases} \tag{6}$$

where $\alpha \geq 0$ is a real valued threshold. In practice, α is chosen to be on the scale of the object to allow robustness to initialization. Finally, for points $\underline{x}_0$ at the zero level set of the surface ϕ we set the component of the evolution speed due to the data energy E_d to be proportional to the negative of the energy gradient based on the approximated delta function:

$$\left[\frac{d\phi}{dt}\bigg|_{\underline{x}=\underline{x}_0}\right]_d = -\sum_{i=1}^N \frac{dE_d}{dg(\underline{x}_i, \phi)}\delta_\alpha(\underline{x}_0 - \underline{x}_i) \tag{7}$$

The use of an approximated delta function allows each data point to contribute directly to the evolution speed of the zero

level set even when the data point is far away from the zero level set. This data-driven property makes our method robust to local minima. Another nice property of our method is that its robustness to clutter increases with the amount of true data since the contribution from the true data points will gradually dominate. More generally, the approximated delta function serves as an interpolation function on the data. By varying the threshold α, we will be able to handle data sets with different sparsity.

To construct the component of the evolution speed for the function ϕ due to the data term over the whole domain, denoted as F_d, so that ϕ will maintain the property of a signed distance function (i.e. incorporate the magnitude 1 gradient constraint), we follow [8] and solve the PDE:

$$\nabla \phi \cdot \nabla F_d = 0 \qquad (8)$$

with the boundary condition

$$F_d(\underline{x}_0) = \left[\left. \frac{d\phi}{dt} \right|_{\underline{x}=\underline{x}_0} \right]_d$$

obtained from Eq.(7). This PDE is derived from the condition $d|\nabla\phi|^2/dt = 0$, so that $|\nabla\phi| = 1$ will remain true as time evolves if we start with a valid signed distance function. Fast marching methods [10] were originally used for the efficient solution of this PDE in [8]. In our experiments, we solve this PDE based a fast sweeping algorithm[11], which has a lower computation cost than the fast marching method.

For the component of the evolution speed due to the smoothing term E_s we use the negative of the gradient of this term with respect to ϕ. Using the results in [9], this component of the evolution speed is given by:

$$\left[\frac{d\phi}{dt} \right]_s = \left(\nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} \right) |\nabla\phi| \qquad (9)$$

where $\left(\nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} \right)$ is the curvature of the level set curves of the function ϕ.

Combining the evolution speed contributions due to E_d and E_s, the final evolution equation for minimizing the energy E is given by:

$$\frac{d\phi}{dt} = F_d|\nabla\phi| + \mu \left(\nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} \right) |\nabla\phi| \qquad (10)$$

Due to the way F_d was constructed, the first term will ensure ϕ satisfies the unit gradient magnitude constraint, so that ϕ will remain a signed distance function. The second term serves to smooth this function, and so gradually evolves ϕ away from the unit magnitude gradient constraint over time. As a result we periodically need to reinitialize ϕ in our implementation.

4. EXPERIMENTAL RESULTS

We present some preliminary experimental results of our method for 3D shape reconstruction in this section. In the examples shown below, we choose noise model mean **u** as zero and the noise model covariance **W** as proportional to an identity matrix.

In the first example, we reconstruct an oil pump from a set of clean sampled point data from the website http://ftp.research.microsoft.com/users/hhoppe/data/thesis. The 3D data set, which consists of 30937 points, is shown in Fig.1(a). Because our method is robust to local minima, using an initial zero level set close to the true shape is helpful, but not essential. The initial zero level set used in this example, shown in Fig.1(b), is a box containing all of the observed points. We reconstructed the final shape, shown in Fig.1(c), on a grid of size $80 \times 95 \times 110$. Our reconstruction is smooth and does an excellent job reconstructing the shape.

In the second example, we demonstrate our method's ability to handle noisy and sparse data with unknown topology, and its robustness to clutter. The true shape to be reconstructed is composed of two interlinked tori. The data set, shown in Fig.2(a), is formed by randomly choosing 1200 points from the surface of the true shape, then adding independent Gaussian noise to each coordinate and adding an additional set of clutters points unrelated to the shape. The resulting data set is very sparse and noisy. The size of the grid over which we reconstruct the shape is $40 \times 40 \times 60$. The initial zero level set used is again a box containing all the data points. The final reconstructed shape is shown in Fig.2(b). The two smooth tori are successfully reconstructed without the need for user specification of shape characteristics, control points, etc.

5. CONCLUSION

In this paper, we presented a constrained energy minimization framework for shape reconstruction from noisy and unorganized point data sets. Variability of the data is directly included in the formulation, as is prior knowledge of shape smoothness. An efficient data-driven level set method is then developed to minimize this energy. Topology of the reconstructed surface is derived automatically. Promising 3D examples have been presented to demonstrate the method.

6. REFERENCES

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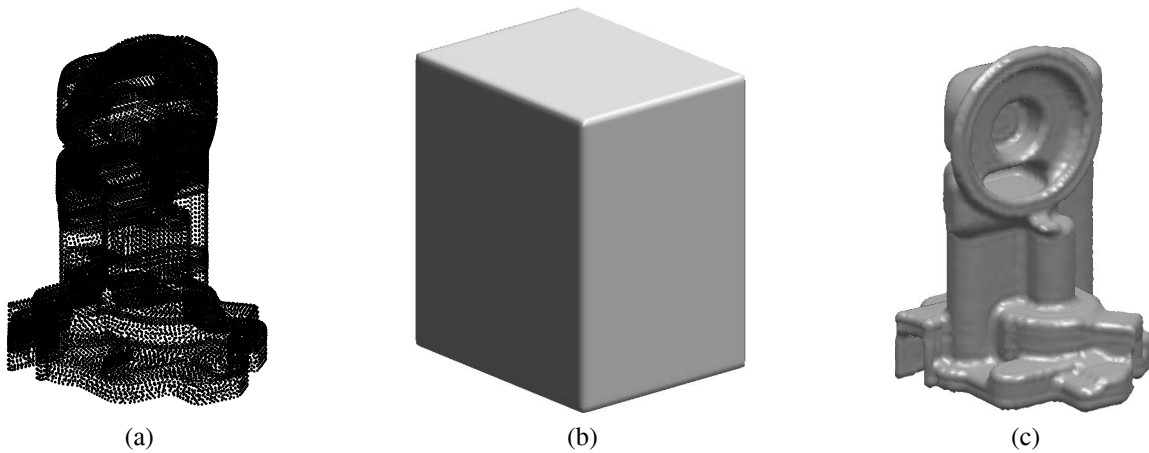[2] G. Taubin, "Estimation of planar curves, surfaces, and nonplanar space curves defined by implicit equations

(a)   (b)   (c)

Fig. 1. Reconstruction of an oil pump from unorganized data points. (a) The 3D data set. (b) The initial zero level set. (c) The reconstructed 3D shape.
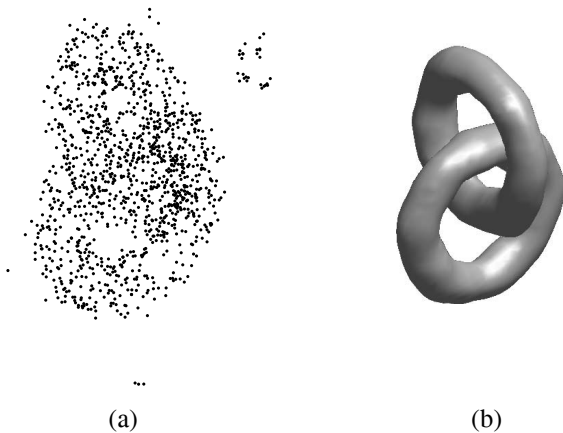

(a)   (b)

Fig. 2. Reconstruction of two tori from noisy, sparse and unorganized data points with clutter. (a) The 3D data set. (b) The reconstructed 3D shape.

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