MMSE JOINT DETECTION FOR AN ASYNCHRONOUS SPREAD-SPECTRUM SYSTEM BASED ON RANDOM PERMUTATIONS

Martial COULON and Daniel ROVIRAS

ENSEEIHT-IRIT/TéSA, 2 Rue Camichel, 31071 Toulouse cedex 7, France

martial.coulon@tesa.prd.fr,daniel.roviras@tesa.prd.fr

ABSTRACT

This paper studies the joint detection of bits for a new spread-spectrum system based on random permutations. The proposed technique can be regarded as a special case of discrete Linear Periodic Time Varying filters. As any spread-spectrum technique, this method generates multi-access interference, which must be mitigated in order to retrieve the users'bits with acceptable bit-error-rate. Thus, different detection strategies must be analysed regarding their performance, their complexity and the available a priori information. In this paper, an asynchronous model is considered, and a linear minimum mean-square error (MMSE) detector fitted to this model is investigated. Theoretical and simulated results are provided, along with a comparison with the corresponding detector for CDMA systems.

1. INTRODUCTION

Spreading techniques are one of the prominent multiple access schemes for 3G mobile cellular systems. DS-CDMA is a classical technique for spreading a data signal. Nevertheless, other multiple access schemes have equivalent spreading capabilities. Among them, Linear Periodic Time Varying (LPTV) filters can be pointed out. The transformation given by an LPTV can be used for multiple access purposes [6]. A special case of LPTV filters called Periodic Clock Changes (PCC) are useful for multiple access because of their spreading properties [5], [6]. In this paper is investigated a sub-class of discrete PCC called permutations. In a permutation, a block of input samples $(x(n))_n$ is transformed into a block of samples of same length with permuted samples: this operation can be regarded as a particular discrete PCC [5]. Random permutation-based Multiple Access (RPMA) together with the matched filter detector has been investigated in previous works [5], [6]. Concerning CDMA systems, many detectors have been proposed in the literature (see for instance [2], [3], [7] and references therein). The purpose of this paper is to study a new detector based on a linear Minimun Mean Square Error (MMSE) approach. Section 2 presents the asynchronous RPMA signal model in its continuous and its discrete forms. Section 3 presents the MMSE detection scheme, where a theoretical analysis is conducted. Section 4 is devoted to simulation results. The near-far effect is investigated and a comparison is carried out between the RPMA system and a DS-CDMA system based on Gold codes.

2. PROBLEM FORMULATION

2.1. Modelling of the asynchronous signal

Denote K as the number of users. The k-th user sends a stream of L bits $\underline{b}_k \triangleq [b_k(1), \ldots, b_k(L)]^T$, where $b_k(l) \in \{-1; +1\}$, $l \in \{1, \ldots, L\}$. The bit duration is equal to T and the waveform pattern with duration T is denoted $m(t)$. It is assumed that an antipodal signaling waveform is used (for instance, an NRZ or Biphase line code). Sampling the waveform pattern $m(t)$ with N points gives $\underline{m} \triangleq [m_1, \ldots, m_N]^T$, where N represents the spreading factor [1]. Without loss of generality, it is imposed that $\|\underline{m}\|^2 = \underline{m}^T \underline{m} = 1$. The coded bits of user k, when sampled with N points per symbol, are given by $[b_k(1)\underline{m}^T, \ldots, b_k(L)\underline{m}^T]^T = M^T \underline{b}_k$, where $M = \underline{m}^T \otimes \mathbf{I}_L$ is the $L \times (NL)$ matrix defined by

$$M = \begin{bmatrix} \underline{m}^T & 0 & \cdots & \\ 0 & \ddots & \ddots & \\ & \cdots & & \underline{m}^T \end{bmatrix}$$

($\otimes$ denotes the Kronecker product, and $\mathbf{I}_n$ is the identity matrix of order n). A different permutation is assigned to each user. Each permutation has a length NL, which corresponds to a random permutation on the set $\{1, \ldots, NL\}$. The permutation is applied on the block of samples $M^T \underline{b}_k$. Let P_k denote the $(NL) \times (NL)$ permutation matrix associated to user k. The transmit scrambled sequence for user k is $P_k M^T \underline{b}_k$. Since the pattern $m(t)$ is binary, the continuous-time signal transmitted by the k-th user is given by:

$$\sum_{j=1}^{NL} \left(P_k M^T \underline{b}_k\right)_j r\left(t - \frac{j-1}{N}T - \tau_k\right)$$

where $(v)_j$ denotes the jth component of any vector v, $r(t)$ is the indicator function over $[0; T/N]$, and $\tau_k \in [0; T]$ is the relative offset of user k. Here, we will consider an asynchronous model, which means that offsets τ_k's can be considered as independent random variables uniformly distributed over the interval $[0, T]$. This model is of particular interest for the uplink channel, where the system's users generally transmit their data in an asynchronous way. It is assumed in this paper that the transmission channel is a flat fading channel. Consequently, the continuous time received signal can be expressed as:

$$y(t) = \sum_{k=1}^{K} A_k \sum_{j=1}^{NL} \left(P_k M^T \underline{b}_k\right)_j r\left(t - \frac{j-1}{N}T - \tau_k\right) + n(t)$$

where $A_k > 0$ is the received amplitude for the k-th user's signal and $n(t)$ is an additive white Gaussian noise (AWGN), independent of the transmitted signals.

2.2. The discrete asynchronous model signal

The first step of the detection consists of passing the continuous received signal $y(t)$ through a filter bank, where filters are matched to the waveforms of all users and all bits. More precisely, one defines the following variables:

$$y_k(j) \triangleq \int y(t) r\left(t-(j-1)\frac{T}{N}-\tau_k\right) dt\,, \quad \begin{array}{l} k=1,\ldots,K, \\ j=1,\ldots,NL \end{array}$$

Define then vectors $\underline{y}_k \triangleq [y_k(1), y_k(2), \ldots, y_k(NL)]^T$, $k = 1,\ldots,K$, and $\mathbf{y} \triangleq \left[\underline{y}_1^T, \underline{y}_2^T, \ldots, \underline{y}_K^T\right]^T$, which has length NKL. Then, $\mathbf{y}$ is a sufficient statistic for the bits $(b_k(j))_{1\leq j\leq L; 1\leq k\leq K}$. Indeed, given the fact that the data are independent and equiprobable, and that the noise is AWGN, the optimal detector consists of minimizing the error function $\int (y(t) - \widetilde{y}(t))^2\, dt$ with respect to $(b_k(j))_{1\leq j\leq L; 1\leq k\leq K}$, where $\widetilde{y}(t)$ denotes the uncorrupted version of $y(t)$. Now, it can be shown that this error function only depends on the received signal $y(t)$ through the variables $(y_k(j))_{k=1,\ldots,K;\, j=1,\ldots,NL}$. Consequently, the proposed detector is based on the statistic $\mathbf{y}$.

Moreover, the statistic $\mathbf{y}$ can be expressed in matrix form as follows. Denote $\tau_k \triangleq n_k\frac{T}{N} + \varepsilon_k$, with $n_k \in \{0,\ldots,N-1\}$ and $\varepsilon_k \in [0; T/N[$. Suppose without loss of generality that the users are numbered in the increasing order of their offset, i.e. $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_K$. Now, define $\beta_{kl}^- \triangleq \frac{N}{T}|\varepsilon_l - \varepsilon_k| 1_{\varepsilon_l > \varepsilon_k}$, $\beta_{kl}^+ \triangleq \frac{N}{T}|\varepsilon_l - \varepsilon_k| 1_{\varepsilon_l > \varepsilon_k}$ (where $1_{a>b} = 1$ if $a > b$ and $1_{a>b} = 0$ otherwise), and $\alpha_{kl} \triangleq 1 - \frac{N}{T}|\varepsilon_l - \varepsilon_k|$. Then, define the $NL \times NL$ matrix Λ_{kl} such that the j-th row of Λ_{kl} is $[0,\ldots,0,\beta_{kl}^-, \alpha_{kl}, \beta_{kl}^+, 0,\ldots,0]$, where α_{kl} occurs at the $(j + n_k - n_l)$*-th* column (in particular, $\Lambda_{kk} = I_{NL}$). Then, it can be shown that $\mathbf{y}$ can be written as:

$$\mathbf{y} = \Lambda \Pi A \underline{b} + \underline{n}$$

where:

- $A = diag([A_1,\ldots,A_K]^T \otimes \mathbf{1}_{NL})$ is a diagonal matrix containing the receivers'amplitudes ($\mathbf{1}_{NL}$ is the NL-vector with all elements equal to 1);

- Π is a $NKL \times KL$ block-diagonal matrix, whose k-th diagonal block is the $NL \times L$ matrix $P_k M^T$;

- Λ is a $NKL \times NKL$ block matrix, whose block $(k,l)_{1\leq k\leq K;\, 1\leq l\leq K}$ is the $NL \times NL$ matrix $\Lambda_{k,l}$ (note in particular that Λ is symmetric);

- $\underline{n} \triangleq [n_1, n_2, \ldots, n_{NKL}]^T$ is a zero-mean Gaussian vector with covariance matrix $\sigma^2\Lambda$.

3. THE MMSE DETECTOR

3.1. Inverse Permutations

The first step of the detection consists of: $i)$ performing the inverse permutation; $ii)$ passing the data through a filter matched to the waveform $m(t)$. This yields to the definition of the L-length vector variables:

$$\begin{aligned} \underline{z}_k &\triangleq M P_k^T \underline{y}_k \\ &= A_k \underline{b}_k + \sum_{l\neq k} A_l R_{kl} \underline{b}_l + M P_k^T \underline{n}_k \end{aligned} \quad (1)$$

where $R_{kl} \triangleq \left(P_k M^T\right)^T \Lambda_{kl} \left(P_l M^T\right)$ (note that $R_{kk} = I_{NL}$ and $R_{kl}^T = R_{lk}$). The elements of R_{kl} can be regarded as the correlations between the bits of the kth and the lth users. Denote R as the $KL \times KL$ block matrix whose block (k,l) is R_{kl}, which can also be written as $R = \Pi^T \Lambda \Pi$ (hence, R is symmetric); then, R represents a measure of the Multiple-Access Interference (MAI). Denoting $\mathbf{z} \triangleq \left[\underline{z}_1^T, \ldots, \underline{z}_K^T\right]^T$, one obtains:

$$\mathbf{z} = \Pi^T \mathbf{y} = \Pi^T \Lambda \Pi A \underline{b} + \Pi \underline{n} = RAb + \widetilde{\underline{n}} \quad (2)$$

where $\widetilde{\underline{n}}$ is a zero-mean Gaussian vector with covariance matrix $\sigma^2 R$. If the MAI was zero, the optimal decision rule for the j-th bit of the k-th user would simply be given by $sign\,(z_k(j))$ (which is the so-called matched-filter detection). In the general case where the MAI is non-zero, this detection scheme is no more optimal since it exhibits an important near-far phenomenon ; consequently, different detection strategies must be developed. For instance, the decorrelating detector has been studied in [1]. In this paper, a linear MMSE detector is proposed. The objective of this detector is to retrieve the MAI from the discrete data $\mathbf{y}$, without increasing dramatically the additive Gaussian noise term.

3.2. Optimization Problem

It is assumed in this section that the user amplitudes $(A_k)_{k=1,\ldots,K}$ are known by the receiver. Usually, the MMSE estimator of bit $b_k(j)$ is the value which minimises $E\left[\left(\widehat{b}_k(j) - b_k(j)\right)^2\right]$ with respect to $\widehat{b}_k(j)$. The estimator is then given by $E\left[b_k(j)\,|\underline{y}\right]$. Now, the problem is that the estimated value obtained for $\widehat{b}_k(j)$ is not in general in the set $\{-1;+1\}$. Instead, an alternative approach consists of deriving the linear MMSE estimator [7], i.e. the estimator $\widehat{b}_k(j)$ is defined by

$$\widehat{b}_k(j) = sign\left(\underline{c}_{k,j}{}^T \underline{y}\right)$$

where $\underline{c}_{k,j}$ is the (NKL)-length vector wich minimizes the mean-square error

$$\varepsilon_k(j) \triangleq E\left[\left(b_k(j) - \underline{c}^T \mathbf{y}\right)^2\right] \quad (3)$$

with respect to $\underline{c} \in \mathbb{R}^{NKL}$. Moreover, it can easily be seen, that Π^T (or equivalently Π) is a full-rank matrix; consequently, Π^T is a regular matrix, and it exists a unique $(NKL) \times (KL)$ matrix $\widetilde{\Pi}$ such that $\widetilde{\Pi}\Pi^T = \mathbf{I}_{NLK}$. Now, since $\mathbf{z} = \Pi^T \mathbf{y}$, we have $\mathbf{y} = \widetilde{\Pi}\mathbf{z}$, and $E\left[\left(b_k(j) - \underline{c}^T\mathbf{y}\right)^2\right] = E\left[\left(b_k(j) - \underline{c}^T\widetilde{\Pi}\mathbf{z}\right)^2\right]$. Consequently, the optimization problem (3) is equivalent to the minimization of

$$E\left[\left(b_k(j) - \widetilde{\underline{c}}^T \mathbf{z}\right)^2\right] \quad (4)$$

with respect to $\widetilde{\underline{c}} \in \mathbb{R}^{KL}$. Since this minimization has to be performed for all $(b_k(j))_{1\leq k\leq K; 1\leq j\leq L}$, the problem can be written in a matrix form as

$$\min_{\widetilde{C}} E\left[\left\|\underline{b} - \widetilde{C}\mathbf{z}\right\|^2\right] \quad (5)$$

where the minimization is with respect to the $(KL) \times (NL)$ matrices. In (5), the matrix norm is defined by $\|A\| \triangleq \left(trAA^T\right)^{1/2}$ where tr is the trace operator. It can then be shown as in [7] that

$$\begin{aligned} Cov\left(\underline{b} - \widetilde{C}\mathbf{z}\right) &= \left(I + \sigma^{-2} ARA\right)^{-1} + \\ &\quad \left(\widetilde{C} - \overline{C}\right)\left(RA^2R + \sigma^2 R\right)\left(\widetilde{C} - \overline{C}\right)^T \end{aligned}$$

where $\overline{C} \triangleq A^{-1}\left(R+\sigma^2 A^{-2}\right)^{-1}$. The solution of (5) is then obtained for $\widetilde{C}=\overline{C}$. Consequently, the decision rule is given by

$$\widehat{\underline{b}} = sign\left(\overline{C}\mathbf{z}\right)$$

i.e.,

$$\widehat{b}_k(j) \triangleq sign\left(\left(\left(R+\sigma^2 A^{-2}\right)^{-1}\mathbf{z}\right)_{(k-1)L+j}\right)$$

It must be pointed out that:

• when the SNR is high, i.e. when $\sigma^2 \longrightarrow 0$, $\left(R+\sigma^2 A^{-2}\right)^{-1}$ can be approximated by R^{-1}, so that $\widehat{b}_k(j) \simeq sign\left(R^{-1}\mathbf{z}\right)_{(k-1)L+j}$. Thus, the MMSE detector behaves in this case as the decorrelating detector (see [1]).

• when the SNR is low, i.e. when $\sigma^2 \longrightarrow +\infty$, the noise term $\sigma^2 A^{-2}$ dominates the multiple-access term R. Therefore, $\widehat{b}_k^{mmse}(j) \simeq sign\left(\left(\left(\sigma^2 A^{-2}\right)^{-1}\mathbf{z}\right)_{(k-1)L+j}\right) = sign(z_k(j)) = \widehat{b}_k(j)$. Hence, one obtains the matched filter-decision.

Thus, the MMSE detector can be seen as a compromise between the matched-filter and the decorrelating detectors.

3.3. Performance

Denoting $\Psi \triangleq \left(R+\sigma^2 A^{-2}\right)^{-1}$, it is straightforward to show, using (2), that:

$$\left(\left(R+\sigma^2 A^{-2}\right)^{-1}\underline{z}\right)_{(k-1)L+j} = A_k\left(\Psi R\right)_{(k-1)L+j,(k-1)L+j} b_k(j) + \sum_{\substack{l=1\\(l,i)\neq(k,j)}}^{K}\sum_{i=1}^{L} A_l\left(\Psi R\right)_{(k-1)L+j,(l-1)L+i} b_l(i) + \left(\Psi\underline{\widetilde{n}}\right)_{(k-1)L+j}$$

Consequently, since $A_k > 0$ and $(\Psi R)_{(k-1)L+j,(k-1)L+j} > 0$, the decision rule is defined by:

$$\widehat{b}_k(j) = sign\left(b_k(j) + \sum_{\substack{l=1\\(l,i)\neq(k,j)}}^{K}\sum_{i=1}^{L}\beta_{k,j}^{l,i} b_l(i) + n_k(j)\right)$$

where

$$\beta_{k,j}^{l,i} \triangleq \frac{A_l\left(\Psi R\right)_{(k-1)L+j,(l-1)L+i}}{A_k\left(\Psi R\right)_{(k-1)L+j,(k-1)L+j}}$$

and

$$n_k(j) \triangleq \frac{\left(\Psi\underline{\widetilde{n}}\right)_{(k-1)L+j}}{A_k\left(\Psi R\right)_{(k-1)L+j,(k-1)L+j}}$$

is a Gaussian random variable $\mathcal{N}\left(0,\sigma_{k,j}^2\right)$ with

$$\sigma_{k,j}^2 \triangleq \sigma^2\frac{\left(\Psi R\Psi\right)_{(k-1)L+j,(k-1)L+j}}{\left(A_k\left(\Psi R\right)_{(k-1)L+j,(k-1)L+j}\right)^2}.$$

It can then be shown using Bayes' formula that the BER for $b_k(j)$ is given by:

$$P_k(j) = 2^{1-KL}\sum_{\underline{q}_1\in\{-1;+1\}^L} .. \sum_{\underline{q}_k\in\{-1;+1\}^{L-1}} .. \sum_{\underline{q}_K\in\{-1;+1\}^L} Q\left(\left[1+\sum_{\substack{l=1\\(l,i)\neq(k,j)}}^{K}\sum_{i=1}^{L}\beta_{k,j}^{l,i}q_l(i)\right]/\sigma_{k,j}\right) \quad (6)$$

where $Q(x) \triangleq \int_x^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-u^2/2}du$. Now, the computational cost of formula (6) grows exponentially with K and/or L: it is therefore useless in practice. However, using the Central Limit theorem, for large K and/or L, it is possible to consider $\sum_{\substack{l=1\\(l,i)\neq(k,j)}}^{K}\sum_{i=1}^{L}\beta_{k,j}^{l,i}b_l(i)$ as a zero-mean Gaussian variable with variance $\sum_{\substack{l=1\\(l,i)\neq(k,j)}}^{K}\sum_{i=1}^{L}\left(\beta_{k,j}^{l,i}\right)^2$.

Consequently, one obtains the following approximation:

$$P_k(j) \simeq Q\left(1/\left(\sigma_{k,j}^2 + \sum_{\substack{l=1\\(l,i)\neq(k,j)}}^{K}\sum_{i=1}^{L}\left(\beta_{k,j}^{l,i}\right)^2\right)^{1/2}\right) \quad (7)$$

Moreover, the probability of error (6) or (7) has been derived for fixed users' offsets $(\tau_1,\ldots,\tau_K)$. Now, these offsets are random variables uniformly distributed over the interval $[0,T]$. Consequently, a mean probability should be obtained by computing the expectation of (6) or (7) with respect to these K random variables. However, a closed-form expression for this mean probability is not available, and Monte-Carlo simulations must be performed in order to estimate it (see section 4).

4. SIMULATION RESULTS

The theoretical results have been confirmed by a large number of Monte-Carlo simulations. Fig. 1 shows the theoretical and simulated BER's obtained with the MMSE detector. The parameters used for these simulations are the following: $K = 4$ users, $L = 4$ bits per user, and a spreading factor equal to $N = 8$; the users' amplitudes A_k are all equal to 1, and the non-orthogonal permutations are randomly selected. The offsets (normalized by T) are: 0.025, 0.3125, 0.4375, and 0.725, respectively. 5000 Monte-Carlo runs have been conducted to yield these results. Clearly, these laters validate the theoretical derivations (obviously, the simulated curves have greater variance for low BER's, since in that case the number of runs required to obtain accurate estimates is very large). The results are very similar for the different bits and the different users (for instance, the curves for $P_1(2)$ and $P_2(1)$ are almost identical). Note however that, even for equal amplitudes, the error probability may slightly vary for different bits and/or different users: indeed, different correlations between bits of the same users, or between bits of different users, are assigned by the random permutations. Now that the theoretical results have been confirmed by simulations, only theoretical curves will be shown in the following figures.

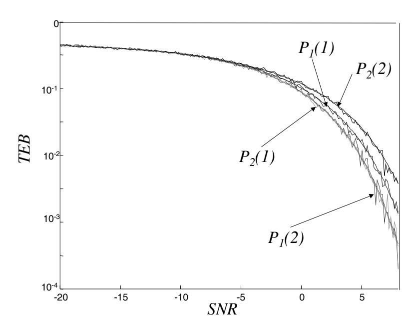

Fig. 1: Theroretical and simulated BER's of the MMSE detector for constant users'energies.

Fig. 2 considered the near-far problem encountered in multiple access systems. Here, the parameters are unchanged with respect to fig. 1, except that the users'amplitudes are now equal to $[10; 1; 5; 8]$. The results are very close for all users. In particular, the BER is not a monotonic decreasing function of the amplitudes: for instance, P_1 is greater than P_3 and P_4, while A_1 is greater than A_3 and A_4. Consequently, this figure proves that the MMSE detector eliminates a significant part of the MAI.

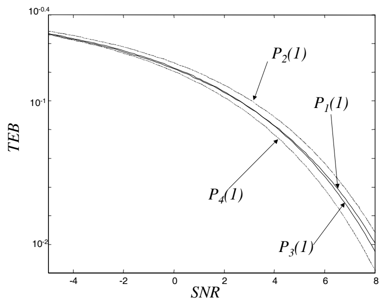

Fig. 2: BER's of the MMSE detector for different users'energies.

Moreover, the theoretical results presented in fig. 1 and 2 are obtained with fixed offset values $(\tau_1, \ldots, \tau_K)$. Fig. 3 shows the BER's obtained with the same parameters as in fig. 1, with different offsets drawn uniformly on the set $[0; T]^K$, as well as the mean BER's. This figure proves that the values of the offsets are not that much significant for the detector performance.

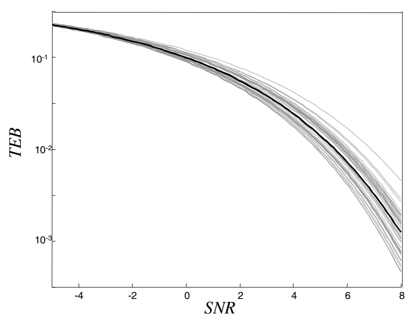

Fig. 3: BER's for different sets of offsets (dotted lines), and mean BER computed from these sets (bold line).

The RPMA system considered in this paper is next compared with the DS-CDMA for the MMSE detectors (designed for the corresponding multiple-access techniques). The CDMA results have been obtained using Gold codes (see [7]). The parameters are identical to those of fig. 1. The Gold codes have length 7, which is comparable to $N = 8$ for the random permutations. Thus, the spreading factor is equal to 8 for permutations and 7 for Gold codes. Here, a set of Gold codes is defined by a particular choice of 4 codes among 9 possible codes, along with a random delay from 0 to 6 chips. Now, the choice of a particular set of permutations or codes influence the performance of the considered multiple access technique. Therefore, it is not very meaningful to compare the BER's given by only few permutations or codes. It has then be decided to present the error probability curves given by 100 random permutations and 100 code sets. Fig. 4 and 5 superimpose the results obtained for the permutations and for the Gold codes, respectively. It can be noted that both multiple access techniques give very similar performance.

5. CONCLUSION

In this paper, the multiple access based on random permutations has been studied in the asynchronous case. A linear MMSE detector has been developed and analysed. The advantage of the MMSE detector is that it attempts to reduce simultaneously the multiple-access interference and the additive noise. The theoretical study has been validated by Monte-Carlo simulations. The choice of the permutations has been discussed in [1]: briefly, it was explained that an optimal set of permutations could be obtained by using particular optimization algorithms such as MCMC algorithm or genetic algorithm. The MMSE approach for RPMA has also been compared with the equivalent detector designed for DS-CDMA, where it has been shown that both methods behave very similarly. Finally, the case of frequency-selective channels is currently under investigation: it is expected that in such cases, the RPMA system perform better than the CDMA system since it includes in its principle an interleaving process, which should make the RPMA system more robust with respect to selectivity.

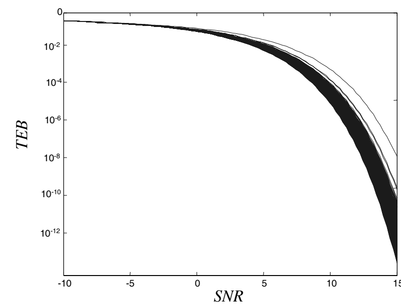

Fig. 4: BER's for 100 different random permutation sets.

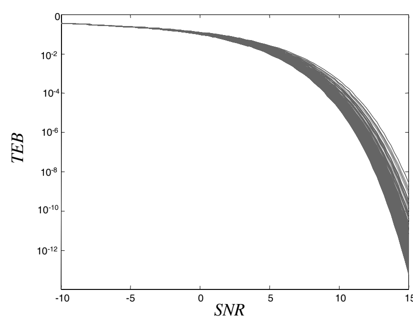

Fig. 5: BER's 100 different Gold code sets.

6. REFERENCES

[1] M. Coulon and D. Roviras, "Multi-User Detection for Random Permutation-Based Multiple Access", submitted to ICASSP'2003, Hong-Kong, April 2003.

[2] "Multiuser Detection Techniques with Application to Wired and Wireless Communications Systems I", IEEE J. on Selected Areas in Comm., Vol. 19, n° 8, August 2001.

[3] "Multiuser Detection Techniques with Application to Wired and Wireless Communications Systems II", IEEE J. on Selected Areas in Comm., Vol. 20, n° 2, February 2002.

[4] B. Lacaze, "Stationary clock changes on stationary processes", Signal processing, 55 (2) (1996) pp. 191-205.

[5] B. Lacaze and D. Roviras, "Effect of random permutations applied to random sequences and related applications", Signal Processing, 82 (2002) pp. 821-831.

[6] D. Roviras, B. Lacaze and N. Thomas, "Effects of Dicrete LPTV on Stationary Signals", Proc. of ICASSP 2002, Orlando, USA, may 2002.

[7] S. Verdu, Multiuser Detection, Cambridge University Press, Cambridge, 1998.