ATTITUDE DETERMINATION OF A SPINNING OBJECT USING DUAL IMAGING SENSORS AND A STAR CATALOG

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ABSTRACT

A bootstrap filter algorithm, which uses a sequence of the number of stars to estimate the attitude of a spinning object, is presented. This choice of measurement makes the algorithm practicable to apply to the spinning object without any initial acquisition lock. The statistical model of measurement is derived and incorporated to make star-density map. The conic fitting method is used to obtain measurements in the star sensor images. The simulation result is presented which demonstrates the ability of attitude estimation and the fast convergence property of the proposed algorithm.

1. INTRODUCTION

The problem of determining the attitude of an object using imaging sensors and star catalog has been researched in the past, using various approaches. The attitude estimation method based on the extended Kalman filter (EKF) using vector measurements from star sensor has been widely used for the attitude determination of a satellite (or spacecraft) [1]. In this method, the attitude represented by a quaternion, which is convertible to the direction cosine matrix (DCM) or the Euler angles, is evaluated with pairs of vector of the stars which are observed in the body frame and pairs of unit vector of corresponding stars in the ECI frame which can be obtained from star catalog by identifying the stars through pattern matching. For a spinning object, however, vector observations are not simple or impossible to obtain due to the rotation of the object. Moreover, a pattern matching process for the identification of stars usually needs high computational power.

A new attitude determination method which does not need vector observations is introduced here. In this method, the number of stars in the field of view (FOV) is used for the measurement. The statistical model of measurement is derived and a substantial measuring method of the number of stars is suggested. The bootstrap filter is used because the system dealt in this work is nonlinear and has non-Gaussian noise model.

The problem considered in this work is stated in Section 2. The measurement obtaining method is described in Section 3. In Section 4 an attitude determination algorithm is presented. Section 5 presents the result of simulation of algorithm, and the conclusions are discussed in Section 6.

2. PROBLEM STATEMENT

We wish to estimate the attitude of a spinning object. For the representation of the attitude of an object, we choose the right ascension, α, and the declination, β, in the Earth-centered inertial (ECI) coordinate. With this representation, we need to know at least two vectors' direction of the body-fixed (BF) frame to determine the attitude.

2.1. State model

We assume that dual star sensors' optical axes are identical to the x-axis and the y-axis of the BF frame as in fig. 1(a). The state vector is

$$\mathbf{x} = \begin{bmatrix} \alpha_{\mathrm{x}} & \beta_{\mathrm{x}} & \alpha_{\mathrm{y}} & \beta_{\mathrm{y}} \end{bmatrix}^{\mathrm{T}}, \tag{1}$$

where subscript x and y represent the x-axis and the y-axis respectively.

In the ECI frame, the unit vector lied along each axis of the BF frame is a function of state elements and expressed as

$$\mathbf{u}_{\mathrm{x}}^{\mathrm{e}} = \begin{bmatrix} \cos\beta_{\mathrm{x}}\cos\alpha_{\mathrm{x}} & \cos\beta_{\mathrm{x}}\sin\alpha_{\mathrm{x}} & \sin\beta_{\mathrm{x}} \end{bmatrix}^{\mathrm{T}}, \tag{2}$$

$$\mathbf{u}_{\mathrm{y}}^{\mathrm{e}} = \begin{bmatrix} \cos\beta_{\mathrm{y}}\cos\alpha_{\mathrm{y}} & \cos\beta_{\mathrm{y}}\sin\alpha_{\mathrm{y}} & \sin\beta_{\mathrm{y}} \end{bmatrix}^{\mathrm{T}}, \tag{3}$$

$$\mathbf{u}_{\mathrm{z}}^{\mathrm{e}} = \frac{\mathbf{u}_{\mathrm{x}}^{\mathrm{e}} \times \mathbf{u}_{\mathrm{y}}^{\mathrm{e}}}{\left\|\mathbf{u}_{\mathrm{x}}^{\mathrm{e}} \times \mathbf{u}_{\mathrm{y}}^{\mathrm{e}}\right\|_2}, \tag{4}$$

where the superscript $(\cdot)^e$ means the quantity in the ECI frame and $\times$ denotes the cross product.

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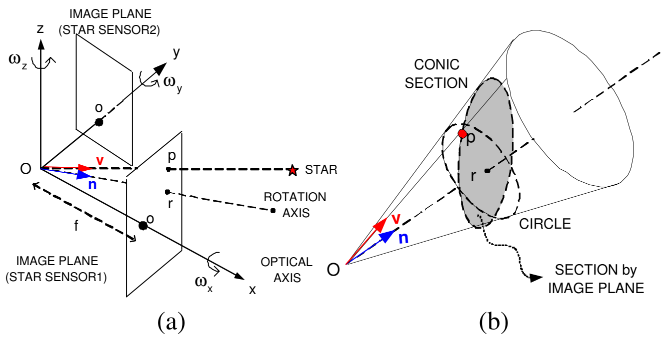


Fig. 1. The BF frame (a) and Star trace (b).

The DCM, C, can be expressed by unit vectors as

$$\mathbf{C} = \left[\begin{array}{ccc} \mathbf{u}_{\mathrm{x}}^{\mathrm{e}} & \mathbf{u}_{\mathrm{y}}^{\mathrm{e}} & \mathbf{u}_{\mathrm{z}}^{\mathrm{e}} \end{array} \right]^{\mathrm{T}}. \tag{5}$$

For a spinning object, the BF frame rotates with respect to the ECI frame. Denote the angular rate vector of this rotation by $\tilde{\omega} = [\omega_{\mathrm{x}}\ \omega_{\mathrm{y}}\ \omega_{\mathrm{z}}]^{\mathrm{T}}$. It is well known that the rate of change of $\mathbf{C}$ is given by

$$\frac{d\mathbf{C}}{dt} = \mathbf{C}\Omega, \tag{6}$$

where

$$\Omega = \left[\begin{array}{ccc} 0 & -\omega_{\mathrm{z}} & \omega_{\mathrm{y}} \\ \omega_{\mathrm{z}} & 0 & -\omega_{\mathrm{x}} \\ -\omega_{\mathrm{y}} & \omega_{\mathrm{x}} & 0 \end{array} \right].$$

It is assumed that the measured angular rate, $\tilde{\omega}$, from the gyros has a zero-mean white-noise, $\delta\omega$; hence

$$\tilde{\omega} = \bar{\omega} + \delta\omega, \tag{7}$$

where $\bar{\omega}$ is the true angular rate vector.

The DCM propagates according to (6). Therefore, the rotation dynamic model can be represented by

$$\mathbf{x}_{\mathrm{k}+1} = f(\mathbf{x}_{\mathrm{k}}, \mathbf{w}_{\mathrm{k}}; \bar{\omega}_{\mathrm{k}}) \tag{8}$$

where $\mathbf{w}_{\mathrm{k}}$is the process noise due to the uncertainty of the angular rate of the covariance $\mathbf{Q} = \sigma_{\omega}^2 \mathbf{I}_{3\times3}$.

2.2. Measurement model

A sequence of the number of stars in the FOV of star sensor is used for the measurement. To obtain statistical response of the star sensor, we assumed the followings. First, the star catalog contains all stars in the sky. Second, the magnitude of each star recorded in star catalog has the normal pdf. Third, the star sensor has the exact detection threshold. Finally, the FOV of the star sensor has a circular shape.

Let N_p be the number of stars in the FOV when the direction of the optical axis of the star sensor points the position p (α right ascension, β declination) and m_{th} be the detection threshold magnitude of the star sensor. From the second assumption, the magnitude of the i_{th} star is normally distributed,$\mathrm{N}(m_i, \sigma^2)$ and the detection probability of the i_{th} star is given by

$$p_i = \int_{-\infty}^{m_{th}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - m_i)^2}{2\sigma^2} \right) dx. \tag{9}$$

The mean, μ_i, and variance, σ_i^2, of the detection probability of the i_{th} star are given by

$$\mu_i = p_i, \tag{10}$$

$$\sigma_i^2 = p_i(1 - p_i). \tag{11}$$

Let $n_p = p_1 + p_2 + \cdots + p_{N_p}$ be a set of N_p independent random variates and each p_i has a binomial distribution with mean of (10) and variance of (11). The random variate n_p is then approximately normally distributed with

$$\mu_{n_p} = \sum_{i=1}^{N_p} p_i, \tag{12}$$

$$\sigma_{n_p}^2 = \sum_{i=1}^{N_p} p_i(1 - p_i). \tag{13}$$

From the above equations (12) and (13), we can know the statistical number of stars detected by star sensor approximately has a normal distribution and obtain the star density maps which are functions of the magnitude threshold m_{th}, the direction of sensor's optical axis and the FOV. Figure 2 (a) and (b) show the star-density mean map $h_\mu(\mathbf{x})$ and the star-density variance map $h_{\sigma^2}(\mathbf{x})$, respectively, obtained from the Smithsonian Astrophysical Observatory (SAO) star catalog assuming that $m_{th} = 6$ and $10°$ FOV. The grid size of star-density map is $1° \times 1°$. For a sub-grid region, the mean and variance values are computed by the bilinear interpolation.

The measured number of stars, finally, can be modeled as

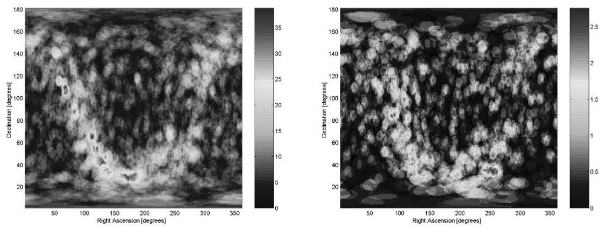
$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k, \tag{14}$$

where

$$h(\mathbf{x}_k) \equiv \left[\begin{array}{c} N(h_\mu(\mathbf{x}_k^{\mathrm{x}}), h_{\sigma^2}(\mathbf{x}_k^{\mathrm{x}})) \\ N(h_\mu(\mathbf{x}_k^{\mathrm{y}}), h_{\sigma^2}(\mathbf{x}_k^{\mathrm{y}})) \end{array} \right],$$

$$\mathbf{v}_k \equiv \left[\begin{array}{c} \eta(\mathbf{x}_k^{\mathrm{x}}) + q_k^{\mathrm{x}} \\ \eta(\mathbf{x}_k^{\mathrm{y}}) + q_k^{\mathrm{y}} \end{array} \right],$$

where the η is the digitization error caused by the bilinear interpolation and q_k is the two level quantization error because the measurement z_k is in finite integer set. Here, we set the state as $\mathbf{x} = [\mathbf{x}^{\mathrm{x}}\ \mathbf{x}^{\mathrm{y}}]$ where $\mathbf{x}^{\mathrm{x}} = [\alpha_{\mathrm{x}}\ \beta_{\mathrm{x}}]^{\mathrm{T}}$ and $\mathbf{x}^{\mathrm{y}} = [\alpha_{\mathrm{y}}\ \beta_{\mathrm{y}}]^{\mathrm{T}}$.



(a) (b)

Fig. 2. Star-density mean map (a) and variance map (b).

3. ACQUISITION OF MEASUREMENT

Because the BF frame rotates with an angular rate in the spinning object, the stars trace some trajectories in the image plane. In our work it is assumed that the angular rate is constant between the measurement interval (i.e. exposure time of the star sensor). Under this assumption, the rotation of the spinning object can be represented with the rotation axis $\mathbf{n} = [n_x \; n_y \; n_z]^{\mathrm{T}}$ and the rotation angle, θ, given by

$$\mathbf{n} = \omega / \left\|\omega\right\|_2, \tag{15}$$

$$\theta = \left\|\omega\right\|_2 \cdot \Delta t, \tag{16}$$

where Δt is the measurement interval time. As shown in the fig. 1(b), the unit vector to a star in the BF frame shaped a cone with the rotation axis $\mathbf{n}$ as the central axis. The intersection of this cone and the image plane, the conic section, determines the star trace. The rotation axis $\mathbf{n}$ also intersects the image plane at point r given by

$$r_{\mathrm{x}} = (m_{\mathrm{x}}, n_{\mathrm{x}}) = \left(\frac{n_y}{n_x} f, \frac{n_z}{n_x} f\right), \tag{17}$$

$$r_{\mathrm{y}} = (m_{\mathrm{y}}, n_{\mathrm{y}}) = \left(-\frac{n_x}{n_y} f, \frac{n_z}{n_y} f\right), \tag{18}$$

where (m,n) represents the position in the image coordinate.

We can measure the number of stars by counting the number of connected conic sections in a radial manner with this point r as a center because conic sections representing star traces do not intersect mutually.

3.1. Pre-processing

A pre-processing of star sensor images is needed before the acquisition of star traces in the image. This process is composed of three steps. We apply an thresholding scheme to distinguish between star traces and background in the star sensor image. Secondly, a morphology-based thinning technique is used to get one-pixel wide traces [2]. Intersections are detected by a window-based probing process and removed. This pre-processing results in multiple connected traces coming from multiple stars.

3.2. Conic fitting

A star trace shapes a conic section which can be described by the following equation:

$$Q(x, y) = Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0, \tag{19}$$

where A, B and C are not simultaneously zero. We should fit a conic section to a set of N connected points $\{\mathbf{x}_i\} = \{(x_i, y_i)\}$ $(i = 1, \ldots, N)$ obtained from the pre-processing. There're several solutions for solving conic fitting problem with these noisy data [3]. Among them, the least-squares fitting with normalization constraint $A + C = 1$ is a common practice for its ease of implementation. We added a constraint that the point r is on the major axis of the conic section (but not a center or a focus) to the basic least-squares fitting. This constraint can be written as $\mathrm{E/D = n/m}$. All conic section can then be described by a vector $\mathbf{p} = [A \; B \; D \; F]^{\mathrm{T}}$. Given N points, we have the following vector equation:

$$\mathbf{Ap} = \mathbf{b}, \tag{20}$$

where

$$\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_{\mathrm{N}}]^{\mathrm{T}},$$
$$\mathbf{b} = [b_1, b_1, \cdots, b_{\mathrm{N}}]^{\mathrm{T}},$$

where

$$\mathbf{a}_i = [x_i^2 - y_i^2, 2x_i y_i, 2(x_i + n/m) y_i, 1]^{\mathrm{T}},$$

$$b_i = -y_i^2.$$

The least-squares solution is given by

$$\mathbf{p} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}. \tag{21}$$

3.3. Overlapping of star traces

Arbitrary two stars having same angle with respect to $\mathbf{n}$ may have overlapped traces according to the angular rate and to the position in the ECI frame. To cope with this case, we introduced a length comparison method. In this method, we compare the angular length γ of detected trace with the rotation angle obtained from gyro measurements with (16). The angular length γ can be obtained by

$$\gamma = \cos^{-1}\left(\frac{\mathbf{s}^{\mathrm{T}}\mathbf{t}}{\left\|\mathbf{s}\right\|_2 \cdot \left\|\mathbf{t}\right\|_2}\right), \tag{22}$$

where $\mathbf{s}$ is the vector from the point r to the start point p of trace and $\mathbf{t}$ from the point r to the end point q. Due to the noise in the angular rate, we should consider the variance of the rotation angle which is modeled as $\sigma_\theta^2 = \sigma_\omega^2$. When there is n overlapped stars in one trace, then, the length comparison yields

$$\gamma \geq \mathrm{n}\theta \pm \mathrm{k}\sigma_\theta, \tag{23}$$

where k is a constant.

4. BOOTSTRAP FILTERING

As can be seen from (8) and (14), the system model is highly nonlinear and the measurement model has a non-Gaussian probability distribution. We used a bootstrap filtering method to estimate the state [4]. The implementation of the bootstrap filter is as follows.

1. *Initialization:* Generate $\{\mathbf{x}_1(i)\}_{i=1}^{N}$ drawn from $p(\mathbf{x}_1|Z_0) = p(\mathbf{x}_1)$.
2. *Measurement Update:* Update the weights by the likelihood of samples according to (14) $\zeta_i = p(\mathbf{z}_k|\hat{\mathbf{x}}_k(i)), i = 1, 2, \cdots, N$, and normalize to $\zeta_i = \zeta_i / \sum_i \zeta_i$.
3. *Resampling Procedure:* Take N samples $\{\mathbf{x}_k(i)\}_{i=1}^{N}$, so that $P\left[\mathbf{x}_k(j) = \hat{\mathbf{x}}_k(i)\right] = \zeta_i$, for any j.
Take, $\hat{\mathbf{x}}_k = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_k(i)$.
4. *Time Update:* Take a $\mathbf{w}_k$ from (8), and get $\hat{\mathbf{x}}_{k+1}(i)$, $(i = 1, 2, \cdots, N)$ by numerically integration of (6).
5. Increase time step and iterate to item 2.

5. SIMULATION

To evaluate the performance of the proposed attitude determination method, we consider a spinning object with an initial true state $[21.7\ \ -38.3\ \ 136.8\ \ 50.7]^{\mathrm{T}}$ $(deg.)$ and a constant angular rate $[-0.01\ \ 0.02\ \ -0.01]^{\mathrm{T}}$ $(deg./sec)$. Each component of the gyro measurement $\tilde{\omega}$ has an additive white noise with 0.5 arc-sec. The time interval between measurements Δt is 200 seconds. The sample size N is 2000. Figure 3(a) shows one of synthetic star sensor images and the detected conic section is given in the fig. 3(b).

To appraise the performance of the estimate, we quote convergence index defined as

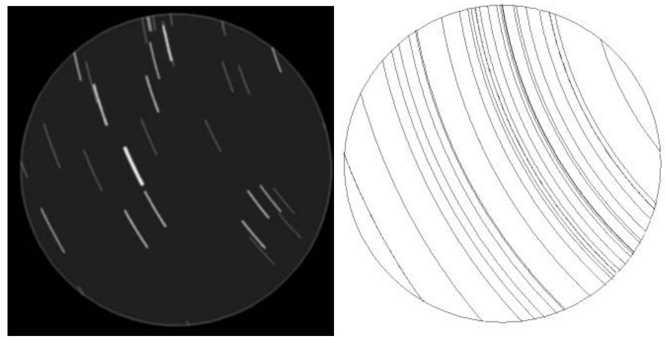
$$J_k \equiv \left\| \mathbf{C}(\mathbf{x}_k) - \mathbf{C}(\hat{\mathbf{x}}_k) \right\|_F^2 , \tag{24}$$

where $\hat{\mathbf{x}}_k$ is the estimated state vector. Figure 4(a) shows the behavior of the convergence index. After converging, the convergence index is under 10^{-4}. Each component of estimation errors is depicted in fig. 4(b).

When lager number of samples are used, the proposed algorithm may give more accurate estimate, on the other hand, computational load increases.

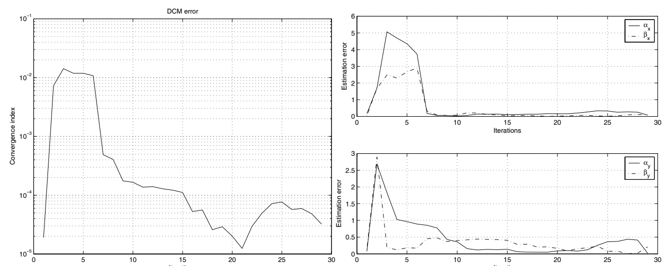
6. CONCLUSION

We have proposed the attitude determination method of a spinning object based on the bootstrap filter using dual star sensors. We derived the appropriate measurement equation which is modeled as a non-Gaussian probability density function. The method for measuring the number of stars in the star sensor image in which stars make traces also was suggested. In the simulation study, the proposed algorithm gave accurate estimates and showed fast convergence properties.



(a) (b)

Fig. 3. Star sensor image (a) and the detected conic sections (b).



(a) (b)

Fig. 4. Behavior of the convergence index (a) and the estimation errors for each state element (b).

The proposed method using dual sensors can easily be modified to a method using only one sensor by choosing the Euler angles (or quaternion)as system state. This modification, however, may need more samples and more computational costs.

7. REFERENCES

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