DYNAMIC POWER ESTIMATION AND PREDICTION IN COMPOSITE FADING-SHADOWING CHANNELS

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ABSTRACT

We present methods for dynamic estimation and prediction of local mean (shadow) powers from instantaneous signal powers in composite fading-shadowing wireless communication scenarios. We adopt a Nakagami-m fading model for the instantaneous signal powers and a first-order autoregressive [AR(1)] model for the shadow process in decibels. Sequential Bayesian analysis is applied to estimate the shadow powers assuming that the Nakagami-m and AR(1) model parameters are known. We also develop a method for jointly estimating both the shadow powers and unknown model parameters. Numerical simulations demonstrate the performance of the proposed methods.

1. INTRODUCTION

In wireless communications, the ability to accurately estimate local mean (shadow) powers is instrumental for adaptive modulation techniques, handoff, channel access, and power control, see e.g. [1]–[3] and references therein. In this paper, we derive sequential Bayesian algorithms for estimating and predicting the shadow powers in composite fading-shadowing channels[1] with a Nakagami-m fading component and a shadowing component that follows a first-order autoregressive [AR(1)] random process.

The measurement model and dynamic estimation algorithm for known model parameters are presented in Section 2. The joint estimation of shadow powers and model parameters is discussed in Section 3. In Section 4, we evaluate the proposed methods using numerical simulations.

2. MEASUREMENT MODEL AND PROPOSED DYNAMIC ESTIMATOR OF SHADOW POWERS

We model the *instantaneous signal powers* $y_k,\ k = 1, 2, \ldots$ received at the mobile station as conditionally independent gamma random variables with the following probability density functions (pdfs):

$$p_{y|\beta}(y_k|\beta_k; m) = \frac{m^m y_k^{m-1}}{10^{m\beta_k/10}\Gamma(m)} \exp\left(-my_k 10^{-\beta_k/10}\right), \tag{2.1}$$

where $\Gamma(\cdot)$ denotes the gamma function, m the Nakagami-m fading parameter, and $\beta_k,\ k = 1, 2, \ldots$ the *local mean (shadow) powers* in decibels. We assume that the shadow powers β_k follow a first-order autoregressive [AR(1)] random process satisfying the following recursion:

$$\beta_k = \alpha_k \beta_{k-1} + \omega_k, \tag{2.2}$$

where ω_k are independent zero-mean random variables with variances $\sigma^2_{\omega,k}$. The AR(1) model in (2.2) is widely used to describe the correlation of the shadow process β_k, see e.g. [2], [9], [10], and references therein. Our goal is to estimate the *unknown* shadow powers β_k assuming that the model parameters (Nakagami-m fading parameter, AR coefficients α_k, and variances $\sigma^2_{\omega,k}$) are *known.* (An extension to the scenario where the model parameters are *unknown* is considered in Section 3.) Estimates of the shadow powers in decibels are being utilized by most handoff algorithms, as well as for channel access and power control, see [2].

Note that we have not specified the distributional form of the random variables ω_k apart from their first two moments; hence the distribution of β_k is also not fully specified. Denote the mean and variance of β_k by μ_k and c_k. Immediately before we observe y_k, all currently available information is described by the mean μ_{k-1} and variance c_{k-1} of the shadow process. At time $k = 1$, these are the starting values μ_0 and c_0, and for all other k will come from the posterior distribution of β_{k-1} given y_{k-1}, denoted by $[\beta_{k-1}|y_{k-1}]$. Using the AR(1) model in (2.2), we compute the mean a_k and variance r_k of the *prior distribution* $[\beta_k|\mu_{k-1}, c_{k-1}]$:

$$a_k = \alpha_k\, \mu_{k-1}, \tag{2.3a}$$
$$r_k = \alpha_k^2\, c_{k-1} + \sigma^2_{\omega,k}. \tag{2.3b}$$

Since $[\beta_k|\mu_{k-1}, c_{k-1}]$ is specified only through the above moments, we are free to choose the form of this distribution

[1]Composite fading-shadowing models are used to describe the statistical properties of wireless communication channels in congested downtown areas [3]–[6], satellite communication systems [7], and distributed antenna systems [8].

as long as it is consistent with (2.3), see also the discussion in [11, p. 526]. Here, we adopt the Gaussian prior pdf with mean and variance given in (2.3):

$$[\beta_k|\mu_{k-1}, c_{k-1}] = g(\beta_k; a_k, r_k) = \frac{1}{\sqrt{2\pi r_k}} e^{-(\beta_k - a_k)^2/(2r_k)} \tag{2.4}$$

Another convenient choice for the prior distribution is the *conjugate prior* on the natural parameter of the observation-model distribution in (2.1), which leads to the *dynamic generalized linear model* in e.g. [11, Ch. 14.3]. Aside from the different choice of the prior distribution, our model in (2.2)–(2.3) can be viewed as a special case of the dynamic model in [11, eqs. (14.11) and (14.21)–(14.22)].

Using the Gaussian prior (2.4), the posterior updating equations (2.5) at the bottom of the page are derived by computing the mean and variance of $[\beta_k|y_k]$. The approximate expressions (2.5b) and (2.6b) follow by applying the change-of-variable transformation $x = (\beta_k - a_k)/\sqrt{2r_k}$ to the numerators and denominators in (2.5a) and (2.6a) and using Gauss-Hermite quadrature to numerically evaluate the obtained integrals. Here, L is the quadrature order (determining approximation accuracy), $x_l,\ l = 1, \dots, L$ are the zeroes of the Lth-order Hermite polynomial and $h_{x_l},\ l = 1, \dots, L$ are the Gauss-Hermite quadrature weight factors tabulated in e.g. [12]. The expressions (2.5b) and (2.6b) are remarkably simple due to the cancellations of the common terms in the numerators and denominators of (2.5a) and (2.6a).

To summarize, we propose a sequential Bayesian method for the dynamic estimation and prediction of shadow powers using the prior cascade equations (2.3) and posterior updating equations (2.5). Assuming that instantaneous signal powers until time k are available, the proposed *estimator* of the shadow power β_k is simply μ_k [computed in (2.5b)] and the corresponding one-step *predictor* of β_{k+1} is [see (2.3a)]:

$$a_{k+1} = \alpha_{k+1}\mu_k. \tag{2.7}$$

3. JOINT ESTIMATION OF SHADOW POWERS AND MODEL PARAMETERS

We develop an iterative method for *jointly* estimating the shadow powers *and* model parameters. (The joint estimation of the shadow powers and model parameters is important in urban environments if the sampling period with which the measurements are collected is relatively large, see [2, Sect. IV].) First, assume that the AR coefficients α_k and variances $\sigma^2_{\omega,k}$ are constant (independent of k) in the interval $\{1, 2, \dots, K\}$, i.e.

$$\alpha_k = \alpha \tag{3.1a}$$

$$\sigma^2_{\omega,k} = \sigma^2_\omega, \quad k = 1, 2, \dots, K, \tag{3.1b}$$

implying *stationarity* of the shadow process. We propose the following *alternating-projection* algorithm for estimating the shadow powers and unknown model parameters in (3.1) using the instantaneous powers $y_1, y_2, \dots, y_K$: iterate between

Step 1: fix α and σ^2_ω and estimate $\beta_1, \beta_2, \dots, \beta_K$ using (2.5b) [where the full recursion is described by (2.3) and (2.5)] and

Step 2: fix $\beta_1, \beta_2, \dots, \beta_K$ and estimate α and σ^2_ω using their asymptotic maximum likelihood (ML) estimates (see e.g. [14, Ex. 7.18]):

$$\widehat{\alpha} = \frac{\sum_{k=1}^{K-1} \beta_k \beta_{k+1}}{\sum_{k=1}^{K} \beta_k^2}, \tag{3.2a}$$

$$\widehat{\sigma}^2_\omega = \frac{1}{K}\Big(\sum_{k=1}^{K} \beta_k^2\Big) \cdot (1 - \widehat{\alpha}^2). \tag{3.2b}$$

Note that Step 1 requires knowledge of the Nakagami-m fading parameter, which can be estimated separately. In the following, we briefly discuss the estimation of m.

$$\mu_k = \mathrm{E}_{\beta|y}[\beta_k|y_k] = \frac{\int_{-\infty}^{\infty} \beta\, p_{y|\beta}(y_k|\beta, m)\, g(\beta; a_k, r_k)\, d\beta}{\int_{-\infty}^{\infty} p_{y|\beta}(y_k|\beta, m)\, g(\beta; a_k, r_k)\, d\beta} \tag{2.5a}$$

$$\approx \frac{\sum_{l=1}^{L} h_{x_l} \cdot (\sqrt{2r_k} \cdot x_l + a_k) \cdot \exp[-my_k/\upsilon_l(a_k, r_k)] \cdot \upsilon_l(a_k, r_k)^{-m}}{\sum_{l=1}^{L} h_{x_l} \exp[-my_k/\upsilon_l(a_k, r_k)] \cdot \upsilon_l(a_k, r_k)^{-m}} \tag{2.5b}$$

$$c_k = \mathrm{var}_{\beta|y}[\beta_k|y_k] = \mathrm{E}_{\beta|y}[\beta_k^2|y_k] - \mu_k^2, \tag{2.5c}$$

where $\upsilon_l(a_k, r_k) = 10^{(\sqrt{2r_k}\cdot x_l + a_k)/10}$ and

$$\mathrm{E}_{\beta|y}[\beta_k^2|y_k] = \frac{\int_{-\infty}^{\infty} \beta^2\, p_{y|\beta}(y_k|10^{\beta/10}, m)\, g(\beta; a_k, r_k)\, d\beta}{\int_{-\infty}^{\infty} p_{y|\beta}(y_k|10^{\beta/10}, m)\, g(\beta; a_k, r_k)\, d\beta} \tag{2.6a}$$

$$\approx \frac{\sum_{l=1}^{L} h_{x_l} \cdot (\sqrt{2r_k} \cdot x_l + a_k)^2 \cdot \exp[-my_k/\upsilon_l(a_k, r_k)] \cdot \upsilon_l(a_k, r_k)^{-m}}{\sum_{l=1}^{L} h_{x_l} \exp[-my_k/\upsilon_l(a_k, r_k)] \cdot \upsilon_l(a_k, r_k)^{-m}}. \tag{2.6b}$$

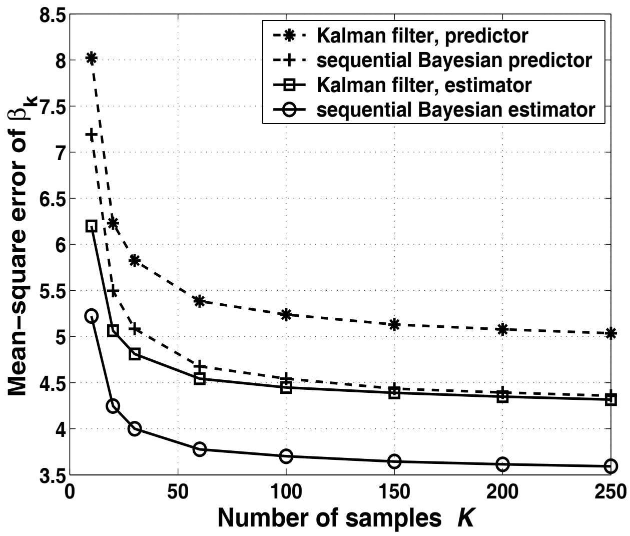

Fig. 1. Mean-square errors for the sequential Bayesian and Kalman-filter based estimators and predictors of the shadow powers as functions of K, assuming known model parameters and $m = 1$ (Rayleigh fading).

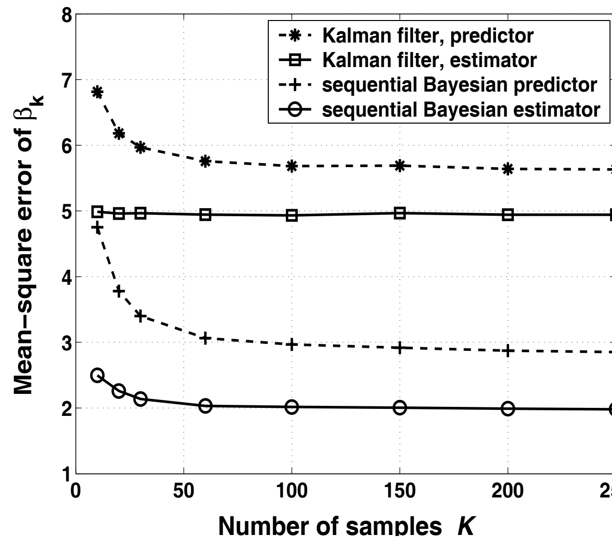

Fig. 2. Mean-square errors for the sequential Bayesian and Kalman-filter based estimators and predictors of the shadow powers as functions of K, assuming known model parameters and $m = 3$.

Estimating the Nakagami-m parameter: The Nakagami-m fading parameter can be estimated using the algorithms proposed in [13], where we derived ML methods for estimating m from the instantaneous powers $y_1, y_2, \ldots, y_K$ [which follow (2.1)] under the piecewise-constant model for shadow powers. In particular, $\beta_1, \beta_2, \ldots, \beta_K$ were assumed to be constant within intervals (windows) of length N but allowed to vary randomly from one interval to another. In [13], we chose $K = LN$ and $\beta_{(l-1)N+1} = \beta_{(l-1)N+2} = \beta_{(l-1)N+N-1} = z_l$, where z_l, $l = 1, 2, \ldots, L$ were modeled as independent, identically distributed Gaussian random variables with unknown mean and variance.

Backward recursion: In addition to the "forward" recursion described in Step 1, we can also estimate the shadow powers by applying (2.3) and (2.5) "backward" to the observations arranged in the reverse order: $y_K, y_{K-1}, \ldots, y_1$, which follows from the stationarity of the shadow process, see (3.1). The shadow-power estimation in Section 2 and Step 1 (above) can be improved by running *both* forward and backward recursions and averaging the obtained forward and backward estimates of $\beta_1, \beta_2, \ldots, \beta_K$. Note, however, that forward-backward averaging requires non-dynamic (batch) processing of the instantaneous powers.

4. NUMERICAL EXAMPLES

The numerical examples presented here assess the estimation accuracy of the proposed methods. The instantaneous powers y_k, $k = 1, 2, \ldots$ were simulated from a *gamma-lognormal* fading channel model (see e.g. [3] and [4]) with correlated shadow powers, described by (2.1) and (2.2) with $w_k, k = 1, 2, \ldots$ generated from a Gaussian distribution. [Consequently, the shadow powers $\beta_k, k = 1, 2, \ldots$ are also Gaussian.] We assume that the AR coefficients α_k and variances $\sigma^2_{\omega,k}$ are constant, equal to $\alpha_k = \alpha = 0.9704$ and $\sigma^2_{\omega,k} = \sigma^2_\omega = 0.9318$, which are typical values in an urban environment, see [2]. Our performance metric is the mean-square error (MSE), calculated using 500 independent trials. The quadrature order of the Gauss-Hermite approximations in (2.5b) and (2.6b) was $L = 20$.

In the first set of simulations, we consider a scenario with known model parameters and apply the dynamic shadow power estimation and prediction algorithm in Section 2. The recursion in (2.3) and (2.5) was initialized using $\mu_0 = 0$ and $c_0 = \sigma^2_\omega/(1 - \alpha^2)$. In Figs. 1 and 2, we show the MSEs of the sequential Bayesian estimator (2.5b) and one-step predictor (2.7) for $m = 1$ (Rayleigh fading) and $m = 3$, respectively, as functions of the number of samples K. Figs. 1 and 2 also show the MSE performances of the Kalman-filter based power estimators and predictors recently proposed in [2]. The sequential Bayesian method outperforms the Kalman filter in both scenarios[2].

In the second set of simulations, we consider the scenario where the model parameters are unknown (in addition to the shadow powers) and apply the iterative algorithm in Section 3 for jointly estimating the shadow powers and model parameters. To improve the shadow power estimation in Step 1 of the iteration, we averaged the forward and backward estimates of the shadow powers, as described in Section 3. In Figs. 3 and 4, we show the MSEs of the model-parameter and shadow-power estimates (respectively) for

[2]Note that the Kalman filtering method in [2] was designed for the Rayleigh-fading scenario only.

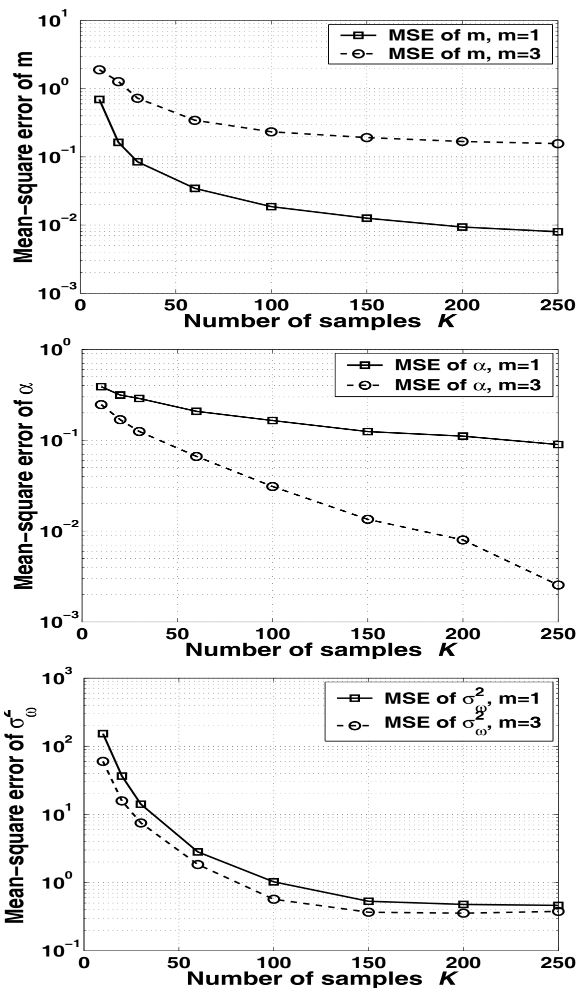

Fig. 3. Mean-square errors for the estimators of the model parameters (m, α, and σ_ω^2, respectively) as functions of K, obtained for $m = 1$ and $m = 3$ using the iterative algorithm in Section 3.

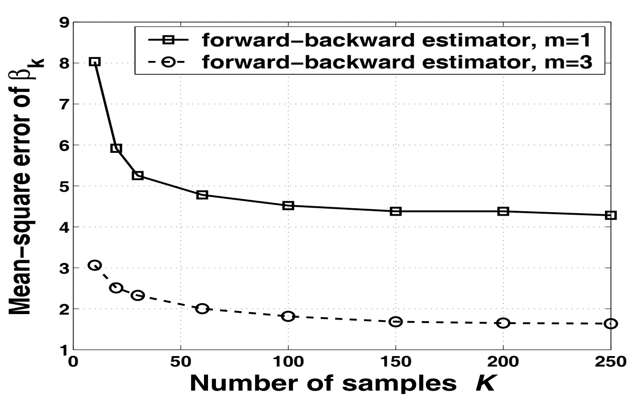

Fig. 4. Mean-square errors for the forward-backward estimators of the shadow powers as functions of K, obtained for $m = 1$ and $m = 3$ using the iterative algorithm in Section 3.

$m = 1$ and $m = 3$, obtained using the method in Section 3. We selected $N = 5$ to be the window length for estimating m. Interestingly, for large K and $m = 3$, the forward-backward shadow-power estimates for unknown model parameters outperform the dynamic shadow-power estimates for known model parameters, see Figs. 4 and 2.

5. REFERENCES

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