SYNTHESIS OF HYBRID FILTER BANKS FOR A/D CONVERSION WITH IMPLEMENTATION CONSTRAINTS - MIXED DISTORTION/ALIASING OPTIMIZATION -

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ABSTRACT

This paper presents two methods of designing Hybrid Filter Banks (HFB) taking into account analog filters constraints resulting from the need of having very simple analog structures. Therefore standard resonators are considered for the analog analysis filters. They are seen as input data of the design process and are kept unmodified throughout the design procedure. Direct resolution of simplified perfect reconstruction conditions give primary synthesis filters which can be further optimized in order to minimize the reconstruction error. Mean aliasing of the designed HFB is below -80 dB and mean distortion is below 0.15 dB.

1. INTRODUCTION

In wireless communication and a number of other domains, the demand for higher data rates together with versatility is always rising. Significant improvements have been achieved in DSP part of telecom systems, but the A/D conversion is still a bottleneck. Low costs, for instance, need higher working frequency whereas higher data rates and versatility need much wider bandwidths. Parallelization of channels is a first idea when trying to build a very wide band ADC. Hybrid Filter Banks (Figure 1) are very good candidates to achieve an analog (continuous time) decomposition together with a digital/discrete time recombination. HFB ADCs were discussed in [1], [2], [3] or [4]. Those authors gave the right analysis formulas taking into account the effective sampling within each path. In [5], the authors present a design algorithm based on the minimization of the noise energy, derived from a time domain analysis of the HFB. Velazquez et al. proposed [1] a design method based on a frequency domain analysis which leads to define both analysis and synthesis filters. In this paper we propose a method which takes into account the need of dealing with available, simple, high speed analog filters that can be found within a given technology. Indeed, considering cost targets, these filters can only be implemented with high-frequency integrated components such as integrated LCs, gmC amplifiers or SAW devices. In any case, only simple transfer functions can be implemented (typically resonators). The set of the possible choices for the analog filters being small, their parameters must be considered as input (prior data) of the design. The proposed approach starts with the knowledge of the analog transfer functions $\{H_m(s)\}$ in order to reach the discrete ones, namely $\{F_m(z)\}$. To do this, several ideas may be found. One could be to find a digital analysis filter bank equivalent (in a given frequency band) to the analog one, then to use the theoretical background of Digital Filter Banks [6], to get the corresponding synthesis filter parameters. Another idea is to globally work out the synthesis filter bank from the knowledge of the analog ones. We will further explore the second idea.

2. HYBRID FILTER BANK BRIEF ANALYSIS

Figure 1 shows an HFB, where $H_0(s)$, $H_1(s)$, ... , $H_{M-1}(s)$ are the analog analysis filters and $F_0(z)$, $F_1(z)$, ... , $F_{M-1}(z)$ are the digital synthesis filters. A sampler and a quantizer (block designated by Q) may be found on each channel. The Fourier transform of the output signal can be written [2]:

$$Y(e^{j\omega}) = \sum_{m=0}^{M-1} F_m(e^{j\omega}) X_m(e^{j\omega M}), \tag{1}$$

with $\omega = \Omega T$ and

$$X_m(e^{j\omega M}) = \frac{1}{MT} \sum_{p=-\infty}^{\infty} X(j\Omega - j\frac{2\pi p}{MT}) H_m(j\Omega - j\frac{2\pi p}{MT}). \tag{2}$$

Equation (1) can be rewritten as follows:

$$Y(e^{j\omega}) = \sum_{p=-\infty}^{\infty} X(j\Omega - j\frac{2\pi p}{MT}) T_p(e^{j\omega}), \tag{3}$$

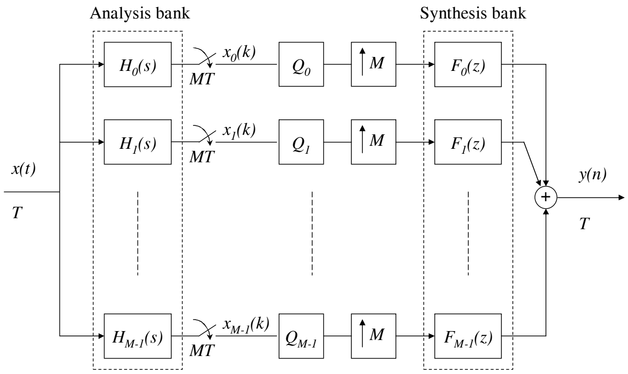

Fig. 1. Hybrid filter bank

where

$$T_p(e^{j\omega}) = \frac{1}{MT}\sum_{m=0}^{M-1} F_m(e^{j\omega}) H_m(j\Omega - j\frac{2\pi p}{MT}). \quad (4)$$

The input signal $x(t)$ is supposed to be band limited to $\frac{\pi}{T}$ by some external filter. Then, within the interval $-\frac{\pi}{T} < \Omega \leq \frac{\pi}{T}$, the output is:

$$Y(e^{j\omega}) = \sum_{p=-(M-1)}^{M-1} X(j\Omega - j\frac{2\pi p}{MT}) T_p(e^{j\omega}). \quad (5)$$

Furthermore, considering $H_m(j\Omega)$ band limited to $\frac{\pi}{T}$ [2]:

$$Y(e^{j\omega}) = \sum_{p=0}^{M-1} X^S(j\Omega - j\frac{2\pi p}{MT}) T_p^S(e^{j\omega}), \quad (6)$$

where

$$T_p^S(e^{j\omega}) = \frac{1}{MT}\sum_{m=0}^{M-1} F_m(e^{j\omega}) H_m^S(j\Omega - j\frac{2\pi p}{MT}), \quad (7)$$

$$X^S(j\Omega) = X(j\Omega) + X(j\Omega + j\frac{2\pi}{T}) \quad (8)$$

and

$$H^S(j\Omega) = H(j\Omega) + H(j\Omega + j\frac{2\pi}{T}). \quad (9)$$

Perfect reconstruction means that the output is simply a sampled, scaled and delayed version of the input. So, taking into account equation (6), the perfect reconstruction conditions are:

$$T_p^S(e^{j\omega}) = \begin{cases} ce^{-j\omega d} & , p = 0 \\ 0 & , p \in \{1, ..., (M-1)\}. \end{cases} \quad (10)$$

$T_0^S(e^{j\omega})$, with $d \in \mathbb{R}$ being the filter bank's delay and $c \in \mathbb{R}$ a scale factor, indicates the HFB's gain and phase and represents the distortion function. $T_p^S(e^{j\omega}), p \in \{1, 2, ..., M-1\}$ are the aliasing functions since they show how the shifted, unwanted versions of the input are present in the output signal. Let's now compare equations (3), (5) and (6). Equation (3) gives the exact form of the Fourier transform of the output signal. It can be noticed that it contains an infinite number of terms, so writing the perfect reconstruction conditions, as in (10) yields a linear system with an infinite number of equations and M variables, the synthesis functions (as the analysis filters are supposed known see Section 1). The input signal being band limited (3) can be transformed into (5). In (5) the sum has a finite number of terms which significantly simplifies the design. Still, the sum has $2M - 1$ terms, almost twice as much as the sum obtained in the discrete filter bank case (M terms) [6]. However, it can be shown that the number of terms in the summation depends on the frequency value considered, in the sense that for a given Ω_0 with $-\pi/T < \Omega_0 \leq \pi/T$, (5) has only M nonzero terms [7], so that the linear system resulting from the perfect reconstruction conditions has a unique solution for every frequency value considered. This idea is exploited by the first synthesis method of the HFB presented here (see Section 3.1). The expression (6) is a compact form that reduces the number of terms to M, which simplifies even more the design. These simplifications result from the band limited approximation of the input signal and of the analog analysis filters.

3. SYNTHESIS METHODS

3.1. Direct synthesis method

Equations (10), (7) are written for each frequency value $\omega_k \in \{\omega_1, \omega_2, ..., \omega_{N_f}\}$, frequency set which covers the $-\pi < \omega \leq \pi$ interval:

$$\mathcal{T}_p^S(\omega_k) = \begin{cases} ce^{-j\omega_k d} & , p = 0 \\ 0 & , p \in \{1, ..., M-1\} \end{cases} \quad (11)$$

$k \in \{1, 2, ..., N_f\}$,
where:

$$\mathcal{T}_p^S(\omega_k) = \frac{1}{MT}\sum_{m=0}^{M-1} \mathcal{F}_m(\omega_k) H_m^S(j\Omega_k - j\frac{2\pi p}{MT}). \quad (12)$$

As explained in Section 2, equation (11) written for each $p \in \{0, 1, ..., M-1\}$ represents an M by M linear equations system. The unknown variables $\mathcal{F}_m(\omega_k)$ are the ideal frequency response values of the M synthesis filters, namely $F_m(e^{j\omega})$ in $\omega = \omega_k$. Solving each of the N_f linear equation systems for $k \in \{1, 2, ..., N_f\}$, N_f values for each of the synthesis functions $F_m, m \in \{0, ..., M-1\}$ are found on $-\pi < \omega \leq \pi$. Then the $F_m(e^{j\omega})$ functions fitting the $\mathcal{F}_m(\omega_k)$ values in a squared error sense are obtained for $m \in \{0, ..., M-1\}$ and $k \in \{1, 2, ..., N_f\}$.

3.2. Error minimization method

Assuming the transfer functions $\{H_m(j\Omega)\}$ are known, the synthesis transfer functions $\{F_m(e^{j\omega})\}$ must be worked out in order to have equations (10) satisfied. Let us consider the following composite criterion:

$$\begin{aligned}\phi &= \left|\int_{-\pi}^{\pi}\left(T_0^S(e^{j\omega}) - ce^{-j\omega d}\right)d\omega\right| + \\ &+ \sum_{p=1}^{M-1} c_p \left|\int_{-\pi}^{\pi} T_p^S(e^{j\omega})d\omega\right|.\end{aligned} \tag{13}$$

It is clear that minimizing the above criterion is equivalent to approximately satisfying equations (10). If $T_0^S(e^{j\omega})$ is close to $ce^{-j\omega d}$, and $T_p^S(e^{j\omega})$ are close to zero, the perfect reconstruction conditions (10) are closely satisfied. In this case the above criterion is close to zero as well. The functions $F_m(e^{j\omega})$ that minimize ϕ are then approximate solutions of (10). The values c_p are used to balance between distortion and aliasing errors. To ensure an efficient optimization, the initial values for the wanted filter parameters are obtained using the direct method. This minimization method leads to significant improvement of the performances as compared to the direct one (see next section).

4. FOUR CHANNEL FILTER BANK SIMULATION RESULTS

A four channel HFB has been designed. As discussed in Section 1 the analog filters chosen for the filter bank are very simple structures (2nd order LC filters) in order to fulfill technology and cost constraints since high frequency operation is requested. The transfer function of the chosen resonators is:

$$H_m(s) = \frac{\frac{\Omega_m}{Q_m}s}{s^2 + \frac{\Omega_m}{Q_m}s + \Omega_m^2}, m \in \{1, 2, ..., M-1\}. \tag{14}$$

This corresponds to a parallel LC structure with

$$\Omega_m = \frac{1}{\sqrt{L_m C_m}},\ Q_m = \frac{R_{pm}}{\Omega_m L_m}.$$

R_{pm} is the parasitic resistance of the inductance L_m. For $H_0(s)$, low pass filters are usually considered. Simple RC circuits are used:

$$H_0(s) = \frac{\Omega_0}{s + \Omega_0},\ \Omega_0 = \frac{1}{R_0 C_0}, \tag{15}$$

where Ω_0 is the cut-off frequency of the filter. 64 length FIR synthesis filters are considered. Optimization was performed using the standard Nelder-Mead Simplex algorithm, the variables of this minimization being the FIR filters coefficients [8]. $N_f = 128$ points were chosen for the discrete frequency domain, for the computation of the integrals in

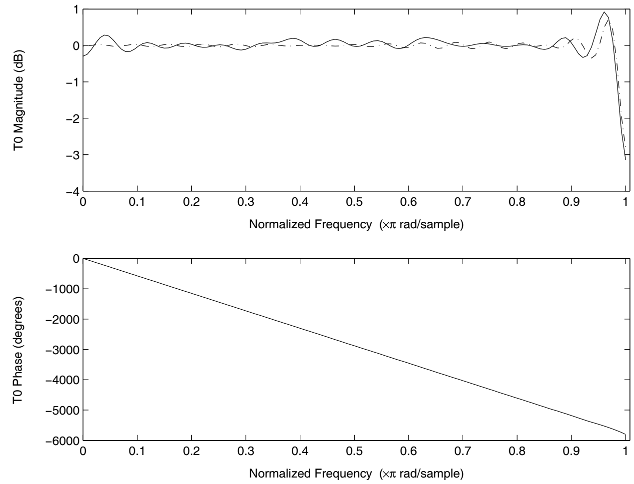

Fig. 2. Magnitude and phase of the distortion function

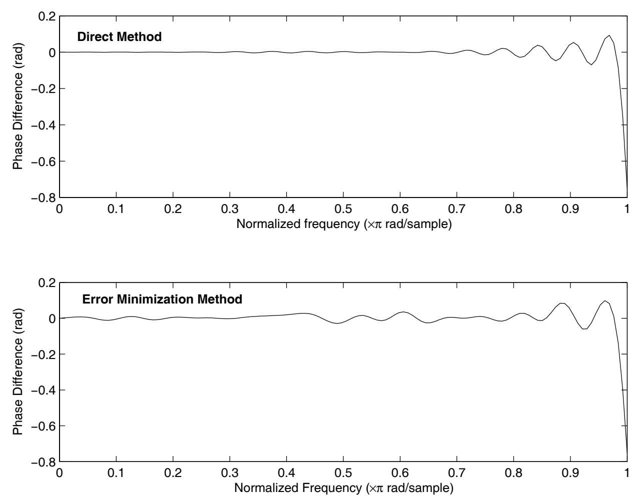

Fig. 3. Phase of the ratio $R = \frac{T_0(e^{j\omega})}{ce^{-j\omega d}}$

(13) and for the resolution of the linear equations systems obtained in the direct method (see Section 3.1). The constants were $c_1 = c_2 = c_3 = 100$ to emphasize the minimization of the aliasing errors rather than of the distortion errors. In Figures 2 and 4, dashed lines stand for the direct method and solid lines stand for the error minimization method. The magnitude and phase of the resulting distortion function T_0 is shown in Figure 2. Due to the choice of c_1, c_2, c_3, the minimization procedure doesn't improve the distortion function as can be observed in Figure 2 and Figure 3. On the other hand, the aliasing functions considerably decrease (to illustrate this, T_1, T_2, T_3 were represented in Figure 4). It may be quoted that the overall resolution of the global A/D conversion is highly dependent of aliasing functions. The Table 1 compares the two methods. Figure 3

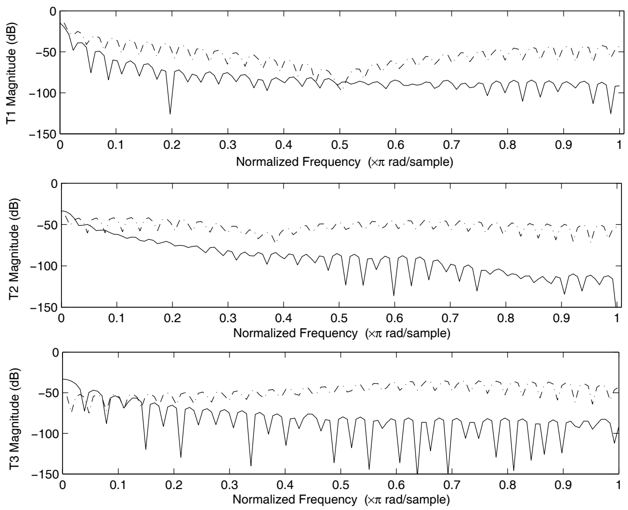

Fig. 4. Magnitude of the aliasing functions

Table 1. Method comparison

	$\|T_0\|$(dB) average	$\|T_1\|$(dB) average	$\|T_2\|$(dB) average	$\|T_3\|$(dB) average
Direct Method	0.09	-54.19	-52.16	-48.17
Error Minimization method	0.15	-82.36	-91.36	-84.69

shows the phase (*Phase difference*) of the ratio

$$R = \frac{T_0(e^{j\omega})}{ce^{-j\omega d}} \tag{16}$$

for the two methods, with $d = 32$. Ideally, for an HFB with no phase distortion, this phase difference must be either 0 (if d is the system delay) or constant (if d is an arbitrary constant). It can be noticed that the filter bank has an almost constant group delay of 32 samples. The average deviation from linear phase is 0.018 radians for the direct method and 0.02 radians for the error minimization method.
Another option for HFB synthesis would be to optimize the analysis filters in order to minimize an error criterion similar to the one in (13) [9]. However, this could raise some sensitivity problems resulting from the imperfections in the analog filters realization.

5. CONCLUSION

This paper describes two methods for obtaining the digital synthesis filters for a HFB, knowing the analog analysis filters. The first one is to solve the perfect reconstruction conditions (10) on a discrete set of frequencies. The second method is to minimize an objective function (13), containing both distortion and aliasing terms. Simulation results were presented for a four channel HFB. The analog analysis filters considered here are simple resonators. Even with these important implementation constraints, the performance of the resulting filter banks is quite satisfactory, the resulting average aliasing error in the second method being below -80 dB, and the average distortion being 0.15 dB.

6. REFERENCES

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