DYADIC-BASED FACTORIZATIONS FOR REGULAR PARAUNITARY FILTER BANKS

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ABSTRACT

M-channel paraunitary filter banks (PUFBs) can be designed and implemented using either degree-one or order-one dyadic-based factorization. This paper summarizes how regularity of a desired degree is *structurally* imposed on such factorizations for any number of channels $M \geq 2$ and any phase responses. The regularity conditions are found to be conveniently expressed in terms of recently reported M-channel lifting structures, which allow fast and reversible implementations. Design examples are presented and they outperform previously reported ones in transform-based image coding.

1. INTRODUCTION

A causal M-channel filter bank with polyphase matrix $\mathbf{E}(z)$ is said to be paraunitary (PU) if $\widetilde{\mathbf{E}}(z)\mathbf{E}(z) = \mathbf{I}$, where the $\widetilde{\ }$ operation stands for conjugate transpose (†) and time-reversal ($z \to z^{-1}$). Namely, $\mathbf{E}(z)$ is unitary on the unit circle $|z| = 1$. To implement $\mathbf{E}(z)$, the minimum number of delay elements required is referred to as the *McMillan degree*, or simply *degree*. Another important but distinct concept is that of the *order* of $\mathbf{E}(z)$, which refers to the highest power of z^{-1} appearing in $\mathbf{E}(z)$. An order-L (causal) PUFB can have degree ranging from L to ML.

Any PUFB $\mathbf{E}(z)$ of degree N always assumes the factorization $\mathbf{E}(z) = \prod_{m=N}^{1} \mathbf{V}_m(z)\mathbf{E}_0$ where $\mathbf{V}_m(z) = \mathbf{I} - \mathbf{v}_m\mathbf{v}_m^\dagger + z^{-1}\mathbf{v}_m\mathbf{v}_m^\dagger$ with $\|\mathbf{v}_m\| = 1$ is the *degree-one paraunitary building block*, and $\mathbf{E}_0$ is unitary [1]. It is called the *dyadic-based* structure as it involves the *dyadic form* $\mathbf{v}_m\mathbf{v}_m^\dagger$. Generalizing the above degree-constrained structure, Gao *et al.* have recently proposed a factorization given the order of the PUFB [2].

Regularity of a filter bank is equivalent to the number of vanishing moments of the M-band wavelets [3]. A PUFB is said to be K*-regular* or have K *degrees of regularity* if its analysis lowpass filter $H_0(z)$ has a zero of multiplicity K at the Mth roots of unity $e^{j2\pi m/M}$ for $m = 1, \dots, M-1$. This is equivalent to

$$\frac{d^\ell}{dz^\ell}\left\{\mathbf{E}(z^M)\begin{bmatrix}1 & z^{-1} & \dots & z^{-(M-1)}\end{bmatrix}^T\right\}\Big|_{z=1} = c_\ell\mathbf{e}_0 \quad (1)$$

where $c_\ell \neq 0$ for $\ell = 0, 1, \dots, K-1$, and $\mathbf{e}_0$ is the 0th unit vector of $\mathbb{R}^M$ [3, 4]. In many applications such as smooth signal interpolation, approximation, and data compression [5–9], regular filter banks are very desirable. It is hence essential to *structurally* guarantee regularity, for both design and implementation.

For the class of M-channel linear-phase PUFBs (a.k.a. GenLOT [10]) with M even, the imposition of up to two degrees of regularity on the lattice structure was discussed in [11]. The regularity conditions were expressed in terms of the rotation angles of the lattice components. On the other hand, for the most general class of M-channel regular PUFBs without the linear-phase constraint, the imposition of structural regularity has not been reported, except when $M = 2$ for which the regularity of degree one is guaranteed if all the rotation angles of the lattice structure sum up to $\pi/4$ [5]. However, the structural conditions for regularity of higher degree have not been reported in the literature. We aim to solve this problem in its most general form by considering a higher degree of regularity and an arbitrary number of channels $M \geq 2$ with unconstrained phase responses. The resulting design outperforms and spans a larger class than the regular GenLOT [11].

Section 2 reviews the complete parameterizations of PUFBs, namely, the *degree-one* and *order-one* factorizations. Then the *structural* conditions for imposing up to two degrees of regularity on these factorizations are summarized in Section 3. These conditions find a natural and convenient expression in the context of the M*-channel lifting factorization*, which allows efficient and reversible implementations of the filter bank. Section 4 presents two design examples along with their performance in a transform-based image codec. Concluding remarks are found in Section 5.

Bold-faced characters denote either a column vector or a matrix. The ith column of a matrix $\mathbf{w}_m$ is denoted as $\mathbf{w}_{m,i}$, while the ith element of an M-vector $\mathbf{v}_m$ as v_i^m. For $i = 0, \dots, M-1$, $\mathbf{e}_i$ is the ith unit vector of $\mathbb{C}^M$. $\mathbf{1}_M$ and $\mathbf{0}_M$ are the M-vectors of all ones and all zeros, respectively, and $\mathbf{I}_M$ denotes the $M \times M$ identity matrix. $\rho(\mathbf{A})$ denotes the rank of $\mathbf{A}$. An $m \times n$ constant matrix $\mathbf{A}$ is said to be unitary if $\mathbf{A}^\dagger\mathbf{A} = \mathbf{I}_n$.

2. PRELIMINARIES

2.1. Householder Factorization of Unitary Matrices

The M-dimensional Householder transformation $\mathbf{H}[\mathbf{p}]$ maps a given vector $\mathbf{x}$ in $\mathbb{C}^M$ to a mirror image $\mathbf{y}$ with respect to a plane E with unit normal $\mathbf{p}$, i.e. $\mathbf{y} = \mathbf{H}[\mathbf{p}]\mathbf{x}$. By simple geometry, it can be derived that $\mathbf{H}[\mathbf{p}] = \mathbf{I} - 2\mathbf{p}\mathbf{p}^\dagger$, with $\|\mathbf{p}\| = 1$. Given a unitary matrix $\mathbf{U}$, there exists a Householder transformation which aligns the 0th column of $\mathbf{U}$ with $\mathbf{e}_0$:

$$\mathbf{H}[\mathbf{p}_0]\,\mathbf{U} = \left[\begin{array}{c|c} e^{j\theta_0} & \mathbf{0}^T \\ \hline \mathbf{0} & \mathbf{U}_1 \end{array}\right], \quad \mathbf{U}_1^\dagger\mathbf{U}_1 = \mathbf{I}.$$

Such a process can be repeated on $\mathbf{U}_1$ and so on, to arrive at $\mathbf{H}[\mathbf{p}_{M-2}]\,\dots\,\mathbf{H}[\mathbf{p}_0]\,\mathbf{U} = \mathbf{D}$ where $\mathbf{D} = \text{diag}(e^{j\theta_0}, \dots, e^{j\theta_{M-1}})$, $\theta_m \in \mathbb{R}$, or equivalently $\mathbf{U} = \mathbf{H}[\mathbf{p}_0]\,\dots\,\mathbf{H}[\mathbf{p}_{M-2}]\,\mathbf{D}$.

2.2. Dyadic-Based Factorizations for Paraunitary Filter Banks

The order-one factorization [2] and the degree-one factorization [1, 2] provide a complete parameterization of PUFBs with or without length constraint. Both will be used in this paper and are summarized below.

Lemma 1 *The dyadic-based structure with parameter vector* $\mathbf{v}_m$

$$\mathbf{V}_m(z) = \mathbf{I} - \mathbf{v}_m\mathbf{v}_m^\dagger + z^{-1}\mathbf{v}_m\mathbf{v}_m^\dagger, \quad \|\mathbf{v}_m\| = 1 \tag{2}$$

is the ***degree-one paraunitary building block****: any degree-*N *paraunitary polyphase matrix* $\mathbf{E}(z)$ *can be factored as*

$$\mathbf{E}(z) = \mathbf{V}_N(z)\,\mathbf{V}_{N-1}(z)\ldots\mathbf{V}_1(z)\,\mathbf{E}_0 \tag{3}$$

where $\mathbf{E}_0$ *is unitary:* $\mathbf{E}_0^\dagger\mathbf{E}_0 = \mathbf{I}$*. This structure is the degree-one factorization and is complete for any given degree* N [1]. ∎

Lemma 2 *The dyadic-based structure with parameter matrix* $\mathbf{w}_m$

$$\mathbf{W}_m(z) = \mathbf{I} - \mathbf{w}_m\mathbf{w}_m^\dagger + z^{-1}\mathbf{w}_m\mathbf{w}_m^\dagger, \quad \mathbf{w}_m^\dagger\mathbf{w}_m = \mathbf{I}_{\gamma_m} \tag{4}$$

is the ***order-one paraunitary building block*** *for some integer* γ_m *with* $1 \le \gamma_m \le M$*. Any order-*L *paraunitary polyphase matrix* $\mathbf{E}(z)$ *can be factored as*

$$\mathbf{E}(z) = \mathbf{W}_L(z)\,\mathbf{W}_{L-1}(z)\ldots\mathbf{W}_1(z)\,\mathbf{E}_0 \tag{5}$$

for some $M \times M$ *unitary* $\mathbf{E}_0$ *and some integers* $\gamma_1, \ldots, \gamma_L$*. This structure is the order-one factorization of* $\mathbf{E}(z)$ *and is complete for any given order* L [2]. ∎

Having reviewed the complete structures for PUFBs, we are ready to present *structural* conditions for regularity.

3. DYADIC-BASED STRUCTURES WITH REGULARITY

3.1. One-Regular Dyadic-based Structures

A degree-0 M-channel PUFB with Type-I analysis polyphase matrix $\mathbf{E}(z) = \mathbf{E}_0$ is, by definition, 1-regular if and only if the 0th row of $\mathbf{E}_0$ has identical elements equal to $\frac{1}{\sqrt{M}}e^{j\phi}$, $\phi \in \mathbb{R}$. The following theorem summarizes the structural conditions for one degree of regularity [12].

Lemma 3 *A degree-*0 *and/or order-*0 M*-channel PUFB with Type-I polyphase matrix* $\mathbf{E}(z) = \mathbf{E}_0$ *is* 1*-regular if and only if* $\mathbf{E}_0^\dagger = \mathbf{H}[\mathbf{p}_0]\ldots\mathbf{H}[\mathbf{p}_{M-2}]\,\mathbf{D}^\dagger$ *is such that* $p_0^0 = \sqrt{\frac{\sqrt{M}-s}{2\sqrt{M}}}\,e^{j\eta}$ *and* $p_i^0 = \frac{-s\,e^{j\eta}}{\sqrt{2(M-s\sqrt{M})}}$*,* $i > 0$*, where* $s = \pm 1$ *and* $\eta \in \mathbb{R}$*. In this case,* $\mathbf{E}\left(z^M\right)\mathbf{1}_M = c_0\mathbf{e}_0$ *with* $c_0 = s\sqrt{M}e^{j\theta_0}$*.* ∎

Theorem 1 *A degree-*N *PUFB* (3) *or an order-*L *PUFB* (5) *is* 1*-regular if and only if* $\mathbf{E}_0$ *is* 1*-regular as in Lemma* 3*.* ∎

3.2. Two-Regular Dyadic-based Structures

Depending on the PUFB structures with or without length constraint, the following structural conditions for two degrees of regularity can be established [12].

Theorem 2 *A degree-*N *PUFB* (3) *is* 2*-regular if and only if*

1. $\mathbf{E}_0$ *is* 1*-regular as in Lemma* 3*, and*
2. *the unit-norm* $\mathbf{v}_m$ *of* $\mathbf{V}_m(z)$ *as in* (2) *satisfy*

$$sM^{3/2}\sum_{m=1}^{N} v_0^{m*}\check{\mathbf{v}}_m = -e^{-j\theta_0}\check{\mathbf{E}}_0\mathbf{b}_M \tag{6}$$

where $\mathbf{b}_M = \begin{bmatrix} 0 & 1 & \ldots & M-1 \end{bmatrix}^T$*,* $\mathbf{v}_m = \begin{bmatrix} v_0^m & \big| & \check{\mathbf{v}}_m^T \end{bmatrix}^T$*, and* $\mathbf{E}_0 = \begin{bmatrix} \frac{se^{j\theta_0}}{\sqrt{M}}\mathbf{1}_M & \big| & \check{\mathbf{E}}_0^T \end{bmatrix}^T$*.* ∎

Theorem 3 (Two-Regular PUFB with Length Constraint) *An order-*L *PUFB* (5) *is* 2*-regular if and only if*

1. $\mathbf{E}_0$ *is* 1*-regular as in Lemma* 3*, and*
2. *the unitary* $\mathbf{w}_m \triangleq \begin{bmatrix} \mathbf{w}_{m,1} & \mathbf{w}_{m,2} & \ldots & \mathbf{w}_{m,\gamma_m} \end{bmatrix}$ *of the order-one PU building blocks* $\mathbf{W}_m(z)$ *satisfy*

$$sM^{3/2}\sum_{m=1}^{L}\sum_{i=1}^{\gamma_m} w_0^{m,i*}\check{\mathbf{w}}_{m,i} = -e^{-j\theta_0}\check{\mathbf{E}}_0\mathbf{b}_M \tag{7}$$

where $\mathbf{w}_{m,i} = \begin{bmatrix} w_0^{m,i} & \big| & \check{\mathbf{w}}_{m,i}^T \end{bmatrix}^T$*.* ∎

Based on the above results, one can show the minimum McMillan degree required for two degrees of regularity to be one [12]. Note that this is consistent with the fact that the minimum order for a 2-regular PUFB is 1, and that the filter length is thus $2M$ [3] which is a stronger requirement. One should also note that, if the linear-phase property is imposed, this minimum length is increased to $3M$ [11].

Although not all choices of unit-norm vectors $\mathbf{v}_m$ satisfy (6) for the degree-one factorization, it is interesting to note that approximately half of the unit-norm vectors $\mathbf{v}_m$ in (6) can be arbitrarily chosen in imposing two degrees of regularity. Similar comments hold for the order-one factorization. Both are summarized below.

Theorem 4 (Two-Regular Feasibility) *Given a* 2*-regular degree-one factorization* (3) *and the corresponding* 2*-regular condition* (6)*, suppose that* $\mathcal{A}_k \triangleq \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ *has been given. Then there always exist unit-norm vectors* $\mathbf{v}_{k+1}, \ldots, \mathbf{v}_N$ *which, together with* $\mathbf{v}_i$ *in* $\mathcal{A}_k$*, satisfy* (6) *regardless of the choice of* $\mathcal{A}_k$*, for any* $k \le \left\lceil \frac{N}{2} \right\rceil - 1$*. Similarly, for a* 2*-regular order-one factorization* (5) *and the corresponding* 2*-regular condition* (7)*, suppose that* $\mathcal{B}_\ell \triangleq \{\mathbf{w}_1, \ldots, \mathbf{w}_\ell\}$ *has been given. Then there always exist unitary matrices* $\mathbf{w}_{\ell+1}, \ldots, \mathbf{w}_L$ *which, together with* $\mathbf{w}_i$ *in* $\mathcal{B}_\ell$*, satisfy* (7) *regardless of the choice of* $\mathcal{B}_\ell$*, for any* $\ell \le \left\lceil \frac{L}{2} \right\rceil - 1$*. It is understood that* $\mathcal{A}_k \equiv \emptyset$ *if* $N \le 2$*, and that* $\mathcal{B}_\ell \equiv \emptyset$ *if* $L \le 2$*.* ∎

3.3. Structural Regularity and Lifting Factorization

The recently reported M-channel lifting factorization allows efficient, reversible, and possibly multiplierless implementations of the given filter banks, even under finite precision and/or nonlinear liftings [13]. As is shown in [12], the M-channel lifting factorization provides a natural parameterization of the problem of regularity imposition—the important quantities $v_0^{m*}\check{\mathbf{v}}_m$ and $w_0^{m,i*}\check{\mathbf{w}}_{m,i}$ in (6) and (7) turn out to be the lifting multipliers of the M-channel lifting factorizations of $\mathbf{V}_m(z)$ and $\mathbf{W}_m(z)$, respectively. For example, $\mathbf{V}_m(z)$ can be lifting-factorized as shown in (8), with the *lifting multipliers* $\alpha_i^m = v_i^{m*}/v_r^{m*}$, $\beta_i^m = v_i^m v_r^{m*}$ for some $r \in \{0, 1, \ldots, M-1\}$ with $v_r^m \neq 0$, and $\underline{x}$ stands for $-x$. Based on this, the lifting factorization of $\mathbf{W}_m(z)$ can be obtained using a parallel form [12]. The lifting representation of the Householder matrix $\mathbf{H}[\mathbf{p}_m]$ is readily available by setting $\mathbf{v}_m = \mathbf{p}_m$ and $z = -1$ in (8), with the associated lifting multipliers $\sigma_i^m = p_i^{m*}/p_r^{m*}$ and $\rho_i^m = p_i^m p_r^{m*}$ for some $r \in \{0, \ldots, M-1\}$ with $p_r^m \neq 0$.

$$\mathbf{V}_m(z) = \begin{bmatrix} 1 & & & \alpha_0^m & & & \\ & \ddots & & \vdots & & & \\ & & 1 & \alpha_{r-1}^m & & & \\ & & & 1 & & & \\ & & & \alpha_{r+1}^m & 1 & & \\ & & & \vdots & & \ddots & \\ & & & \alpha_{M-1}^m & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ \underline{\beta_0^m} & \cdots & \underline{\beta_{r-1}^m} & z^{-1} & \underline{\beta_{r+1}^m} & \cdots & \underline{\beta_{M-1}^m} \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ \beta_0^m & \cdots & \beta_{r-1}^m & 1 & \beta_{r+1}^m & \cdots & \beta_{M-1}^m \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \underline{\alpha_0^m} & & & \\ & \ddots & & \vdots & & & \\ & & 1 & \underline{\alpha_{r-1}^m} & & & \\ & & & 1 & & & \\ & & & \underline{\alpha_{r+1}^m} & 1 & & \\ & & & \vdots & & \ddots & \\ & & & \underline{\alpha_{M-1}^m} & & & 1 \end{bmatrix} \quad (8)$$

It is convenient to define the vectors of lifting multipliers:

$$\boldsymbol{\alpha}^m \triangleq \begin{bmatrix} \alpha_1^m & \alpha_2^m & \ldots & \alpha_{M-1}^m \end{bmatrix}^T \quad \text{and} \quad (9)$$

$$\boldsymbol{\beta}^m \triangleq \begin{bmatrix} \beta_1^m & \beta_2^m & \ldots & \beta_{M-1}^m \end{bmatrix}^T. \quad (10)$$

One can show that they are related by $\boldsymbol{\beta}^m = \frac{\boldsymbol{\alpha}^{m*}}{1+\|\boldsymbol{\alpha}^m\|^2}$ or $\boldsymbol{\alpha}^m = |v_0^m|^{-2}\boldsymbol{\beta}^{m*}$, with $|v_0^m|^2 = \frac{1}{2}\left(1 \pm \sqrt{1-4\|\boldsymbol{\beta}^m\|^2}\right)$ or $|v_0^m|^2 = \left(1+\|\boldsymbol{\alpha}^m\|^2\right)^{-1}$ as a result of paraunitariness [12]. In this way, one has $0 \leq \|\boldsymbol{\beta}^m\| \leq 1/2$ and $\|\boldsymbol{\alpha}^m\|^2 = \frac{1}{|v_0^m|^2}\sum_{i=1}^{M-1}|v_i^m|^2 = \frac{1}{|v_0^m|^2} - 1 \geq 0$. As for $\mathbf{W}_m(z)$, one can similarly define lifting multipliers $\boldsymbol{\alpha}^{m,i}$ and $\boldsymbol{\beta}^{m,i}$ for $i = 1, \ldots, \gamma_m$. However, as dictated by the unitariness of the parameter matrix $\mathbf{w}_m$ of $\mathbf{W}_m(z)$, $\boldsymbol{\alpha}^{m,i}$ and $\boldsymbol{\beta}^{m,i}$ must also satisfy the "*obtuse*-angle" condition as described in [12].

3.3.1. *One-Regular Lifting Structure*

The one-regular condition can be expressed in terms of the lifting multipliers. Consider a PUFB in either the degree-one factorization (3) or the order-one factorization (5), with the unitary matrix $\mathbf{E}_0$ parameterized as $\mathbf{E}_0^\dagger = \mathbf{H}[\mathbf{p}_0] \ldots \mathbf{H}[\mathbf{p}_{M-2}]\,\mathbf{D}^\dagger$. For $i = 1, \ldots, M-1$, let σ_i^0 and ρ_i^0 be the lifting multipliers of $\mathbf{H}[\mathbf{p}_0]$ as defined above. Then one can show that the PUFB is 1-regular if and only if the lifting multipliers are such that $\sigma_i^0 = (1-s\sqrt{M})^{-1}$ and $\rho_i^0 = -s(2\sqrt{M})^{-1}$ for $i = 1, 2, \ldots, M-1$.

3.3.2. *Two-Regular Lifting Structures*

Similarly, the two-regular conditions can be expressed in terms of the vectors $\boldsymbol{\beta}^m$. The second condition for 2-regularity (6) is conveniently written as

$$\sum_{m=1}^{N} \boldsymbol{\beta}^m = -sM^{-3/2}e^{-j\theta_0}\breve{\mathbf{E}}_0\mathbf{b}_M \quad (11)$$

for the degree-one factorization, and (7) becomes

$$\sum_{m=1}^{L}\sum_{i=1}^{\gamma_m} \boldsymbol{\beta}^{m,i} = -sM^{-3/2}e^{-j\theta_0}\breve{\mathbf{E}}_0\mathbf{b}_M \quad (12)$$

for the order-one factorization. To summarize, consider a PUFB as in (5) or (3) and let the unitary matrix $\mathbf{E}_0$ be parameterized by the above one-regular lifting structure. Then, one can show that the PUFB is two-regular with or without length constraint if and only if the lifting multipliers satisfy (12) or (11), respectively.

It is in this way that the M-channel lifting factorization lends itself to a natural and convenient parameterization of the problem of regularity imposition.

Table 1: PUFBs Used in the Image Compression Experiment. LPvn=n-Regular PULP in [11]; PUvn=n-Regular PUFB in Sec.4.

	8×8 DCT	8×16 LOT	8×24 LPv1	8×24 LPv2	8×24 PUv1	8×24 PUv2
Reg. K	1	1	1	2	1	2
G (dB)	8.83	9.22	9.36	9.33	9.49	9.43
C_{stop}	3.09	.211	.133	.374	.088	.078

4. DESIGN EXAMPLES AND EVALUATIONS

The proposed regularity theory is implemented in this section. Based on the proposed structures for regular PUFBs, optimal designs can be obtained by unconstrained optimizations for some design criteria such as stopband energy C_{stop} and coding gain G [1], which are based to design the following one- and two-regular 8×24 PUFBs ($M = 8$ and order $L = 2$), denoted as PUv1 and PUv2, respectively. The order-one building blocks $\mathbf{W}_m(z)$ are such that $\rho(\mathbf{w}_1) = \rho(\mathbf{w}_2) = 4$. Fig. 1(b) and Fig. 2(b) plot the zeros of the lowpass filters $H_0(z)$ and verify that the resulting designs are indeed one- and two-regular, respectively. For PUv1, $G = 9.49$dB and $C_{\text{stop}} = 0.0876$; for PUv2, $G = 9.43$dB and $C_{\text{stop}} = 0.0780$.

We now evaluate the performance of PUv1 and PUv2 with a transform-based image codec, where each input image is block-transformed using PUvn, and the transform coefficients are quantized, zigzag scanned (runlength coding), and Huffman coded. Table 1 summarizes the properties of PUvn and some other PUFBs in the literature. Note that the proposed PUvn's are the most general designs and thus achieve the highest objective performance.

The coding results are tabulated in Table 2. As PUv1 and PUv2 are the most general PUFBs, they almost always result in higher PSNRs than their linear-phase counterparts (LPv1 and LPv2) [11], with an exception for the image *Goldhill* at 8:1 compression using one-regular PUFBs. It is observed in this experiment that the compressed images obtained by using PUv1 and PUv2 have fewer aliasing artifacts in the texture regions and that PUv1 and PUv2 result in smoother approximation (less blocky) in the smooth regions than those obtained by using LPv1 and LPv2, respectively.

5. CONCLUSION

We have presented the theory and structures of the most general PUFBs with up to two degrees of regularity. No constraint on the

Table 2: Compression PSNRs in dB for the chosen transforms.

Barbara	PSNR(dB)					
Comp. ratio	8×8 DCT	8×16 LOT	8×24 LPv1	8×24 LPv2	8×24 PUv1	8×24 PUv2
8:1	35.38	36.49	37.05	36.66	37.22	37.22
16:1	30.24	31.83	32.23	31.81	32.50	32.48
32:1	26.42	27.86	28.18	27.90	28.53	28.41
64:1	23.77	24.88	25.11	25.00	25.43	25.33

Lena	PSNR(dB)					
Comp. ratio	8×8 DCT	8×16 LOT	8×24 LPv1	8×24 LPv2	8×24 PUv1	8×24 PUv2
8:1	38.83	38.96	39.29	39.18	39.34	39.33
16:1	35.51	35.79	36.31	36.12	36.41	36.42
32:1	32.08	32.66	33.07	32.76	33.24	33.22
64:1	28.91	29.60	29.94	29.65	30.16	30.16

Goldhill	PSNR(dB)					
Comp. ratio	8×8 DCT	8×16 LOT	8×24 LPv1	8×24 LPv2	8×24 PUv1	8×24 PUv2
8:1	35.29	35.63	35.77	35.64	35.72	35.74
16:1	31.97	32.36	32.49	32.37	32.49	32.46
32:1	29.31	29.76	29.87	29.72	29.90	29.86
64:1	27.12	27.56	27.70	27.56	27.72	27.68

number of channels M is assumed, and the phase responses of the filters are not constrained. Both dyadic-based and M-channel lifting structures are considered and the corresponding regular structures are proposed, whereby the M-channel lifting structure lends itself to a natural and convenient parameterization of the problem of imposing regularity. The resulting PUFBs are guaranteed to be regular as the regularity conditions are *structurally* imposed. Regular PUFBs with or without length constraint are readily obtained. Design examples have been presented and are found to outperform previously published PUFBs in the literature.

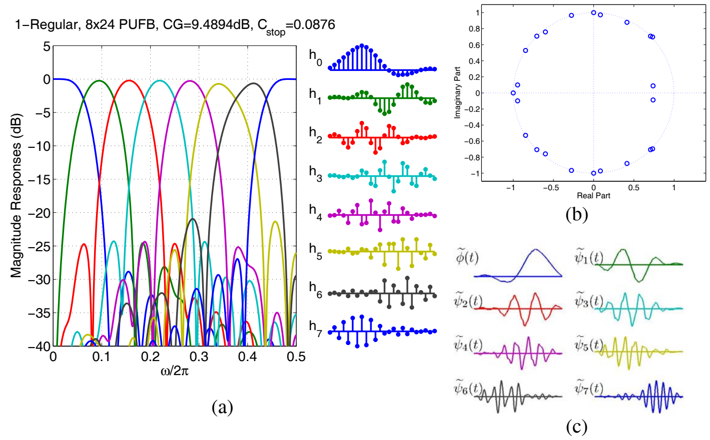

Figure 1: The one-regular 8×24 PUFB: (a) frequency response and basis functions, (b) zeros of $H_0(z)$, (c) the wavelet basis.

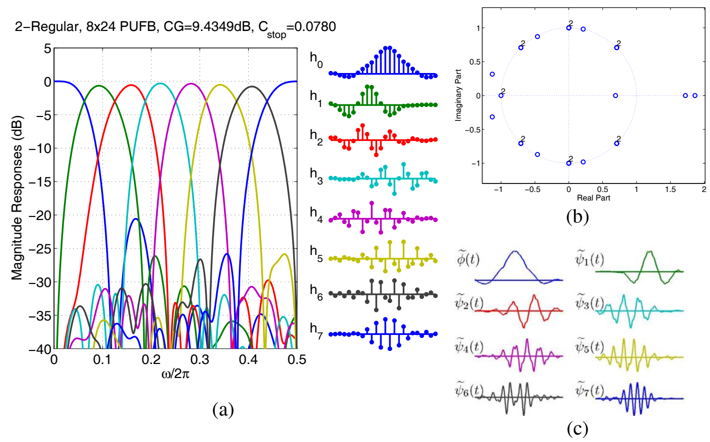

Figure 2: The two-regular 8×24 PUFB: (a) frequency response and basis functions, (b) zeros of $H_0(z)$, (c) the wavelet basis.

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