SHARPENED COMB DECIMATOR WITH IMPROVED MAGNITUDE RESPONSE

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ABSTRACT

A new structure for the realization of a comb decimation filter with a sharpened magnitude response is advanced. The proposed structure consists of two main sections: a comb section and a sharpening comb section with the latter section operating at a lower rate than the high input rate. Using a polyphase decomposition, the sub-filters of the first section can also be operated at this lower rate. The improved magnitude response has been obtained by using the filter sharpening approach of Kaiser and Hamming. The proposed filter has much less passband droop and better attenuation than the equivalent comb filter.

1. INTRODUCTION

A commonly used decimation filter is the cascaded-integrator-comb (CIC) filter, which consists of two main sections: an integrator and a differentiator section separated by a down-sampler [1]. The transfer function of the resulting decimation filter is given by

$$H(z) = \left[\frac{1}{M}\left(\frac{1-z^{-M}}{1-z^{-1}}\right)\right]^K, \tag{1}$$

where M is the decimation ratio, and K is the number of stages. The differentiator section operates at the lower data rate, while the integrator section works at the higher input data rate resulting in a higher chip area and higher power dissipation for this section. In order to resolve this problem, the non-recursive structure of Eq. (1) can be used [2]-[3]. As a reduction of the sampling rate at an early stage helps in the reduction of the power consumption, a polyphase decomposition is usually applied to realize the non-recursive section. More details on a comparison of the performances of the recursive and non-recursive implementations are given in [2].

The magnitude response of the above filter exhibits a linear-phase characteristic given by

$$\left|H(e^{j\omega})\right| = \left|\frac{\sin(\omega M/2)}{M\sin(\omega/2)}\right|^K. \tag{2}$$

The above characteristic has a large droop in the desired passband with the amount of droop being dependent upon the decimation factor M and the number K of sections of the cascade. Several schemes have been proposed to design comb filters with improved magnitude response [4]-[7].

The methods outlined in [4] and [6] use the filter sharpening technique which was originally introduced to improve both the passband and stop-band of a symmetric finite impulse response (FIR) filter by using multiple copies of the same filter. The main drawback of the structures [4] and [6] is that the sharpening carried out at the high input rate.

Recently, an alternate structure consisting of two main sections was proposed in which sharpening section operates at half of the input rate [7]. The main idea of this paper is to generalize further this approach with the aim of obtaining a structure that can operate at a lower sampling rate while achieving better performances than the original comb filter. In Section 2 we first reexamine the modified comb filter and in Section 3 we outline the frequency improvement of the proposed modified comb filter. Finally in Section 4 we outline the new efficient sharpening structure.

2. MODIFIED COMB FILTER

Let $M = M_1 M_2$. We can then rewrite Eq. (1) as

$$H(z) = \left[\frac{1}{M_2}\left(\frac{1-z^{-M_1M_2}}{1-z^{-M_1}}\right)\cdot\frac{1}{M_1}\left(\frac{1-z^{-M_1}}{1-z^{-1}}\right)\right]^K \tag{3}$$

or

$$H(z) = \left[H_1(z^{M_1})H_2(z)\right]^K, \tag{4}$$

where

$$H_1(z^{M_1}) = \frac{1}{M_2}\left(\frac{1-z^{-M_1M_2}}{1-z^{-M_1}}\right), \tag{5}$$

$$H_2(z) = \frac{1}{M_1}\left(\frac{1-z^{-M_1}}{1-z^{-1}}\right). \tag{6}$$

The corresponding magnitude responses are, respectively, given by

$$\left|H_1(e^{j\omega M_1})\right| = \left|\frac{1}{M_2}\left(\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega M_1}{2}}\right)\right|, \tag{7}$$

$$\left|H_2(e^{j\omega})\right| = \left|\frac{1}{M_1}\left(\frac{\sin\frac{\omega M_1}{2}}{\sin\frac{\omega}{2}}\right)\right|. \tag{8}$$

We define the modified comb filter $H_m(z)$ as

$$H_m(z) = \left[H_1(z^{M_1})\right]^{K_1}\left[H_2(z)\right]^{K_2}, \tag{9}$$

where $H_1(z^{M_1})$ and $H_2(z)$ are given by Eqs. (5) and (6), respectively, and K_i , i = 1, 2 are the corresponding stages of the filters.

Using Eq. (9) and the cascade equivalence property, we obtain a two-stage realization of the decimator with decimation factors M_1 and M_2, where the comb filter H_2 operates at the high input rate and the comb filter H_1 operates at the lower rate which is M_1 times less than the high input rate, as shown in Fig. 1.

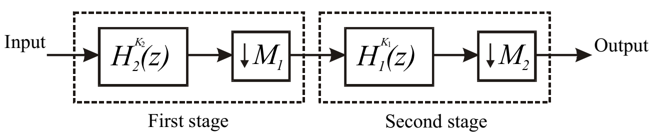


Fig.1. Modified comb decimator

Our aims are thus:

- To improve the magnitude characteristic of the modified comb filter of Eq. (9).
- To eliminate filtering at the high input rate in the first stage.

The first problem is revised in the next section while the Section 4 considers the second problem.

3. FREQUENCY RESPONSE IMPROVEMENT OF THE MODIFIED COMB FILTER

The procedure is based on realizing the comb filter H_1 in the second stage of Fig. 1 using the filter sharpening approach based on the Amplitude Change Function (ACF) technique [8]. An ACF is a polynomial relationship of the form $H_0 = f(H)$ between the amplitude responses of the overall and the prototype filters, H_0 and H, respectively. The improvement in the passband, near H = 1, or in the stopband, near H = 0, depends on the order of tangency of the ACF m and n at H = 1 or at H = 0, respectively. The expression proposed by Kaiser and Hamming for an ACF giving mth and nth order tangencies at H = 1 and H = 0, respectively, is given by

$$\begin{aligned} H_0 &= H^{n+1}\sum_{k=0}^{m}\frac{(n+k)!}{n!\,k!}(1-H)^k \\ &= H^{n+1}\sum_{k=0}^{m}C(n+k,k)(1-H)^k \end{aligned}, \tag{10}$$

where $C(n+k,k)$ is the binomial coefficient. The values of ACF for some typical values of m and n are given in Table I.

TABLE I: ACF polynomials for m = 1,…,3 and n = 1,…,3

m	n	H
1	1	$3H^2$-$2H^3$
1	2	$4H^3$-$3H^4$
1	3	$5H^4$-$4H^5$
2	1	$3H^4$-$8H^3$+$6H^2$
2	2	$6H^5$-$15H^4$+$10H^3$
2	3	$10H^6$-$24H^5$+$15H^4$
3	1	-$4H^5$+$15H^4$-$20H^3$+$10H^2$
3	2	-$10H^6$+$36H^5$-$45H^4$+$20H^3$
3	3	-$20H^7$+$70H^6$-$84H^5$+$35H^4$

In [4] is considered sharpening using $m = n = 1$, while in [6] are also considered cases for $m = n = 2$ and $m = n = 3$. In both cases the sharpening is achieved on the comb filter of the length M so that the sharpening is at the high input rate. We propose here to apply the sharpening only to the comb filter H_1 at the second stage of the Fig.1.

Using the first polynomial in the Table 1, $m = n = 1$, and Eqs. (7)-(8) we have the following magnitude responses for the original sharpening filter H_{sh} and the proposed sharpening filter $H_{sh.m}$.

$$\left|H_{sh}(e^{j\omega})\right| = \left|3\left(\frac{1}{M}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega}{2}}\right)^{2K} - 2\left(\frac{1}{M}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega}{2}}\right)^{3K}\right|. \tag{11}$$

$$\left|H_{sh,m}(e^{j\omega})\right| = \left|\left\{3\left(\frac{1}{M_2}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega M_1}{2}}\right)^{2K_1} - 2\left(\frac{1}{M_2}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega M_1}{2}}\right)^{3K_1}\right\}\left\{\frac{1}{M_1}\frac{\sin\frac{\omega M_1}{2}}{\sin\frac{\omega}{2}}\right\}^{K_2}\right|. \tag{12}$$

The corresponding magnitude responses for M = 16, K = 2, K_1 = 2, and K_2 = 4 are shown in Fig. 2(a).

Similarly for the second polynomial in Table 1, m = 1, n = 2 we have

$$\left|H_{sh}(e^{j\omega})\right| = \left| 4\left(\frac{1}{M}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega}{2}}\right)^{3K} - 3\left(\frac{1}{M}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega}{2}}\right)^{4K} \right|. \tag{13}$$

$$\left|H_{sh,m}(e^{j\omega})\right| = \left| \left\{ 4\left(\frac{1}{M_2}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega M_1}{2}}\right)^{3K_1} - 3\left(\frac{1}{M_2}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega M_1}{2}}\right)^{43K_1} \right\} \left\{\frac{1}{M_1}\frac{\sin\frac{\omega M_1}{2}}{\sin\frac{\omega}{2}}\right\}^{K_2} \right|. \tag{14}$$

For the third polynomial, with m=1, n=3 we obtain

$$\left|H_{sh}(e^{j\omega})\right| = \left| 5\left(\frac{1}{M}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega}{2}}\right)^{4K} - 4\left(\frac{1}{M}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega}{2}}\right)^{5K} \right|. \tag{15}$$

$$\left|H_{sh,m}(e^{j\omega})\right| = \left| \left\{ 5\left(\frac{1}{M_2}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega M_1}{2}}\right)^{4K_1} - 4\left(\frac{1}{M_2}\frac{\sin\frac{\omega M}{2}}{\sin\frac{\omega M_1}{2}}\right)^{53K_1} \right\} \left\{\frac{1}{M_1}\frac{\sin\frac{\omega M_1}{2}}{\sin\frac{\omega}{2}}\right\}^{K_2} \right|. \tag{16}$$

The magnitude plots for the second and third polynomial are shown Figs. 2 (b) and 2(c). Figure 2 demonstrate that we can apply the sharpening only on the filter H_1 and still obtain a magnitude response that is very close to the case when sharpening is applied on the whole comb filter.

4. THE PROPOSED STRUCTURE

According to the results demonstrated in Section 3 we apply the sharpening in the second stage of the modified comb filter. The proposed structure is shown in Figure 3, where $f(H_1^{K_1}(z))$ denotes the sharpening of the comb filter H_1. Depending of the polynomial used for the sharpening, the sharpened section can be further modified moving the comb section after down-sampling by M_2, as is shown in [4] for m=n=1, and in [6] for n=m=2.

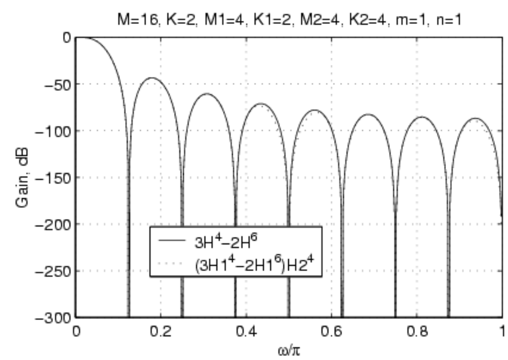
We can notice that in the Structure in Figure 3 the filter H_2 is still working at the high input rate. To resolve this problem this comb filter can be realized as CIC filter or in non-recursive form. Using the proposal from [3] we can obtain a polyphase decomposition of the first stage. Applying the cascade equivalence the down-sampler can be placed before filtering, and as a result, the polyphase filters in the first section are moved to a lower rate, which is M_1 times lower than the input rate.

Example 1:

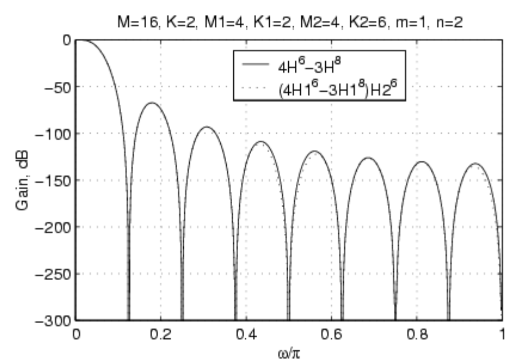
We consider M=32 and choose M_1=4, K_2=8 and M_2=8, K_1=2. In this case, we have in the first stage a cascade of eight length-4 comb filters and in the second stage we have sharpening of a cascade of two length-8 comb filters. In the stopband this cascade is equivalent to a cascade of $2K_1$ length-8 comb filters. An approximate number of stages of an equivalent length-M comb filter is

$$N_e = \left\lceil \frac{M_1 K_2 + 2K_1 M_2}{M} \right\rceil = 2. \tag{17}$$

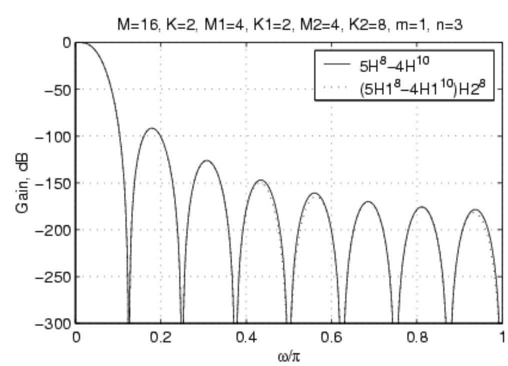
where $\lceil\ \rceil$ means the closest integer.



(a) The first polynomial



(b) The second polynomial



(c) Third polynomial

Fig.2. Magnitude responses

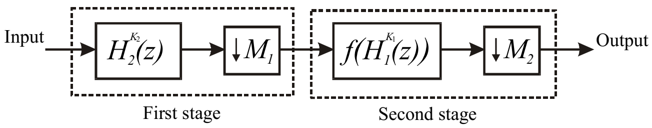
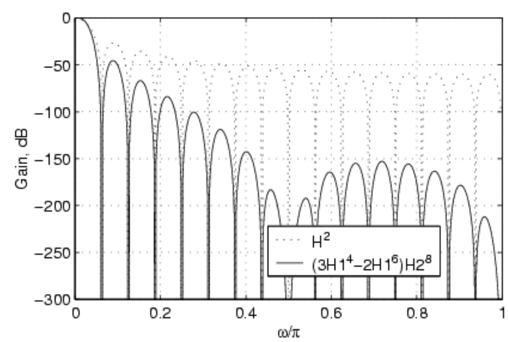
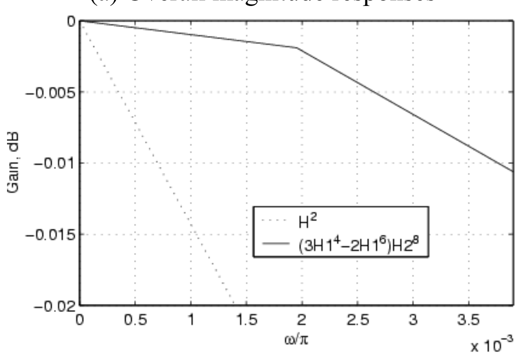


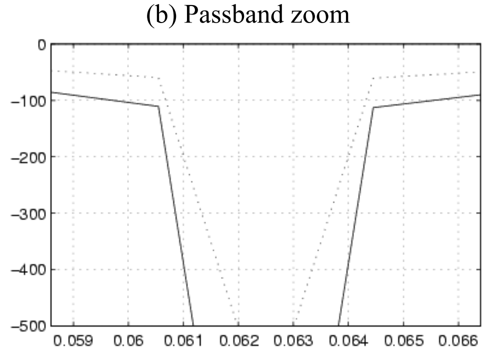
Fig. 3. The proposed structure



(a) Overall magnitude responses



(b) Passband zoom



(c) A detailed view around the first null

Fig.4. Magnitude responses

Figure 4 shows the magnitude responses of the proposed filter and the equivalent comb filter. It should be noted that the proposed filter has a very small passband droop and a high attenuation near the first null.

5. CONCLUSIONS

A new efficient structure for a sharpened comb factor-of-*M* decimation filter is proposed. The structure consists of two main sections: a cascade of comb filters followed by down-sampling with a factor M_1, and a sharpened comb filter followed by down-sampling with a factor M_2, where $M = M_1 M_2$. This arrangement allows the sharpening operation to move to a lower rate which is M_1 times less than the high input rate and to operate on a comb filter of length M_2. The sharpening operation is performed using the method proposed by Kaiser an Hamming. Using the cascade equivalence, for a given polynomial, further simplification of the second stage can be achieved. The first stage can be implemented as a CIC filter or in a non-recursive form. The first section can also be operated at M_1 times lower rate by realizing it in a polyphase form. The alias rejection of the new structure does not depend on M_1 for a given *M*. However, the passband droop increases with an increase in M_1. The new decimation filter has a much smaller passband droop and a much better alias rejection than the conventional comb filter.

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