MODELING AND ESTIMATION OF A CLASS OF DYNAMIC MULTISCALE SYSTEM SUBJECT TO COLORED NOISE

Peiling Cui*, Quan Pan, Hongcai Zhang and Junhong Li
Department of Automatic Control, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China,
*E-Mail: plcui@sina.com

ABSTRACT

In this paper, modeling and estimation of a class of dynamic multiscale system subject to colored state equation noise and measurement equation noise is proposed. The state equation is whitened firstly and then the measurement equation. The state space projection equation is used to link the scales, then a new system model is built. The new model is in a form suitable for the application of the Kalman filter equations. Haar-wavelet-based model and estimation algorithm are given. Monte Carlo simulation results demonstrate that the proposed algorithm is effective and powerful in this kind of multiscale estimation problem.

1. INTRODUCTION

Many advanced systems are mostly observed by several sensors independently at different scales. The resolution and sampling frequencies of the sensors are supposed to decrease from sensor 1 to sensor J, the real state at each scale is $x_j, j=1,2,\ldots,J$, respectively. The state equation is described by a partial differential equation at the finest scale. An important practical problem in the above systems is to find a state estimator given the observations. This problem has been studied in recent years due to the numerous applications associated with it [1]-[10].

In [9], an algorithm for optimal and dynamic multiresolutional distributed filtering is derived. The wavelet transform is utilized as a bridge linking the signals at different resolution levels. In [10], an optimal estimation of a class of dynamic multiscale systems is discussed. The sampling frequencies of the sensors are supposed to decrease by a factor of two. That paper introduces the state space projection equation, and fuses the information at all scales by the measurement equation augmentation.

In aforementioned approaches, they all assumed that measurement equation noise is white. In practice, due to the scintillation of the target, the measurement noise may not be white. Typically, in many radar systems, the measurement frequency is high enough so that the correlation of the successive samples of the noise cannot be ignored without degrading the tracking performance. Very little work has been proposed for solving the estimation problem of dynamic multiscale system subject to colored measurement noise. It is the main focus of this paper.

The state equation is whitened firstly and then the measurement equation. The colored state noise vector is augmented in the system state variables, and a new measurement is introduced at each scale to decorrelate the colored measurement noise. The state space projection equation is used to link the scales, and then a new system model is built. This model is in a form suitable for the application of the standard Kalman filter equations [11]-[12], in which the process and the measurement are presumed white. Haar-wavelet-based model and estimation algorithm are given. Experimental results are reported to demonstrate the effectiveness of the new algorithm.

2. MODELING AND ESTIMATION ALGORITHM BASED ON HAAR WAVELET

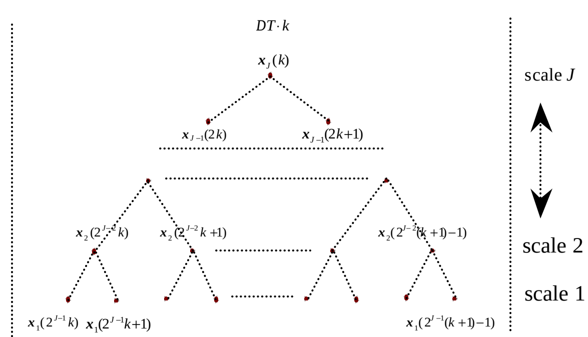


Fig.1 Tree structure of the dynamic multiscale system state nodes at time interval $k\Delta T$

For convenience, let the sampling rate decrease from sensor 1 to sensor J by a factor of two. Obviously, sensor 1 corresponds to the finest scale. The state at all scales in time interval ΔT is called a state block, and the measurement a data block. In every ΔT, the state estimation must be updated when a new data block is available. We hope the approximation of any node at any scale is accomplished in time interval ΔT, not using the state nodes outside of it. We choose the Haar wavelet [13]. This choice is motivated by the particularly simple realization of the Haar wavelet transform in our multiscale framework by using a bintree structure. Haar wavelet is the simplest and most widely used one with low-pass filter $\left[\sqrt{2}/2,\ \sqrt{2}/2\right]$.

For clarity, we unify the notations of state nodes firstly. Fig.1 shows the system bintree structure at time interval $k\Delta T$. $x_J(k)$ is denoted as the state of scale J, $x_{J-1}(2k)$ and $x_{J-1}(2k+1)$ the states at scale $J-1$. Analogically, at scale j there are 2^{J-j} state nodes, which are denoted as $x_j(2^{J-j}k)$, $x_j(2^{J-j}k+1)$ … $x_j\left(2^{J-j}(k+1)-1\right)$. Assuming the multiscale system state structure satisfies the dyadic structure of Haar wavelet, the node $x_j(2^{J-j}k+m_j)$ can be expressed with the finest scale nodes as follows

$$x_j(2^{J-j}k+m_i)=\left(\sqrt{2}/2\right)^{j-1}\sum_{i=0}^{2^{j-1}-1}x_1(2^{J-1}k+2^{j-1}m_j+i)$$

where $m_j=0,1,\dots,2^{J-j}-1$.

2.1 State equation

For simplicity, we assume the system to be time-invariant. The discrete state transition equation of the finest scale at time interval $(k+1)\Delta T$ is

$$x_1(2^{J-1}(k+1))=Ax_1(2^{J-1}(k+1)-1)+B\eta(2^{J-1}(k+1)-1)$$

where $x_1(\bullet)\in R^{N_x}$, state transition matrix $A\in R^{N_x\times N_x}$, noise stimulus matrix $B\in R^{N_x\times u}$, the dimension of colored noise $\eta(\bullet)$ is u, and

$$\eta(2^{J-1}(k+1))=\Phi\eta(2^{J-1}(k+1)-1)+w(2^{J-1}(k+1)-1)$$

where $w(\bullet)\in R^u$ is a Gaussian white process with covariance q.

Define a N_x+u state vector such that it includes the original system state variables and u elements of the colored measurement noise vector, then

$$\begin{bmatrix}x_1(2^{J-1}(k+1))\\ \hline \eta(2^{J-1}(k+1))\end{bmatrix}=\left[\begin{array}{c|c}A&B\\ \hline O&\Phi\end{array}\right]\begin{bmatrix}x_1(2^{J-1}(k+1)-1)\\ \hline \eta(2^{J-1}(k+1)-1)\end{bmatrix}+\begin{bmatrix}O\\ \hline w(2^{J-1}(k+1)-1)\end{bmatrix}$$

Denoting

$$x_1^a(2^{J-1}(k+1))=\begin{bmatrix}x_1(2^{J-1}(k+1))\\ \hline \eta(2^{J-1}(k+1))\end{bmatrix},\ A^a=\left[\begin{array}{c|c}A&B\\ \hline O&\Phi\end{array}\right],$$

$$w^a(2^{J-1}(k+1))=\begin{bmatrix}O\\ \hline w(2^{J-1}(k+1))\end{bmatrix}$$

where $x_1^a(\bullet)\in R^{N_x+u}$, $A^a\in R^{(N_x+u)\times(N_x+u)}$, $w^a(\bullet)\in R^{N_x+u}$. The system state equation by augmenting the state is

$$x_1^a(2^{J-1}(k+1))=A^ax_1^a(2^{J-1}(k+1)-1)+w^a(2^{J-1}(k+1)-1) \tag{1}$$

and $Cov\left(w^a(2^{J-1}(k+1)-1)\right)=q^a=\left[\begin{array}{c|c}O&O\\ \hline O&q\end{array}\right]$.

From Eq. (1), we know

$$\begin{aligned}&x_1^a(2^{J-1}(k+1)+1)\\&=(A^a)^2x_1^a(2^{J-1}(k+1)-1)+A^aw^a(2^{J-1}(k+1)-1)+w^a(2^{J-1}(k+1))\end{aligned}$$

$$\vdots$$

$$x_1^a(2^{J-1}(k+1)+m_1)=(A^a)^{m_1+1}\bullet x_1^a(2^{J-1}(k+1)-1)+\sum_{i=-2}^{m_1-2}(A^a)^{m_1-i-1}w^a(2^{J-1}(k+1)+i)$$

where $m_1=0,1,\dots,2^{J-1}-1$.

Letting

$$\overline{x}(k)=col(x_1^a(2^{J-1}k),x_1^a(2^{J-1}k+1),\cdots,x_1^a(2^{J-1}(k+1)-1))$$

$$\overline{w}(k)=col\left(w^a(2^{J-1}(k+1)-1),w^a(2^{J-1}(k+1)),\dots,w^a(2^{J-1}(k+2)-2)\right)$$

$$\overline{A}(m_1)=(A^a)^{m_1+1},\ \overline{B}(m_1)=[(A^a)^{m_1},(A^a)^{m_1-1},\dots,I,O,\dots,O]$$

where $\overline{x}(k)\in R^{2^{J-1}(N_x+u)\times 1}$, col denotes arranging the data in the bracket into column vector. $\overline{w}(k)\in R^{2^{J-1}(N_x+u)\times 1}$, $\overline{A}(m_1)\in R^{(N_x+u)\times(N_x+u)}$, $\overline{B}(m_1)\in R^{(N_x+u)\times 2^{J-1}(N_x+u)}$ is with zero elements on the last $(2^{J-1}-m_1-1)\times(N_x+u)$ columns.

Letting

$$\overline{A}=\begin{bmatrix}O&\cdots&O&\overline{A}(0)\\O&\cdots&O&\overline{A}(1)\\ \vdots&\ddots&\vdots&\vdots\\O&\cdots&O&\overline{A}(2^{J-1}-1)\end{bmatrix},\quad \overline{B}=\begin{bmatrix}\overline{B}(0)\\ \overline{B}(1)\\ \vdots\\ \overline{B}(2^{J-1}-1)\end{bmatrix}$$

where $\overline{A}\in R^{2^{J-1}(N_x+u)\times 2^{J-1}(N_x+u)}$, $\overline{B}\in R^{2^{J-1}(N_x+u)\times 2^{J-1}(N_x+u)}$. Then we have

$$\overline{x}(k+1)=\overline{A}\,\overline{x}(k)+\overline{B}\,\overline{w}(k) \tag{2}$$

2.2 Measurement equation

From the state space projection equation [10], we have

$$x_j=P_jx_1$$

where P_j is the state space projection operator, discrete measurement equation of node $x_j(2^{J-j}k+m_j)$ of scale j at time interval $k\Delta T$ is

$$\Delta_j(2^{J-j}k+m_j)=D_jx_j(2^{J-j}k+m_j)+\xi_j(2^{J-j}k+m_j),\ j=1,2,\cdots,J \tag{3}$$

where the dimension of $\Delta_j(\bullet)$ is N_z, $\xi_j(\bullet)$ is colored noise, and

$$\xi_j(2^{J-j}k+m_j)=\psi_j\xi_j(2^{J-j}k+m_j-1)+e_j(2^{J-j}k+m_j-1)$$

where $e_j(\bullet)$ is a Gaussian white process with covariance R_{ej}, and $e_j(\bullet)$ and $w(\bullet)$ are mutually independent. Then Eq.(3) becomes

$$\Delta_j(2^{J-j}k+m_j)=\left[\begin{array}{c|c}D_jP_j&O\end{array}\right]\begin{bmatrix}x_1(2^{J-j}k+m_j)\\ \hline \eta(2^{J-j}k+m_j)\end{bmatrix}+\xi_j(2^{J-j}k+m_j)$$

where $O\in R^{N_z\times u}$ is zero element matrix. Denoting $D_j^a=\left[\begin{array}{c|c}D_jP_j&O\end{array}\right]$, then

$$\Delta_j(2^{J-j}k+m_j)=D_j^ax_1^a(2^{J-j}k+m_j)+\xi_j(2^{J-j}k+m_j)$$

To decorrelate the colored measurement noise $\xi_j(\bullet)$, we introduce the pseudo measurement $z_j(2^{J-j}k+m_j)$, and

$$\begin{aligned}z_j(2^{J-j}k+m_j)&=\Delta_j(2^{J-j}k+m_j)-\psi_j\Delta_j(2^{J-j}k+m_j-1)\\&=[D_j^aA^a-\psi_jD_j^a]x_1^a(2^{J-j}k+m_j-1)\\&\quad+[D_j^aw^a(2^{J-j}k+m_j-1)+e_j(2^{J-j}k+m_j-1)]\end{aligned}$$

From the special structure of D_j^a and $w^a(\bullet)$, we know that $D_j^aw^a(2^{J-j}k+m_j-1)=0$. Then

$$z_j(2^{J-j}k+m_j)=[D_j^a A^a-\gamma_j D_j^a]x_1^a(2^{J-j}k+m_j-1)+e_j(2^{J-j}k+m_j-1)$$

Denoting

$$C_j=D_j^a A^a-\gamma_j D_j^a$$

$$v_j(2^{J-j}k+m_j)=e_j(2^{J-j}k+m_j-1)$$

and

$$Cov\left(v_j(2^{J-j}k+m_j)\right)=R_{ej}$$

then we have

$$\begin{aligned} &z_j(2^{J-j}k+m_j)\\ &=C_j x_1^a(2^{J-j}k+m_j-1)+v_j(2^{J-j}k+m_j),\ j=1,2,\dots,J \end{aligned} \qquad (4)$$

Denoting

$$\Theta_j(m_j)=\Big[\underbrace{0\times S,\dots,0\times S}_{m_j 2^{j-1}}\ \ \underbrace{S,\dots,S}_{2^{j-1}}\ \ \underbrace{0\times S,\dots,0\times S}_{(2^{J-j}-m_j-1)2^{j-1}}\Big]$$

where S is $(N_x+u)\times(N_x+u)$ identity matrix, $\Theta_j(m_j)\in R^{(N_x+u)\times 2^{J-1}(N_x+u)}$.

Denoting

$$\bar{z}_j(k)=col\left(z_j(2^{J-j}k+1),z_j(2^{J-j}k+2),\dots,z_j(2^{J-j}(k+1))\right)$$

$$\bar{C}_j=\begin{bmatrix} C_j\Theta_j(0)\\ C_j\Theta_j(1)\\ \vdots\\ C_j\Theta_j(2^{J-j}-1)\end{bmatrix},\quad \bar{v}_j(k)=\begin{bmatrix} v_j(2^{J-j}k+1)\\ v_j(2^{J-j}k+2)\\ \vdots\\ v_j(2^{J-j}(k+1))\end{bmatrix}$$

the covariance of $\bar{v}_j(k)$ is

$$\bar{R}_j(k)=diag[R_{ej},R_{ej},\dots,R_{ej}]$$

then

$$\bar{z}_j(k)=\bar{C}_j\bar{x}(k)+\bar{v}_j(k)$$

Denoting

$$\bar{\boldsymbol{z}}(k)=col(\bar{\boldsymbol{z}}_J(k),\bar{\boldsymbol{z}}_{J-1}(k),\dots,\bar{\boldsymbol{z}}_1(k))$$

$$\bar{C}=col\left(\bar{C}_J,\bar{C}_{J-1},\dots,\bar{C}_1\right)$$

$$\bar{\boldsymbol{v}}(k)=col(\bar{\boldsymbol{v}}_J(k),\bar{\boldsymbol{v}}_{J-1}(k),\dots,\bar{\boldsymbol{v}}_1(k))$$

then we have

$$\bar{z}(k)=\bar{C}\bar{x}(k)+\bar{v}(k) \qquad (5)$$

the covariance of $\bar{\boldsymbol{v}}(k)$ is

$$\bar{R}(k)=diag[\bar{R}_J(k),\bar{R}_{J-1}(k),\dots,\bar{R}_1(k)]$$

2.3 Kalman filter

Eqs. (2) and (5) can be written together

$$\begin{cases}\bar{\boldsymbol{x}}(k+1)=\bar{A}(k)\bar{\boldsymbol{x}}(k)+\bar{B}(k)\bar{\boldsymbol{w}}(k)\\ \bar{\boldsymbol{z}}(k)=\bar{C}(k)\bar{\boldsymbol{x}}(k)+\bar{\boldsymbol{v}}(k)\end{cases} \qquad (6)$$

where $\bar{\boldsymbol{w}}(k)$ and $\bar{\boldsymbol{v}}(k)$ are white noise. Assuming that the filter is stable, the Linear Minimum Mean-square Error (LMMSE) $\hat{\bar{x}}(k)$ of state $\bar{x}(k)$ can be obtained by performing Kalman filtering. In the following, the optimal estimation of the state node at the finest scale is given.

Denoting

$$\mathrm{T}_j(m_j)=\Big[\underbrace{0\times\Lambda,\dots,0\times\Lambda}_{m_j 2^{j-1}}\ \ \underbrace{\Lambda,\dots,\Lambda}_{2^{j-1}}\ \ \underbrace{0\times\Lambda,\dots,0\times\Lambda}_{(2^{J-j}-m_j-1)2^{j-1}}\Big]$$

where $\Lambda=[\mathrm{I}\ \ \mathrm{O}]$, I is $N_x\times N_x$ identity matrix, O is $N_x\times u$ zero matrix. $\mathrm{T}_j(m_j)$ is $N_x\times 2^{J-1}(N_x+u)$ matrix, then the LMMSE estimation of state node $x_j(2^{J-j}k+m_j)$ is $\mathrm{T}_j(m_j)\cdot\hat{\bar{x}}(k)$.

2.4 Filtering outputs at each scale

Filtering outputs at each scale can be obtained according to the theorem 1 shown below. The detailed proof can be referred to reference [10].

Theorem 1 Suppose $\hat{\bar{\mathbf{x}}}(k)$ is the LMMSE of $\bar{\mathbf{x}}(k)$, then the LMMSE of node $\mathbf{x}_j(2^{J-j}k+m_j)$ is $\left(\sqrt{2}/2\right)^{j-1}\bullet\mathrm{T}_j(m_j)\bullet\hat{\bar{x}}(k)$.

3 SIMULATION RESULTS

For verifying the validity of our algorithm, consider the following constant-velocity dynamic system with position-only measurements at two scales.

$$\begin{cases}x_1(2k+2)=Ax_1(2k+1)+Bh(2k+1)\\ z_j(2^{2-j}k+2^{2-j}-1)=D_j x_j(2^{2-j}k+2^{2-j}-1)+x_j(2^{2-j}k+2^{2-j}-1),\ j=1,2\end{cases}$$

where $h(2k+1)$ and $x_j(2^{2-j}k+2^{2-j}-1)$ are colored noise, and

$$h(2k+1)=\Phi h(2k)+w(2k)$$

$$x_j(2^{2-j}k+2^{2-j})=\gamma_j x_j(2^{2-j}k+2^{2-j}-1)+e_j(2^{2-j}k+2^{2-j}-1)$$

$w(\bullet)$ and $e_j(\bullet)$ are Gaussian white noises with zero mean, and

$$\begin{cases}E(w(k)w(l)^T)=q\delta_{kl},\ E(e_j(k)e_j(l)^T)=r_j\delta_{kl}\\ E(e_1(k)e_2(l)^T)=0,\ E(w(k)e_j(l)^T)=0\end{cases}$$

Letting

$$A=\begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix},\ B=\begin{bmatrix}\tfrac{1}{2}T^2 & T\end{bmatrix}^T,\ D_1=D_2=\begin{bmatrix}1 & 0\end{bmatrix},$$

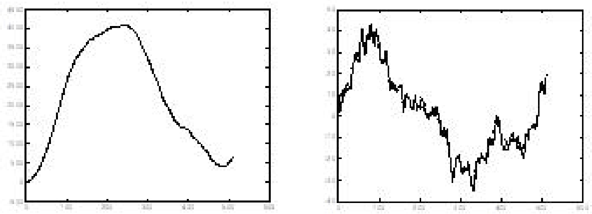
$$T=1,\ q=1,\ r_1=46,\ r_2=42,\ \Phi=0.5,\ \gamma_1=\gamma_2=0.5.$$

T is the sampling rate, and $x_1=[\text{position, velocity}]^T$.

Fig.2 shows a sequence of the true state and the estimated state at scale 1. Fig.3 shows a sequence of the true state and the estimated state at scale 2. Fig.4 and Table 1 give the results of Monte Carlo Simulation (100 runs). Fig.4 compares the measurement noise RMS with the estimation error RMS at two scales. The noise compression ratio at scale 1 and scale 2 are 4.9993dB and 1.8016dB, respectively.

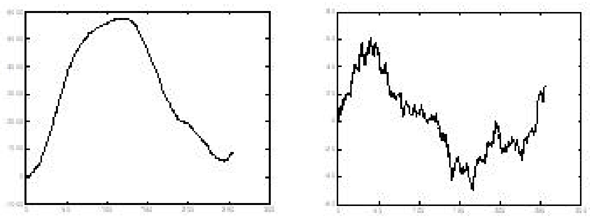
Table 1 shows the influence of q on the noise compression ratio. We can see that, the noise compression ratios at two scales decrease with the increasing of q and Φ. Table 2 shows the influence of γ_1 and γ_2 on the noise compression ratio.

Table 3 shows the influence of r_1 and r_2 on the noise compression ratio. It can be seen that, the noise compression ratio increases at scale 1 and decreases at scale 2 with the decreasing of r_2 while r_1 is unchanged. When r_2 remains unchanged and r_1 decreases, the ratio increases at scale 2 and decreases at scale 1. The ratio at two scales decrease with the decreasing of r_1 and r_2.



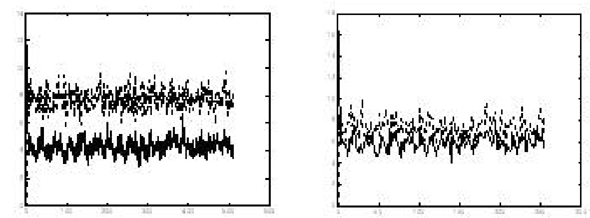
(a)displacement (b) velocity

Fig. 2 True state (dotted) and the estimated state (solid) at scale 1



(a) displacement (b) velocity

Fig. 3 True state (dotted) and the estimated state (solid) at scale 2



(a) scale 1 (b) scale 2

Fig. 4 Measurement noise RMS (dotted) and the estimation error RMS (solid)

Table 1 Influence of q on the noise compression ratio (dB)

Parameters							Noise compression ratio	
Φ	q	T	y_1	y_2	r_1	r_2	Scale 1	Scale 2
0.5	1	1	0.5	0.5	46	43	5.0387	1.9962
0.5	1.5	1	0.5	0.5	46	43	4.5235	1.4910
0.5	2	1	0.5	0.5	46	43	4.1491	1.0808
0.5	2.5	1	0.5	0.5	46	43	3.6724	0.6351
0.5	3	1	0.5	0.5	46	43	3.3379	0.3561

4 CONCLUSION

In this paper, modeling and estimation of a class of dynamic multiscale system is proposed. The system state is described with a partial differential equation, and is observed by multiple sensors in a closed subspace sequence of the state space. The noise in the state equation and measurement noise is colored noise. The state space projection equation is used to link the scales. The state equation is whitened firstly and then the measurement equation. The model of new system satisfies the Kalman filter condition. Haar-wavelet-based model and estimation algorithm are given. Monte Carlo simulation results verify the validation of our algorithm.

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