FURTHER INSIGHTS ON THE EQUIVALENCE OF AVF AND MSWF

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ABSTRACT

The auxiliary vector filter (AVF) and the multistage Weiner filter (MSWF) are two important categories of reduced-rank filters and have been widely applied to the adaptive signal processing domain. The relationship between AVF and MSWF has also drawn a lot of attention. It has been indicated that AVF is equivalent to MSWF due to the identical reduced-rank subspace in the literature. However, except for the same subspace, the structures and the corresponding parameters of AVF are considerably different to that of MSWF. In order to gain further insights on the equivalence between AVF and MSWF, a computation scheme of parameters and a nested structure are presented for AVF in this paper. According to the identical nested structure, it can be proven that AVF has the same parameters as MSWF, which are calculated in different ways. As a consequence, it can be claimed that AVF and MSWF are two alternative computation schemes for the same reduced-rank filter.

1. INTRODUCTION

In adaptive filtering domain, reduced-rank filters have attracted a considerable amount of research due to their satisfactory adaptive performance and low complexity, where the auxiliary vector filter (AVF) [1] and the multistage Weiner filter (MSWF) [2] are two most attractive reduced-rank filters proposed recently. In contrast with eigen-subspace based reduced-rank filters, AVF and MSWF require no eigen decomposition and thus lower computational complexity and are able to provide better performance with the same rank of reduced-dimension subspace. On the other hand, AVF and MSWF can obtain the same performance by employing dissimilar structure and different computation methods of parameters. Consequently, their relationship has evoked much research interest. The equivalence of their reduced-rank subspace has been identified in [3]. Nevertheless, further insights on the relationship between AVF and MSWF is necessary. To this end, a calculation method for the coefficients of auxiliary vectors and a nested structure are presented for AVF in this paper. On the basis of the same nested structure, it is proven that the parameters of AVF are identical to that of MSWF. That is to say, through DSP implementation AVF and MSWF can be thought of as two different schemes for the computation of the same reduced-dimension filter.

2. BACKGROUND

In this section, the basic principles of MSWF and AVF are firstly reviewed. Without loss of generality, a general discrete-time baseband signal model is used since the studies of AVF and MSWF in this paper won't be restricted to one special application domain. Assume $b(i)$ to be the desired signal and $\mathbf{x}(i)$ the $N\times 1$ received signal vector, ($i=0,1,2,\cdots$), which contains the desired signal, interferences and additive white Gaussian noise (AWGN). Then, $\mathbf{R}_x = E\{\mathbf{x}(i)\cdot\mathbf{x}^H(i)\}$ denotes the covariance matrix of the received signal vector and $\mathbf{r}_{xb} = E\{\mathbf{x}(i)b^*(i)\}$ the cross-correlation vector between the desired signal and the received signal vector, where $[\ ]^H$ and $[\ ]^*$ respectively denote the conjugation transpose operator and the conjugation operator.

2.1 The Multistage Wiener Filter

The general structure of the rank $D+1$ MSWF is depicted in Fig.1, which is divided into an analysis stage and a synthesis stage and given by the following set of recursions.

For $d=0,1,\cdots,D$ (Analysis Stage)

$$\tilde{\mathbf{g}}_d = \frac{E\{\mathbf{x}_d(i)\cdot\tilde{z}_d^*(i)\}}{\left\|E\{\mathbf{x}_d(i)\cdot\tilde{z}_d^*(i)\}\right\|}, \tag{1}$$

$$\tilde{z}_{d+1}(i) = \tilde{\mathbf{g}}_d^H\cdot\mathbf{x}_d(i), \tag{2}$$

$$\mathbf{B}_{d+1} = \mathbf{I}_N - \tilde{\mathbf{g}}_d\cdot\tilde{\mathbf{g}}_d^H, \tag{3}$$

$$\mathbf{x}_{d+1}(i) = \mathbf{B}_{d+1}^H\cdot\mathbf{x}_d(i). \tag{4}$$

where $\tilde{z}_0(i)=b(i)$, $\mathbf{x}_0(i)=\mathbf{x}(i)$, and $\mathbf{I}_N$ denotes an $N\times N$ identity matrix.

Decrement $d=D,\cdots,0$ (Synthesis Stage)

$$\tilde{\eta}_d = E\{\tilde{z}_{d+1}^*(i)\cdot\tilde{\varepsilon}_d(i)\}\Big/E\{|\tilde{\varepsilon}_d(i)|^2\}, \tag{5}$$

$$\tilde{\varepsilon}_{d-1}(i) = \tilde{z}_d(i) - \tilde{\eta}_d^* * \tilde{\varepsilon}_d(i). \qquad (d>0) \tag{6}$$

where $\tilde{\varepsilon}_D(i) = \tilde{z}_{D+1}(i)$.

According to Formula (1), $\tilde{\mathbf{g}}_0 = \mathbf{r}_{xb}/\|\mathbf{r}_{xb}\|$, and when $d\ge 1$ $\tilde{\mathbf{g}}_d$ can also be described by [4]

$$\tilde{\mathbf{g}}_d = \frac{E\left\{\mathbf{x}_d(i)\cdot\left[\tilde{\mathbf{g}}_{d-1}^H\cdot\mathbf{x}_{d-1}(i)\right]^*\right\}}{\left\|E\left\{\mathbf{x}_d(i)\cdot\left[\tilde{\mathbf{g}}_{d-1}^H\cdot\mathbf{x}_{d-1}(i)\right]^*\right\}\right\|}$$

$$= \frac{\mathbf{B}_d^H\cdot E\left\{\mathbf{x}_{d-1}(i)\cdot\mathbf{x}_{d-1}^H(i)\right\}\cdot\tilde{\mathbf{g}}_{d-1}}{\left\|\mathbf{B}_d^H\cdot E\left\{\mathbf{x}_{d-1}(i)\cdot\mathbf{x}_{d-1}^H(i)\right\}\cdot\tilde{\mathbf{g}}_{d-1}\right\|}$$

$$= \frac{\left(\mathbf{I}_N-\tilde{\mathbf{g}}_{d-1}\cdot\tilde{\mathbf{g}}_{d-1}^H\right)\cdot\mathbf{R}_{d-1}\cdot\tilde{\mathbf{g}}_{d-1}}{\left\|\left(\mathbf{I}_N-\tilde{\mathbf{g}}_{d-1}\cdot\tilde{\mathbf{g}}_{d-1}^H\right)\cdot\mathbf{R}_{d-1}\cdot\tilde{\mathbf{g}}_{d-1}\right\|}$$

$$= \frac{\left(\mathbf{I}_N-\sum_{i=0}^{d-1}\tilde{\mathbf{g}}_i\cdot\tilde{\mathbf{g}}_i^H\right)\cdot\mathbf{R}\cdot\left(\mathbf{I}_N-\sum_{i=0}^{d-2}\tilde{\mathbf{g}}_i\cdot\tilde{\mathbf{g}}_i^H\right)\cdot\tilde{\mathbf{g}}_{d-1}}{\left\|\left(\mathbf{I}_N-\sum_{i=0}^{d-1}\tilde{\mathbf{g}}_i\cdot\tilde{\mathbf{g}}_i^H\right)\cdot\mathbf{R}\cdot\left(\mathbf{I}_N-\sum_{i=0}^{d-2}\tilde{\mathbf{g}}_i\cdot\tilde{\mathbf{g}}_i^H\right)\cdot\tilde{\mathbf{g}}_{d-1}\right\|}$$

$$= \frac{\left(\mathbf{I}_N-\sum_{i=0}^{d-1}\tilde{\mathbf{g}}_i\cdot\tilde{\mathbf{g}}_i^H\right)\cdot\mathbf{R}\cdot\tilde{\mathbf{g}}_{d-1}}{\left\|\left(\mathbf{I}_N-\sum_{i=0}^{d-1}\tilde{\mathbf{g}}_i\cdot\tilde{\mathbf{g}}_i^H\right)\cdot\mathbf{R}\cdot\tilde{\mathbf{g}}_{d-1}\right\|}, \quad (7)$$

where $\mathbf{R}_d=\mathbf{B}_d^H\cdot\mathbf{R}_{d-1}\cdot\mathbf{B}_d=\prod_{i=1}^{d}\mathbf{B}_i^H\cdot\mathbf{R}\cdot\prod_{i=1}^{d}\mathbf{B}_i$. If $i\neq j$, $\tilde{\mathbf{g}}_i^H\cdot\tilde{\mathbf{g}}_j=0$. Thus $\prod_{i=1}^{d}\mathbf{B}_i\cdot\mathbf{g}_d=\mathbf{g}_d$.

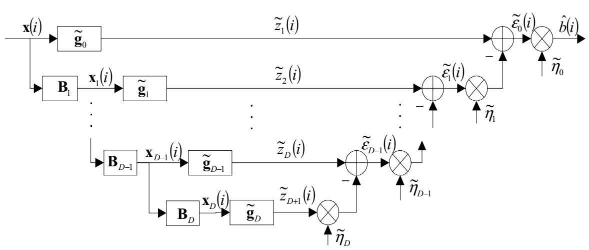

Fig.1 The MSWF structure I

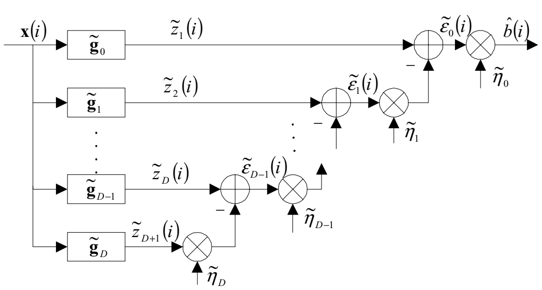

Fig.2 The MSWF structure II

Let $p_0=\|\mathbf{r}_{xb}\|$, $p_d=\tilde{\mathbf{g}}_{d-1}^H\mathbf{R}\tilde{\mathbf{g}}_d$ ($d=1,\cdots,D$), and $q_d=\tilde{\mathbf{g}}_d^H\mathbf{R}\tilde{\mathbf{g}}_d$ ($d=0,1,\cdots,D$), based on (5) and (6), $\{\tilde{\eta}_d\}_{d=0}^{D}$ can be calculated as follows:

$$\tilde{\eta}_d=p_d/\xi_d, \quad (d=D,\cdots,1,0) \quad (8)$$

$$\xi_d=E\left\{\left|\tilde{\varepsilon}_d(i)\right|^2\right\}=q_d-p_{d+1}^2/\xi_{d+1}. \quad (d=D-1,\cdots,1,0) \quad (9)$$

where $\xi_D=E\left\{\left|\tilde{\varepsilon}_D(i)\right|^2\right\}=q_D$.

Since $\prod_{i=1}^{d}\mathbf{B}_i\cdot\mathbf{g}_d=\mathbf{g}_d$, the general structure of MSWF can be simplified as the MSWF structure II, which is shown in Fig.2. The corresponding MSWF is given by

$$\mathbf{w}_{MSWF}^H=\tilde{\eta}_0\cdot\left(\tilde{\mathbf{g}}_0^H-\tilde{\eta}_1\cdot\left(\tilde{\mathbf{g}}_1^H-\tilde{\eta}_2\cdot(\cdots)\right)\right). \quad (10)$$

2.2 The Auxiliary Vector Filter

The general structure of the rank $D+1$ AVF is drawn in Fig.3, where $\mathbf{g}_0=\mathbf{r}_{xb}/\|\mathbf{r}_{xb}\|$, $\{\mathbf{g}_d\}_{d=1}^{D}$ are the auxiliary vectors and computed by [3]

$$\mathbf{g}_1=\frac{\mathbf{R}\mathbf{g}_0-\mathbf{g}_0\left(\mathbf{g}_0^H\mathbf{R}\mathbf{g}_0\right)}{\left\|\mathbf{R}\mathbf{g}_0-\mathbf{g}_0\left(\mathbf{g}_0^H\mathbf{R}\mathbf{g}_0\right)\right\|}, \quad (d=1) \quad (11)$$

$$\mathbf{g}_d=\frac{\mathbf{R}\mathbf{g}_{d-1}-\sum_{j=d-2}^{d-1}\mathbf{g}_j\left(\mathbf{g}_j^H\mathbf{R}\mathbf{g}_{d-1}\right)}{\left\|\mathbf{R}\mathbf{g}_{d-1}-\sum_{j=d-2}^{d-1}\mathbf{g}_j\left(\mathbf{g}_j^H\mathbf{R}\mathbf{g}_{d-1}\right)\right\|}. \quad (2\le d\le D) \quad (12)$$

Define $\mathbf{G}=[\mathbf{g}_0\ \mathbf{g}_1\ \cdots\ \mathbf{g}_D]$ and $\mathbf{u}=[u_0\ u_1\cdots\ u_D]$, then, AVF, which corresponds to the AVF structure I, is given by

$$\mathbf{w}_{AVF}^H=\sum_{d=0}^{D}u_d\cdot\mathbf{g}_d^H=\mathbf{u}\cdot\mathbf{G}^H. \quad (13)$$

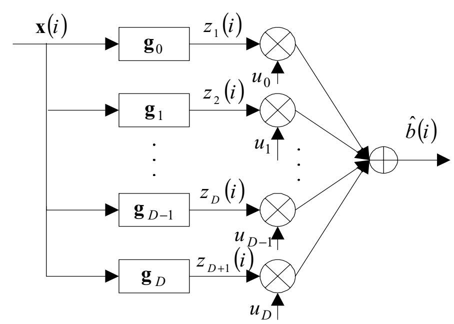

Fig.3 The AVF structure I

According to the MMSE criterion, the optimum $\mathbf{u}$, which minimizes $E\left\{\left|b(i)-\mathbf{u}\cdot\mathbf{G}^H\cdot\mathbf{x}(i)\right|^2\right\}$, can be obtained by solving Equation (14).

$$\mathbf{R}\cdot\mathbf{G}\cdot\mathbf{u}^H=\mathbf{r}_{xb}. \quad (14)$$

Since $\mathbf{g}_i^H\mathbf{R}\mathbf{g}_j=0$ if $|i-j|>1$, which is indicated in [3], we can get the unified form of (3) and (4) as

$$\mathbf{g}_d=\frac{\left(\mathbf{I}_N-\sum_{i=0}^{d-1}\mathbf{g}_i\cdot\mathbf{g}_i^H\right)\cdot\mathbf{R}\cdot\mathbf{g}_{d-1}}{\left\|\left(\mathbf{I}_N-\sum_{i=0}^{d-1}\mathbf{g}_i\cdot\mathbf{g}_i^H\right)\cdot\mathbf{R}\cdot\mathbf{g}_{d-1}\right\|}. \quad (d\ge 1) \quad (15)$$

Comparing (15) with (7), the result that $\mathbf{g}_d=\tilde{\mathbf{g}}_d$ ($d=0,1,\cdots,D$) can be obtained, i.e., AVF has the equivalent reduced-rank subspace as MSWF [3]. In order to gain further insights on the equivalence of AVF and MSWF, the relationship between the structure and parameters of AVF and that of MSWF will be addressed in the later sections.

3. THE CALCULATION SCHEME OF AVF'S PARAMETERS

A low-complexity calculation scheme of AVF's parameters is presented in this section before analyzing the relationship between the structure of AVF and that of MSWF.

Since the vectors in $\mathbf{G}$ are orthogonal to each other [3], by multiplying the two sides of Equation (14) by $\mathbf{G}^H$ we can get

$$\left(\mathbf{G}^H \cdot \mathbf{R} \cdot \mathbf{G}\right) \cdot \mathbf{u}^H = \mathbf{y} \,, \qquad (16)$$

where $\mathbf{y} = \mathbf{G}^H \cdot \mathbf{r}_{xb} = \left[\left\|\mathbf{r}_{xb}\right\| \;\; 0 \;\; \cdots \;\; 0\right]^T$ and $[\;]^T$ denotes the transpose operator.

It is easy to verify that $\mathbf{G}^H\mathbf{R}\mathbf{G}$ is a real symmetric tridiagonal matrix [3]. Hence, the computation of $\mathbf{u}$ can be simplified by using LU factorization and Gaussian elimination to solve Equation (16) [5].

The matrix $\mathbf{G}^H\mathbf{R}\mathbf{G}$ has the following form,

$$\mathbf{G}^H\mathbf{R}\mathbf{G} = \begin{bmatrix} q_0 & p_1 & & & \mathbf{0} \\ p_1 & q_1 & p_2 & & \\ & p_2 & \ddots & \ddots & \\ & & \ddots & q_{D-1} & p_D \\ \mathbf{0} & & & p_D & q_D \end{bmatrix}, \qquad (17)$$

where $q_d = \mathbf{g}_d^H\mathbf{R}\mathbf{g}_d$ and $p_d = \mathbf{g}_{d-1}^H\mathbf{R}\mathbf{g}_d = \mathbf{g}_d^H\mathbf{R}\mathbf{g}_{d-1}$.

By LU factorization, $\mathbf{G}^H\mathbf{R}\mathbf{G} = \mathbf{L}\mathbf{U}$ [5], where $\mathbf{L}$ and $\mathbf{U}$ are respectively given by

$$\mathbf{L} = \begin{bmatrix} 1 & & & \mathbf{0} \\ \tilde{p}_1 & 1 & & \\ & \tilde{p}_2 & \ddots & \\ & & \ddots & 1 \\ \mathbf{0} & & \tilde{p}_D & 1 \end{bmatrix} \text{ and } \mathbf{U} = \begin{bmatrix} \tilde{q}_0 & p_1 & & \mathbf{0} \\ & \tilde{q}_1 & p_2 & \\ & & \ddots & \ddots \\ & & \tilde{q}_{D-1} & p_D \\ \mathbf{0} & & & \tilde{q}_D \end{bmatrix}.$$

In $\mathbf{L}$ and $\mathbf{U}$, let $\tilde{q}_0 = q_0$, $\{\tilde{p}_d\}_{d=1}^D$ and $\{\tilde{q}_d\}_{d=1}^D$ are respectively obtained as follows.

$$\tilde{p}_d = p_d / \tilde{q}_{d-1} \,, \qquad (d = 1,2,\cdots,D) \qquad (18)$$

$$\tilde{q}_d = q_d - \tilde{p}_d p_d \,. \qquad (d = 1,2,\cdots,D) \qquad (19)$$

Due to the features of the lower triangular matrix $\mathbf{L}$ and the upper triangular matrix $\mathbf{U}$, the solution $\mathbf{u}$ of Equation (16) can be attained by a forward substitution and a back substitution [5].

Forward substitution, let $\tilde{y}_0 = \left\|\mathbf{r}_{xb}\right\|$,

$$\tilde{y}_d = -\tilde{p}_d \tilde{y}_{d-1} \,. \qquad (d = 1,2,\cdots D) \qquad (20)$$

Back substitution, let $u_D = \tilde{y}_D \tilde{q}_D^{-1}$,

$$u_d = \left(\tilde{y}_d - p_{d+1}u_{d+1}\right)\tilde{q}_d^{-1} \,. \qquad (d = 0,1,\cdots D-1) \qquad (21)$$

It is obvious that the computation complexity is only $O(N)$ rather than $O(N^3)$ required by direct matrix inversion.

4. FURTHER INSIGHTS ON THE RELATIONSHIP BETWEEN AVF AND MSWF

In order to further understand the relationship between AVF and MSWF after the proof of the equivalence of their reduced-rank subspaces [3], it will be proven that AVF can have the same structure and the corresponding parameters as MSWF.

In AVF, let $\eta_0 = u_0$ and $\eta_d = -u_d / u_{d-1}$ ($d = D,\cdots,2,1$), we can derive the nested structure shown in Fig.4, which is identical to the structure II of MSWF. Thus, AVF can also be expressed by

$$\mathbf{w}_{AVF}^H = \eta_0 \cdot \left(\mathbf{g}_0^H - \eta_1 \cdot \left(\mathbf{g}_1^H - \eta_2 \cdot (\cdots)\right)\right). \qquad (22)$$

It is clear that AVF in (22) have the same form as MSWF in (10). Since it has been known that $\mathbf{g}_d = \tilde{\mathbf{g}}_d$, we can further prove that AVF is completely equivalent to MSWF only if it is true that $\eta_d = \tilde{\eta}_d$ ($d = 0,1,\cdots,D$).

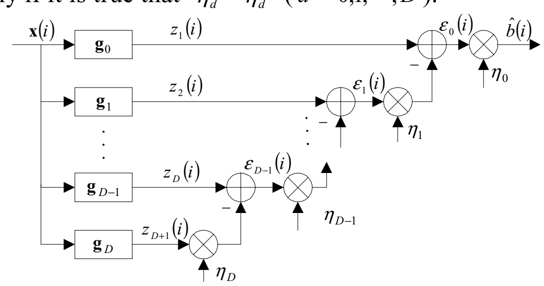

Fig. 4 The AVF structure II

According to the process of the calculation of $\mathbf{u}$ in (18-21), we can get

$$\eta_d = -u_d / u_{d-1}$$

$$= \frac{-u_d}{\left(\tilde{y}_{d-1} - p_d u_d\right)\tilde{q}_{d-1}^{-1}} = \frac{p_d}{p_d\left(p_d - \tilde{y}_{d-1}/u_d\right)\tilde{q}_{d-1}^{-1}}$$

$$= \frac{p_d}{\tilde{p}_d\left(p_d - \tilde{y}_{d-1}/u_d\right)} \,. \quad (d = D,\cdots,2,1) \qquad (23)$$

It is seen from (8) that the proof of $\tilde{\eta}_d = \eta_d$ ($1 \le d \le D$) is equivalent to prove that

$$\xi_d = \tilde{p}_d\left(p_d - \tilde{y}_{d-1}/u_d\right). \qquad (24)$$

In addition, when $d = 0$, it is required to make sure that $\tilde{\eta}_0 = \eta_0 = u_0$.

Proof:

When $d = D$,

$$\tilde{p}_D\left(p_D - \tilde{y}_{D-1}/u_D\right) = \tilde{p}_D\left(p_D - \frac{\tilde{y}_{D-1}}{\tilde{y}_D\tilde{q}_D^{-1}}\right)$$

$$= \tilde{p}_D \left(p_D + \frac{\tilde{q}_D}{\tilde{p}_D} \right) = q_D = \xi_D .$$

When $d = D-1$, consider $\eta_D = p_D/\xi_D = -u_D/u_{D-1}$, we have

$$\tilde{p}_{D-1}\left(p_{D-1} - \tilde{y}_{D-2}/u_{D-1}\right) = \tilde{p}_{D-1}\left(p_{D-1} + \frac{\tilde{y}_{D-2}p_D}{u_D\xi_D} \right)$$

$$= \tilde{p}_{D-1}\left(p_{D-1} + \frac{\tilde{y}_{D-2}p_D}{\tilde{y}_D\tilde{q}_D^{-1}\xi_D} \right) = \tilde{p}_{D-1}\left(p_{D-1} + \frac{p_D\tilde{q}_D}{\tilde{p}_D\tilde{p}_{D-1}\xi_D} \right)$$

$$= \tilde{p}_{D-1}p_{D-1} + \frac{\tilde{q}_{D-1}\tilde{q}_D}{\xi_D}$$

$$= q_{D-1} - \tilde{q}_{D-1} + \frac{\tilde{q}_{D-1}\left(q_D - \tilde{p}_D p_D\right)}{\xi_D}$$

$$= q_{D-1} + \frac{p_D^2}{\xi_D} = \xi_{D-1} .$$

Assume that $\tilde{\eta}_m = \eta_m$ ($m = d+1, d+2, \cdots, D$), then, we have

$$\frac{1}{u_d} = \prod_{m=d+1}^{D}\left(-\eta_m\right) \Big/ u_D = \prod_{m=d+1}^{D}\left(-p_m\right) \Big/ \left(u_D \prod_{m=d+1}^{D} \xi_m \right)$$

$$= \tilde{q}_D \prod_{m=d+1}^{D}\left(-p_m\right) \Big/ \left(\tilde{y}_D \prod_{m=d+1}^{D} \xi_m \right),$$

and

$$\frac{\tilde{y}_{d-1}}{\tilde{y}_D} = \frac{1}{\prod_{m=d}^{D}\left(-\tilde{p}_m\right)} .$$

Thus,

$$\tilde{p}_d\left(p_d - \tilde{y}_{d-1}/u_d\right) = \tilde{p}_d p_d + \frac{\tilde{q}_D \prod_{m=d+1}^{D}\left(-p_m\right)}{\prod_{m=d+1}^{D}\left(-\tilde{p}_m\right) \prod_{m=d+1}^{D} \xi_m}$$

$$= \tilde{p}_d p_d + \frac{\prod_{m=d}^{D}\tilde{q}_m}{\prod_{m=d+1}^{D}\xi_m} = q_d - \tilde{q}_d + \frac{\prod_{m=d+1}^{D}\xi_m\left(\tilde{q}_d - p_{d+1}^2/\xi_{d+1}\right)}{\prod_{m=d+1}^{D}\xi_m} = \xi_d ,$$

where $\prod_{m=d}^{D}\tilde{q}_m = \prod_{m=d+1}^{D}\xi_m\left(\tilde{q}_d - p_{d+1}^2/\xi_{d+1}\right)$, which is derived as follows,

$$\tilde{q}_D = q_D - p_D^2/\tilde{q}_{D-1} = \xi_D - p_D^2/\tilde{q}_{D-1}$$

$$\Rightarrow \tilde{q}_{D-1}\tilde{q}_D = \xi_D\left(\tilde{q}_{D-1} - p_D^2/\xi_D\right)$$

$$\Rightarrow \tilde{q}_{D-1}\tilde{q}_D = \xi_D\left(q_{D-1} - p_{D-1}^2/\tilde{q}_{D-2} - p_D^2/\xi_D\right)$$

$$\Rightarrow \tilde{q}_{D-1}\tilde{q}_D = \xi_D\left(\xi_{D-1} - p_{D-1}^2/\tilde{q}_{D-2}\right)$$

$$\Rightarrow \tilde{q}_{D-2}\tilde{q}_{D-1}\tilde{q}_D = \xi_{D-1}\xi_D\left(\tilde{q}_{D-2} - p_{D-1}^2/\xi_D\right)$$

$$\vdots$$

$$\Rightarrow \prod_{m=d}^{D}\tilde{q}_m = \prod_{m=d+1}^{D}\xi_m\left(\tilde{q}_d - p_{d+1}^2/\xi_{d+1}\right).$$

Finally,

$$u_0 = \left(\tilde{y}_D \prod_{m=1}^{D}\xi_m\right) \Big/ \left(\tilde{q}_D \prod_{m=1}^{D}\left(-p_m\right)\right),$$

$$= \left(\tilde{y}_0 \prod_{m=d}^{D}\left(-\tilde{p}_m\right) \prod_{m=1}^{D}\xi_m\right) \Big/ \left(\tilde{q}_D \prod_{m=1}^{D}\left(-p_m\right)\right)$$

$$= \left(\tilde{y}_0 \prod_{m=1}^{D}\xi_m\right) \Big/ \prod_{m=1}^{D}\tilde{q}_m = p_0/\xi_0 .$$

This completes the proof.

It is seen from the above proof that AVF in (13) is an alternative computation scheme of MSWF in (10).

5. CONCLUSIONS

A nested structure of AVF and a computation method of its corresponding parameters are presented in this paper. It is proven that AVF and MSWF can get the same coefficients by different methods in the same nested structure. Therefore, AVF and MSWF can be regarded as two alternative computation schemes for the same reduced-rank filter. Moreover, it can be easily verified that AVF and MSWF have close computational complexity.

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