SOFT CONSTRAINT SATISFACTION MULTIMODULUS BLIND EQUALIZATION ALGORITHMS

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ABSTRACT

In this work, a new algorithm based on minimum-disturbance principle with relaxation is presented for the blind equalization of complex signals. This algorithm combines the benefits of the well-known reduced constellation algorithm (RCA) and constant modulus algorithm (CMA). The convergence characteristics of the proposed algorithm is demonstrated by way of simulations. In addition, closed form expressions are obtained for the statistical (dispersion) constants used in these algorithms.

1. INTRODUCTION

In most digital communication systems, intersymbol interference (ISI) occurs due to bandwidth limited channels or multipath propagation. Channel equalization is one of the techniques to mitigate the effect of ISI. Adaptive algorithms are used to initialize and adjust equalizer coefficients when a channel is unknown and possibly time-varying. Conventionally, initial setting of the equalizer tap weights is achieved by a training sequence before data transmission.

However, when sending a training sequence is impractical or impossible, it is desirable to equalize a channel without the aid of a training sequence. Equalizing a channel without training mode is known as blind equalization. Sato [1] introduced the idea of blind equalization for multilevel pulse amplitude modulation (PAM). Godard [2] and Benveniste at al. [3] generalized the Sato's algorithm for quadrature amplitude modulation (QAM); their algorithms are known as constant modulus algorithm (CMA) and reduced constellation algorithm (RCA), respectively. CMA is the most widely studied algorithm. It provides reliable convergence, but increases the complexity of implementation of the receiver in steady-state operation because of the need to add a rotator at the output of the equalizer. Rotator removes any possible phase offset error and facilitates the switching from blind mode to decision-directed (DD) mode. However, it is analytically shown in [4] that RCA and a variant of CMA, known as multimodulus algorithm (MMA) [5, 6, 7] exhibits an inherent property of phase recovery. That is, in the presence of little phase and/or frequency offset error(s), they will recover the constellation automatically.

In [8], Lin defined blind equalization as a constrained optimization problem. He used the error terms of MMA scheme as constraints and minimized the squared Euclidean norm of the change in the tap-weight vector. He finally obtained a normalized version of RCA. In this work, we used the same cost function proposed in [8]; and solved the deterministic optimization criterion with a *soft* constraint to obtain an update equation which contains a normalized gradient vector and a particular non-linearity similar to MMA.

2. LIN'S MINIMUM-DISTURBANCE ALGORITHM PRIMER

Consider the baseband representation for digital data transmission in Fig. 1, where $a(n)$ is the transmitted symbols, $\nu(n)$ is the channel noise, $x(n)$ is the equalizer input and $\hat{a}(n)$ is the output of the decision device. The equalizer tap-

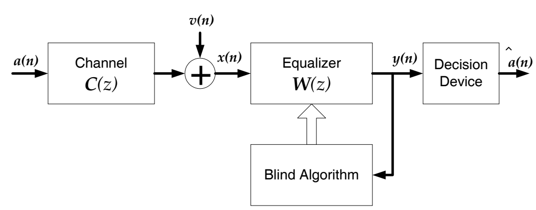

Fig. 1. Blind equalization in the baseband.

weight vector and tap-input vector are respectively defined as $\mathbf{W}(n) = [w_0(n), w_1(n), \cdots, w_{N-1}(n)]^T$ and $\mathbf{X}(n) = [x(n), x(n-1), \cdots, x(n-N+1)]^T$. We define $y(n) = \mathbf{W}^H(n)\mathbf{X}(n)$ and $s(n) = \mathbf{W}^H(n+1)\mathbf{X}(n)$ respectively as the actual and *a posteriori* equalizer outputs [9]. The objective is to achieve $\hat{a}(n) = e^{\jmath\theta}a(n-\Delta)$ without using a training signal available at the receiver. Lin proposed to formulate the following deterministic optimization problem to achieve this object

$$J = \min_{\mathbf{W}(n+1)} \Big\{ \|\mathbf{W}(n+1) - \mathbf{W}(n)\|_2^2 + \underbrace{\lambda_1 s_R(n)(s_R^2(n) - R_R^2) + \lambda_2 s_I(n)(s_I^2(n) - R_I^2)}_{\text{constraints}} \Big\} \quad (1)$$

where λ_1 and λ_2 are the Lagrange multipliers and R_R and R_I are dispersion constants defined in the sequel. We also define

$w_m(n) = a_m(n) + \jmath\, b_m(n)$ and $x(n-m) = u(n-m) + \jmath\, v(n-m)$ for $m = 0, 1, \cdots, N-1$. The constraints in (1) are used to make sure that any change in the tap-weight vector is so smooth that the (blind mode) error signal caused by updating the tap-weight vector can approach to zero. By differentiating (1) with respect to real and imaginary parts of $\mathbf{W}(n+1)$, and then setting the results equal to zero, one can obtain the following equations with substitutions $s_R^2(n) = R_R^2$ and $s_I^2(n) = R_I^2$,

$$\begin{aligned} a_m(n+1) - a_m(n) + \lambda_1 R_R^2 u(n-m) \\ +\lambda_2 R_I^2 v(n-m) = 0, \end{aligned} \quad (2)$$

$$\begin{aligned} b_m(n+1) - b_m(n) + \lambda_1 R_R^2 v(n-m) \\ -\lambda_2 R_I^2 u(n-m) = 0. \end{aligned} \quad (3)$$

Multiplying (2) and (3) with $u(n-m)$ and $v(n-m)$, respectively; the resulting equations are then added and subtracted to yield the following expressions for the optimum Lagrange multipliers λ_{1*} and λ_{2*},

$$\begin{cases} \lambda_{1*} = -\dfrac{1}{R_R^2 \|\mathbf{X}(n)\|_2^2}(s_R(n) - y_R(n)), \\ \lambda_{2*} = -\dfrac{1}{R_I^2 \|\mathbf{X}(n)\|_2^2}(s_I(n) - y_I(n)). \end{cases} \quad (4)$$

and the corresponding update equation is $\mathbf{W}(n+1) = \mathbf{W}(n) - \left(\lambda_{1*} R_R^2 - \jmath\, \lambda_{2*} R_I^2\right) \mathbf{X}(n)$. At each n, *hard constraints* in (1) enforce

$$\begin{cases} s_R(n) = R_R \operatorname{sign}(y_R(n)) \\ s_I(n) = R_I \operatorname{sign}(y_I(n)), \end{cases} \quad (5)$$

which correspond to the *exact* solution of (1). The continued use of these results will lead to the algorithm proposed in [8], as follows

$$\begin{aligned} \mathbf{W}(n+1) = \mathbf{W}(n) - \frac{\mathbf{X}(n)}{\|\mathbf{X}(n)\|_2^2}\, [\, y(n) - \\ \{R_R \operatorname{sign}(y_R(n)) + \jmath\, R_I \operatorname{sign}(y_I(n))\}\,]^*. \end{aligned} \quad (6)$$

where $R_R = \frac{E[a_R^2]}{E[|a_R|]}$ and $R_I = \frac{E[a_i^2]}{E[|a_I|]}$ are defined respectively, as the dispersion constants for the real and imaginary parts of the transmitted signal. It can easily be observed that the Lin's algorithm is actually the normalized version of RCA; however, it is obtained by minimizing the so-called *a posteriori error* as mentioned in Eq. (1).

3. PROPOSED MODIFICATION

In [10], authors presented a method to apply the constraints on a deterministic cost function for blind equalization in a *soft* manner. They introduced a controlling parameter μ to control the degree of constraint satisfaction. Inspired with this technique, we introduce a similar parameter (the step-size μ) in Lin's algorithm (Eq. (6)) to relax the control over the convergence speed.

Incorporating a step-size in (2) and (3), we retain the constraint on $s_R(n)$ and $s_I(n)$ as a *soft constraint*. From (5) one can obtain (allowing a little deviation from our initial optimization statement in Eq. (1))

$$\begin{cases} \dfrac{s_R(n)}{y_R(n)} = \dfrac{R_R}{|y_R(n)|}, \\ \dfrac{s_I(n)}{y_I(n)} = \dfrac{R_I}{|y_I(n)|}, \end{cases} \quad (7)$$

which modifies the Lagrange multipliers λ_{1*} and λ_{2*} (in Eq. (4)) to,

$$\begin{cases} \lambda_{1*} = -\dfrac{s_R(n)}{R_R^2 \|\mathbf{X}(n)\|_2^2}\left(1 - \dfrac{|y_R(n)|}{R_R}\right), \\ \lambda_{2*} = -\dfrac{s_I(n)}{R_I^2 \|\mathbf{X}(n)\|_2^2}\left(1 - \dfrac{|y_I(n)|}{R_I}\right). \end{cases} \quad (8)$$

Substituting the new values of λ_{1*} and λ_{2*} into (2) and (3) and incorporating μ, we get

$$\begin{aligned} a_m(n+1) - a_m(n) &+ \mu[\lambda_{1*} R_R^2 u(n-m) \\ &+ \lambda_{2*} R_I^2 v(n-m)] = 0, \end{aligned} \quad (9)$$

$$\begin{aligned} b_m(n+1) - b_m(n) &+ \mu[\lambda_{1*} R_R^2 v(n-m) \\ &- \lambda_{2*} R_I^2 u(n-m)] = 0. \end{aligned} \quad (10)$$

Solving (9) and (10), one can obtain

$$\begin{cases} s_R(n) = \dfrac{y_R(n)}{1 - \mu\left(1 - \frac{|y_R(n)|}{R_R}\right)}, \\ s_I(n) = \dfrac{y_I(n)}{1 - \mu\left(1 - \frac{|y_I(n)|}{R_I}\right)}. \end{cases} \quad (11)$$

Therefore, instead of using $s_{R,I}(n) = R \operatorname{sign}(y_{R,I}(n))$ which is dictated by the criterion in (1), if the result obtained in (11) is used then the following weight update will be obtained

$$\begin{aligned} \mathbf{W}(n+1) = \mathbf{W}(n) + \frac{\mu}{\|\mathbf{X}(n)\|_2^2}\left[\frac{y_R(n)\left(1 - \frac{|y_R(n)|}{R_R}\right)}{1 - \mu\left(1 - \frac{|y_R(n)|}{R_R}\right)}\right. \\ \left. +\jmath\, \frac{y_I(n)\left(1 - \frac{|y_I(n)|}{R_I}\right)}{1 - \mu\left(1 - \frac{|y_I(n)|}{R_I}\right)}\right]^* \mathbf{X}(n) \end{aligned} \quad (12)$$

This update equation is consistent with (6) in the sense that for $\mu = 1$, the update equation in (12) reduces to (6). However, if μ is small, then around the desirable local solution, we have $1 - \mu\left(\frac{|y_{R,I}(n)|}{R}\right) \approx 1$; thus simplifying (12) to

$$\begin{aligned} \mathbf{W}(n+1) = \mathbf{W}(n) + \frac{\mu}{\|\mathbf{X}(n)\|_2^2}\left[y_R(n)\left(1 - \frac{|y_R(n)|}{R_R}\right)\right. \\ \left. +\jmath\, y_I(n)\left(1 - \frac{|y_I(n)|}{R_I}\right)\right]^* \mathbf{X}(n) \end{aligned} \quad (13)$$

The weight update equations in (12) and (13) are named SCS-MMA and SCS-MMA-I, respectively; where SCS-MMA stands for soft constraint satisfaction multi-modulus algorithm. The updates in SCS-MMA-I is easier to analyze compared with SCS-MMA; though, they are not vastly different for small μ. The real-time implementation of SCS-MMA-I is also straight forward as it doesn't have any division operation. Following the derivation in [3], it is easy to show that for SCS-MMA-I, we have
$R_R = E[|a_R(n)|^3]/E[a_R^2(n)]$ and $R_I = E[|a_I(n)|^3]/E[a_I^2(n)]$.

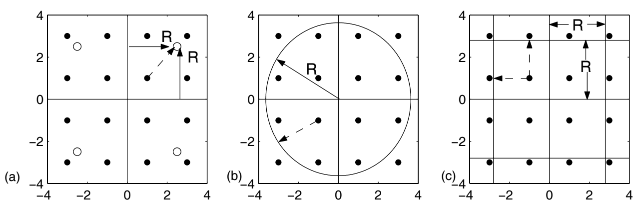

Fig. 2. Graphical representation of (a) RCA, (b) CMA, (c) SCS-MMA-I.

Let us compare the error terms used in Eqs.(6) and (13). Eq.(6) tries to minimize the distance between the equalizer outputs and the statistical points $\pm R \pm \jmath R$. Thus those outputs which fall in the first quadrant are compared with $R+\jmath R$ and those which fall in the second quadrant are compared with $-R+\jmath R$. Similarly, for third and forth quadrants, the equalized values are compared with $-R-\jmath R$ and $R-\jmath R$, respectively. A graphical representation of this behavior is illustrated in Fig. 2(a). In contrast, Eq.(6) tries to move the real and imaginary points of the equalizer output to reside on the points of value $+R$ or $-R$ on real and imaginary axes, respectively. This behavior is shown in Fig. 2(c). In Fig. 2(b), the behavior of CMA algorithm is illustrated, which attempts to derive the equalizer output to lie on the circle of radius R. In Table 1, we provided closed form expressions to obtain the exact values of dispersion constants (Rs) used in RCA, CMA and SCS-MMA-I schemes for QAM constellations. Table 2 shows the numerical values of Rs for some QAM constellations.

4. SIMULATION RESULTS

In this section, we illustrate the performance of the proposed blind equalization algorithm. In all simulations, a complex seven taps transversal equalizer was used and it was initialized so that the center tap was set to one and other taps were set to zero. The channel used in the simulation was taken from [11]. The signal to noise ratio (SNR) was taken as 30dB at the input of the equalizer. The residual ISI is measured and compared as performance parameter. The residual ISI at the equalizer output at *n*-th iteration is defined as $ISI(n) = (\sum |s(n)|^2 - |s(n)|^2_{max})/|s(n)|^2_{max}$, where $\{s(n)\} = \{c(n)\} \otimes \{w(n)\}$ is the overall system impulse response of the transmission channel and the equalizer. $|s(n)|^2_{max}$ is the component of the $\{s(n)\}$ having the maximum absolute value and $\otimes$ denotes convolution. At perfect equalization, the $ISI(n)$ becomes zero. In simulation, normalization factor is removed from all algorithms; as high oscillations were observed in weight adaptation process due to their use, in the case of complex channels with higher constellation sizes. Instead, we employed fixed step-sizes μ in all algorithms to ensure the stability of adaptation process. A modified normalization factor that doesn't affect the stability is under study, and will be presented as a separate future work.

Now, we examine the ability of equalization of the three algorithms - RCA, CMA and SCS-MMA-I on steady-state constellation space, obtained with *same* convergence speed. In CMA, we used a rotator at the output of the equalizer to estimate and remove the rotation of the constellation. The DD-based phase-recovery algorithm used in simulation, is given by $\theta(n+1) = \theta(n) - \mu_\theta \mathtt{Im}[\hat{a}^*(n) y_\theta(n)]$, where μ_θ is the step-size, $y_\theta(n) = e^{-\jmath\theta(n)} y(n)$, $y(n)$ is the equalizer output, and $\hat{a}(n)$ is the decision made on $y_\theta(n)$. Fig. 4 shows the signal constellations before and after the equalization. Each of the constellations shows 300 data points. It can be seen that the CMA had removed the ISI but was unable to remove the constant phase-shift introduced by the channel. Fig. 3 depicts that the rotator converged to $-11°$, and failed to recover the 45° phase ambiguity. Unlike CMA, RCA and SCS-MMA-I mitigated the ISI and phase-shift without needing any rotator. However, it can be seen that the data points are more aggregate in SCS-MMA-I than those in RCA. It also shows that the proposed algorithm performs better than the joint scheme of CMA and DD-based phase recovery, even with reduced complexity.

Fig. 5 depicts the traces of the ISI convergence for the three algorithms obtained with same convergence speed for 16-QAM signaling. It can be observed that SCS-MMA-I outperforms both RCA and CMA by achieving the lowest residual ISI floor. Fig. 6 and Fig. 7 depict the traces of the symbol-error rate (SER) for RCA and SCS-MMA-I for SNR equal to 20dB and 30dB, respectively, for 64-QAM signaling obtained from 50 independent simulation runs. It can be observed that SCS-MMA-I performed better than RCA by achieving lower, faster and less dispersive SER in both two cases. Our simulations show that the new algorithm, SCS-MMA-I, results in performance enhancement in convergence speed with lower SER and residual ISI, irrespective of constellation size, than those of RCA and CMA. Future investigation will focus on the modification of normalization factor and the use of multiple constraints.

5. CONCLUSION

In this work we devised a new blind equalization algorithm by using the minimum disturbance principle with relaxation. The proposed algorithm is able to correct the phase error and removes the ISI simultaneously while maintaining the simplicity. With the help of computer simulations, we have shown that SCS-MMA-I results in performance improvement in convergence speed and residual ISI than those of RCA and CMA.

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Table 1. Closed form expressions for Rs.

Scheme	Square QAM	Symmetric QAM
RCA: $R_{\mathtt{RCA}}$	$\frac{2M-2}{3\sqrt{M}}$	$\frac{31M-32}{66}\sqrt{\frac{2}{M}}$
SCS-MMA-I: $R_{\mathtt{SCS\_MMA\_I}}$	$\frac{3\sqrt{M}}{4}\left(\frac{M-2}{M-1}\right)$	$\frac{3\sqrt{2M}}{16}\left(\frac{89M-176}{31M-32}\right)$
CMA: $R^2_{\mathtt{CMA}}$	$\frac{14M-26}{15}$	$\frac{3251M^2-9920M+6656}{120(31M-32)}$

Table 2. Values of Rs for some QAM constellation.

M	4	16	32	64	128	256
$R_{\mathtt{RCA}}$	1	2.5	3.636	5.25	7.455	10.625
$R_{\mathtt{SCS\_MMA\_I}}$	1	2.8	4.175	5.905	8.549	11.953
$R^2_{\mathtt{CMA}}$	1	13.2	26.2	58	110.1	237.2

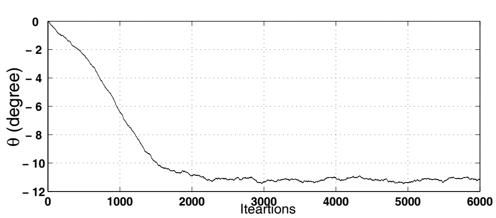

Fig. 3. Rotator phase estimation in CMA.

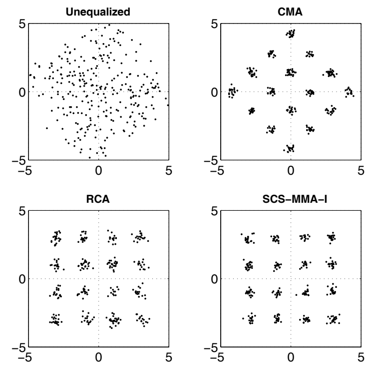

Fig. 4. Signal constellation of 16-QAM (a) Unequalized, (b) CMA, (c) RCA, and (d) SCS-MMA-I. The step-sizes are 8.0×10^{-4}, 4.0×10^{-4} and 5.0×10^{-5} for RCA, SCS-MMA-I and CMA, respectively.

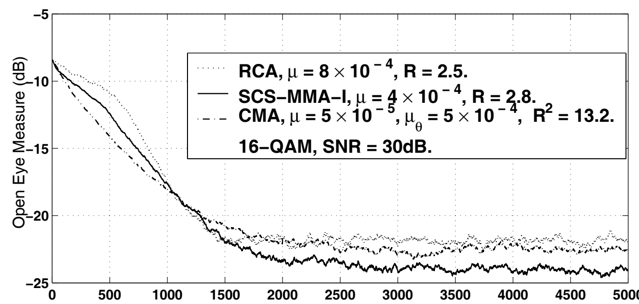

Fig. 5. Ensemble average ISI for 16-QAM. $\mu = 8.0 \times 10^{-4}$ for RCA, $\mu = 4.0 \times 10^{-4}$ for SCS-MMA-I and $\mu = 5.0 \times 10^{-5}$ for CMA.

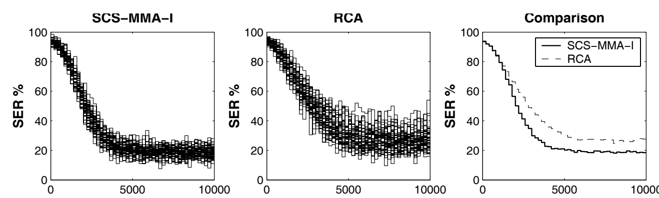

Fig. 6. Ensemble average SER for 64-QAM, SNR = 20dB. $\mu = 8.0\times10^{-5}$ for RCA and $\mu = 2.0\times10^{-5}$ for SCS-MMA-I. SCS-MMA-I is not only less dispersive but its steady-state SER is less than that of RCA by 10 units.

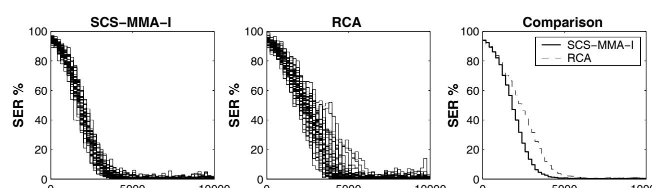

Fig. 7. Ensemble average SER for 64-QAM, SNR=30dB. All parameters are same as those in Fig. 6.