LINKING SEQUENCE BEHAVIOR IN ANC

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ABSTRACT

It has been known for some time that adaptive filters can sometimes provide performance exceeding that of a Wiener filter in cases where there is a difference in frequency between the reference signal and the primary of an adaptive noise canceller. A recent model to explain this behavior was based on the concept of linking sequences. In this paper the characteristics of these linking sequences will be evaluated further. It will be shown that performance improvement of the adaptive filter is paired with low variation of the linking sequences.

1. INTRODUCTION

Non-Wiener behavior has been observed to occur in adaptive filtering, in particular when narrowband signals are involved [1]. The behavior has been termed non-linear, or non-Wiener, as the performance of the adaptive filter is better than that produced by the time-invariant Wiener filter – of the same structure, for the wide-sense stationary scenario at hand – and also because the adaptive filter weights behave in a time-varying manner.

We have offered an explanation for such effects, concentrating on the normalized least-mean-square (NLMS) algorithm [2]. The explanation centered on replacing the auxiliary channel signals of the optimal time-invariant two-channel Wiener filter with linear combinations of the usual reference signals, using the linking sequences. The latter provide a sample-by-sample link between the auxiliary signals and any of the particular reference inputs.

Here we explore the linking sequence behavior itself, with the idea of connecting enhanced adaptive filter performance with specific linking sequence characteristics.

The paper is organized as follows. Section 2 reviews the adaptive noise canceling (ANC) setup, the NLMS algorithm, and some of its properties and relevant interpretations. Section 3 reviews the linking sequence concept, connecting the optimal two-channel Wiener filter with the target of NLMS adaptation. Section 4 looks at the behavior of the linking sequences in the ANC application, under different signal-to-noise ratio (SNR) conditions. A summary is provided in Section 5.

2. NLMS-ANC SETUP

An important application of adaptive filtering is in adaptive noise, or interference, canceling (ANC). The desired or primary signal d_n contains narrowband noise or interference, which we desire to mitigate using a filtered version of the reference signal r_n, in the form of a linear combination of r_n and some of its delayed versions. The usual assumption is that reference and interference are correlated. We therefore assume the signal generator structure in Fig. 1 for our later simulations.

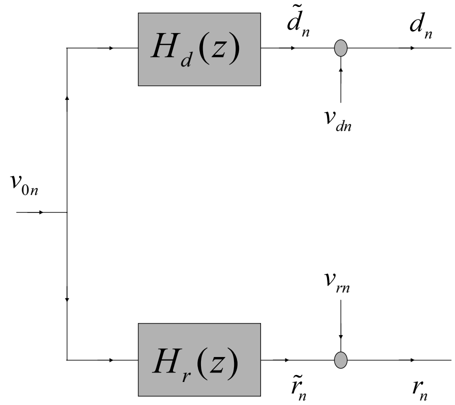

Fig. 1 Signal Generation.

Considering the jointly wide-sense stationary Gaussian process $\{d_n, r_n\}$, the optimal (minimum mean square error) causal estimate of d_n is given by a linear operation on the causal past of $\{d_{n-1}, r_n\}$. Both the Wiener and the NLMS filtering operations are represented in the two-channel filtering structure in Fig. 2, with the auxiliary channel signal x_n defined as d_{n-1}.

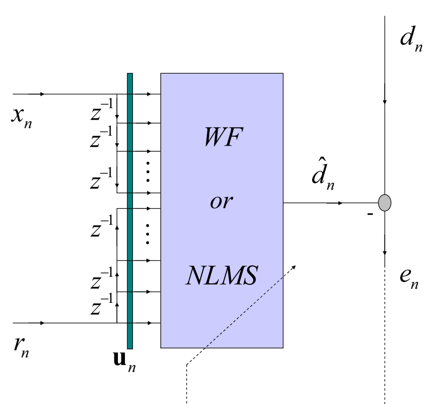

Fig. 2 Two-Channel Filtering.

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Note that in the usual ANC implementation a single-channel filter is used, with the auxiliary channel signal x_n – for all practical purposes – set to zero. To simultaneously cover the single-channel case and the two-channel case, we define the reference vector input to the NLMS algorithm as follows:

$$\mathbf{u}_n = \begin{bmatrix} \mathbf{x}_n \\ \mathbf{r}_n \end{bmatrix} \tag{1}$$

The NLMS adaptation algorithm is the usual one.

$$\begin{aligned} \hat{d}_n &= \mathbf{w}_n^H \mathbf{u}_n \\ e_n &= d_n - \hat{d}_n \\ \mathbf{w}_{n+1} &= \mathbf{w}_n + \bar{\mu} \frac{e_n^*}{\mathbf{u}_n^H \mathbf{u}_n} \mathbf{u}_n \end{aligned} \tag{2}$$

When the model for the estimate of the desired signal $\hat{d}_n$ – used in adaptation, as expressed by the first line of (2) – is the same as the structure of the desired signal d_n, i.e.

$$d_n = \mathbf{w}_o^H \mathbf{u}_n \tag{3}$$

for some ideal weight vector $\mathbf{w}_o$, then the NLMS weight vector converges to the latter. The *a posteriori* errors will be zero and the weight vector increment norms will be zero. If some noise is present in (3), the NLMS weight vector converges to a neighborhood of $\mathbf{w}_o$. For small step-sizes the NLMS weight vector will be close to the best possible constant weight vector.

3. LINKING SEQUENCE CONCEPT

As indicated above, the optimal estimate for the desired signal $\hat{d}_n^0$, with its corresponding optimal error ε_n, allows us to express the desired signal as follows.

$$\begin{aligned} d_n &= \hat{d}_n^o + \varepsilon_n \\ &= \mathbf{w}_o^H \mathbf{u}_n + \varepsilon_n \\ &= \begin{bmatrix} \mathbf{w}_x^H & \mathbf{w}_r^H \end{bmatrix} \begin{bmatrix} \mathbf{x}_n \\ \mathbf{r}_n \end{bmatrix} + \varepsilon_n \end{aligned} \tag{4}$$

Using single-channel NLMS adaptation, mean-square error (MSE) has been observed to sometimes come close to its performance bound [3], i.e. the optimal two-channel MSE. For this to happen, the following single-channel NLMS model

$$\begin{aligned} d_n &= \mathbf{w}_r^H \mathbf{r}_n + e_n \\ &= \hat{d}_n + e_n \end{aligned} \tag{5}$$

must be connected, or linked, to the inherent model or structure of the desired signal, as expressed in (4). The linking sequence concept was invented to facilitate writing (4) in the form of the model structure of (5), by expressing x_{n-m} – elements of the auxiliary channel – in terms of r_{n-l}, elements of the reference channel. The linking sequences were defined as follows [2,4].

$$\rho_{n-l}^{(-m+l)} = \frac{x_{n-m}}{r_{n-l}} = \frac{x_{n-l+(-m+l)}}{r_{n-l}} \tag{6}$$

Note that an element of $\mathbf{x}_n$ can be written in terms of any element of $\mathbf{r}_n$ this way, and that an affine combination (all α_l summing to 1) of these possibilities is equally valid. This leads to the following equivalent of (4),

$$d_n = \left(\sum_{m=0}^{M-1} w_{x,m+1}^* \sum_{l=0}^{L-1} \alpha_l \rho_{n-l}^{(-m+l)} \mathbf{1}_{l+1}^H + \mathbf{w}_r^H \right) \mathbf{r}_n + \varepsilon_n \tag{7}$$

where $\mathbf{1}_{l+1}$ is a zero-vector except for its (l+1)st element of 1.

Non-Wiener effects have been observed at large step-size, for example $\bar{\mu} = 1$, where adaptation is fastest. For the latter, the *a posteriori* error equals zero [5], i.e.

$$d_n = \mathbf{w}_{n+1}^H \mathbf{r}_n \tag{8}$$

The latter suggests that the term in parentheses in (7) represents the general form of $\mathbf{w}_{n+1}$, i.e. the manifold of *a posteriori* NLMS weight vectors from which that vector is selected which minimizes the norm of the weight vector increment [5]. The conditions for the latter interpretation to hold are that (4) represents the optimal two-channel solution, i.e. the number of taps in the auxiliary and reference channels is sufficiently high, and that the minimum mean square error approaches zero. For the first scenario in Section 4 both conditions are met.

Considering that the NLMS adaptation mechanism picks the appropriate affine combination, we see that the behavior of the *a posteriori* weight vector is determined by the behavior of the linking sequences. Generally, by their definition in (6), the linking sequences are time-varying.

4. ANC LINKING SEQUENCE BEHAVIOR

The signal generation process uses AR(1) (first order autoregressive) filters driven by the same white noise. The poles are $p_d = 0.99\exp(j\frac{\pi}{3})$ and $p_r = 0.99\exp\{j(\frac{\pi}{3} + \frac{2\pi}{20})\}$. We add white noise to the AR(1) outputs so that the desired signal-to-noise (SNR) ratios result. In our examples SNR_d=SNR_r, for simplicity's sake, and – to emphasize non-linear effects – a step-size of $\bar{\mu} = 1$ is used throughout.

For our first case SNR equals 80 dB. It can be shown that in this case, the optimal filter in (4) is reached with one tap ($M = 1$) in the auxiliary channel, i.e. $\mathbf{x}_n = d_{n-1}$, and two taps ($L = 2$) in the reference channel. With these choices, the expression in (7) becomes relatively simple, using only two linking sequences. Fig. 3 shows the error performance of the adaptive filter AF(0,2), using zero auxiliary taps and two reference taps, in comparison with the optimal time-invariant Wiener filter WF(0,2) of the same structure. We observe that the adaptive filter on an almost sample-by-sample basis produces smaller errors than its optimal time-invariant counter part. Fig. 4 shows the corresponding weight behavior (real part shown, imaginary part behaves similarly), which exhibits the pseudo- periodic nature (period 20) reflected in the time-varying weights of (7) [2]. Figs. 5 and 6 show the behavior of the magnitude and phase, respectively, of the sample to sample change in the two linking sequences. We observe that the largest performance improvement coincides with relatively small changes in the linking sequence differences, i.e. their magnitudes and phases are about the same from sample to sample. We further observe that the phase difference is relatively constant, small, and non-zero. These characteristics are responsible for the pseudo-periodic weight behavior. For SNR down to 20 dB the results are similar. At SNR equal to 10 dB deterioration sets in, as shown in Figs. 7 through 10.

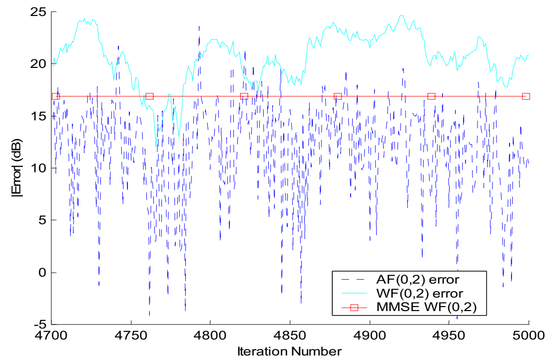

Fig. 3 ANC Performance Comparison for SNR=80 dB.

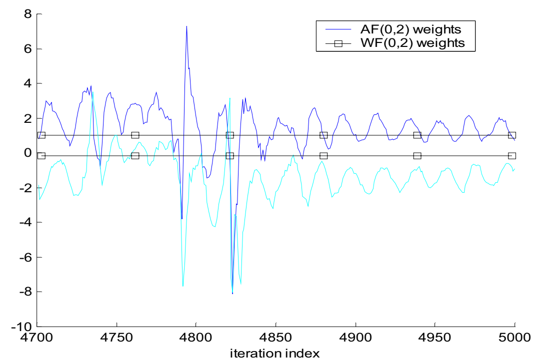

Fig. 4 ANC Re(weight) Behavior for SNR=80 dB.

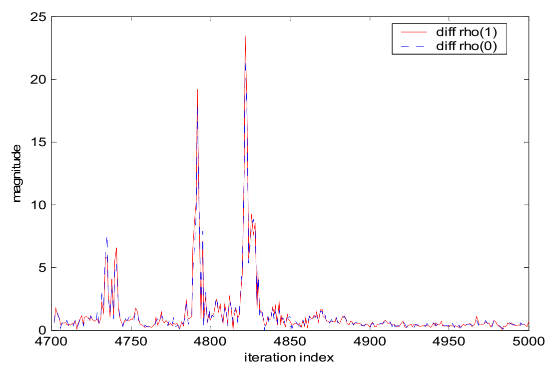

Fig. 5 Magnitude Linking Difference for SNR=80 dB.

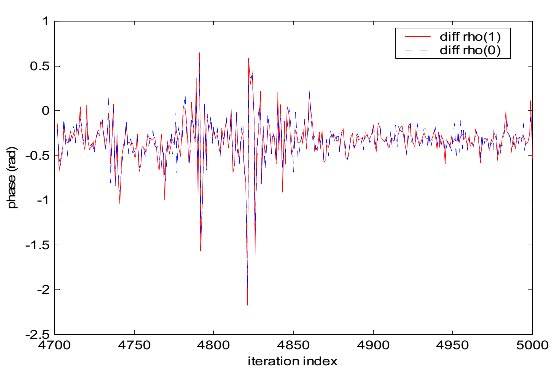

Fig. 6 Phase Linking Difference for SNR=80 dB.

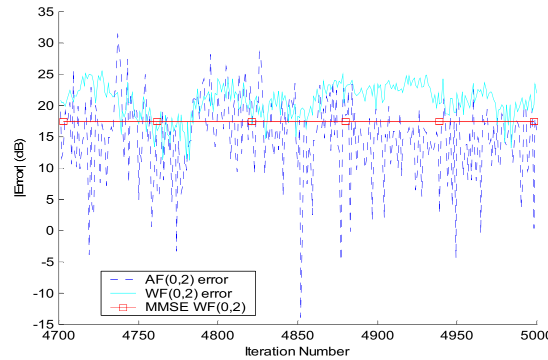

Fig. 7 ANC Performance Comparison for SNR=10 dB.

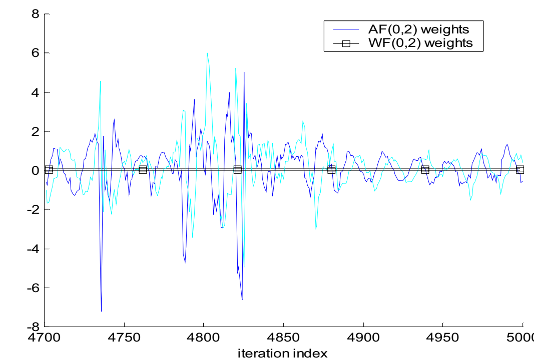

Fig. 8 ANC Re(weight) Behavior for SNR=10 dB.

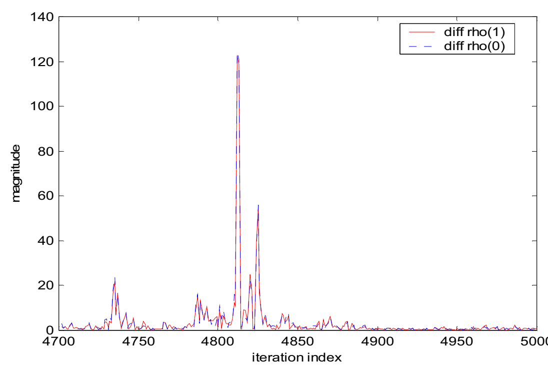

Fig. 9 Magnitude Linking Difference for SNR=10 dB.

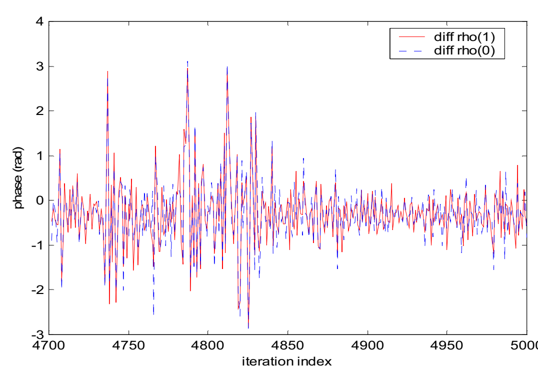

Fig. 10 Phase Linking Difference for SNR=10 dB.

At SNR equal to 10 dB there is – at times – still performance improvement of AF(0,2) over WF(0,2). There is also still semi-periodic weight behavior during the interval of most significant improvement. However, the variation in the linking sequences has increased substantially. The latter is clearly observed in the phase of the linking sequence differences. The interval over which the variational behavior is most prominent corresponds directly to the interval where performance is reduced and semi-periodic time-varying weight behavior has vanished.

Lowering SNR to 0 dB leads to performance of the optimal time-invariant filter being better than that of the adaptive filter for almost any interval. Semi-periodic behavior of the weights is observed during rare and short intervals only, as seen in Fig. 11 around iteration index 4910.

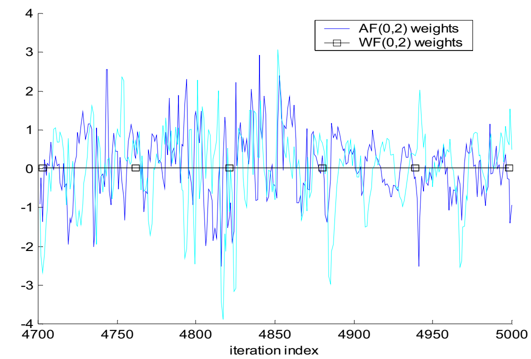

Fig. 11 ANC Re(weight) Behavior for SNR=0 dB.

The linking sequences become more random and highly variational, in particular with respect to their changes in phase from sample to sample, as shown in Fig. 12.

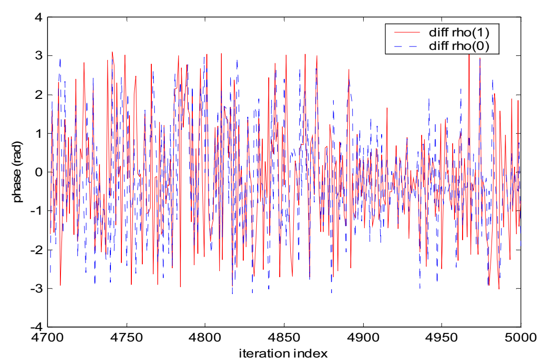

Fig. 12 Phase Linking Difference for SNR=0 dB.

Note from (7) that the desired signal consists of a component that can be linked to the reference input and a component consisting of noise. In the above examples we went from the reference-linked component being dominant and slowly varying to the noise component being comparable in strength together with a much quicker varying reference-linked component. Recall that the manifold determines the *a posteriori* weight vector, and that NLMS incurs a lag error. Under noisier and quicker varying conditions the lag error becomes relatively large.

In the final example we return to SNR of 80 dB, but now we set the pole radii to 0.7, i.e. the AR(1) processes are now wideband rather than narrowband. As in the previous case, here too the optimal WF(0,2) performs better than the AF(0,2). Fig. 13 shows that the linking sequences change phase rather rapidly. The error component affecting the manifold is rather small here.

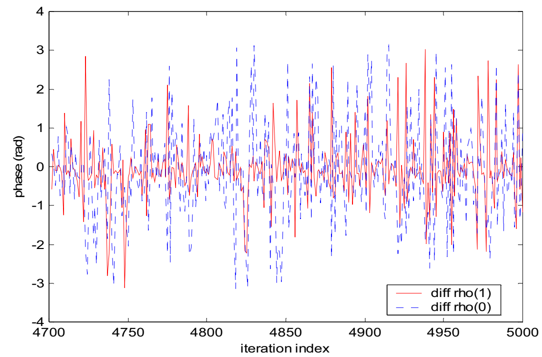

Fig. 13 Phase Linking Difference for SNR=80 dB Wideband AR(1) Processes.

The performance of the theoretical Wiener filters for the above cases is given in Table 1.

Table 1 Theoretical Wiener Filter Performance.

	NARROW			WIDE
SNR dB	80	10	0	80
MMSE (1,2)	1.99_{10}-6	10.41	75.83	5.84_{10}-8
MMSE (0,2)	49.05	54.99	100.36	0.1496

For the 10 dB narrowband case AF(0,2) improved over WF(0,2), while for the 80 dB wideband case no advantage was realized. The difference lies in the variability of the linking sequences.

5. SUMMARY

We investigated the behavior of the linking sequences defined earlier to explain the occurrence of non-Wiener effects in adaptive NLMS filtering. These experiments show that in the adaptive noise canceling application the occurrence of non-Wiener effects is paired with relatively constant behavior of the linking sequences, which happens for narrowband processes.

6. REFERENCES

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