ENHANCEMENT OF SECOND-ORDER CYCLOSTATIONARY SIGNALS: APPLICATION TO VIBRATION ANALYSIS

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ABSTRACT

This paper addresses the issue of extracting the pure second-order cyclostationary (CS2) part of a signal. This proves very useful in many situations where the CS2 part actually contains most of the information in a signal, such as in communications or in vibration analysis. The proposed method exploits the spectral redundancy induced by the pure CS2 part and tries to reconstruct it by combining several filtered frequency-shifted versions of the signal. The derivation of the optimal filters is described in detail. The effectiveness of the method is finally demonstrated on both simulated and real industrial examples.

1. INTRODUCTION

Cyclostationarity has proved extremely useful for modelling communication signals and has led to many breakthroughs in that field. Recently, it has been demonstrated that cyclostationarity also provides powerful tools for analysing vibration signals captured on rotating machinery, for noise control or diagnosis purposes. Just like communication signals, vibration signals are cyclostationary at different orders. Depending on the user's objectives, a given order of cyclostationarity may be scrutinized in priority. Indeed, numerous analyses are mainly concerned with either the periodic part, which belongs to first-order cyclostationarity (CS1), or the second-order cyclostationary (CS2) part of a signal. The former is usually examined with classical Fourier analysis whereas the latter is scrutinized through the spectral correlation or the Wigner-Ville spectrum [1]. The interest of separately processing the periodic part and the CS2 part is illustrated in [1, 2]. Moreover, each part may have a different physical significance. For example, in a gearbox, the periodic part can be assigned to the gear signal whereas the CS2 part rather relates to the bearing signal [3]. The extraction of the periodic part from a signal can efficiently be achieved by various algorithms, which are all different means of filtering a signal: synchronous averaging [4], adaptive line enhancer [5], zeroing the discrete components in the Fourier Transform, etc. As a by-product, the residual component resulting from this operation contains the *pure* CS2 part. However, the latter is usually strongly corrupted by additive noise sources. The purpose of this paper is to propose a technique that can extract the pure CS2 part from the residual signal. To the authors' knowledge, this issue has never been addressed in this form in the signal processing literature. Although the SCORE algorithm introduced by Agee, Shell and Gardner [6] achieves a very similar goal, it uses spatial filtering on a array of sensors and therefore was not suited to our application. We propose herein an approach based on time filtering which equally applies to the multi- and mono-sensor cases. The key idea will first be introduced in the frequency domain where it can intuitively be understood as exploiting the spectral coherence that exists between the spectral components of a CS2 signal. A set of optimal linear periodically-time-varying filters (LPTV) will then be derived in the time domain where it is better adapted to numerical implementation.

The authors would like to thank the Region Rhone Alpes (France) and the DSTO (Australia) for funding.

2. PROBLEM STATEMENT

2.1. Definitions

A signal is said to be purely cyclostationary at order N if its cumulants of order N are all (quasi) periodic functions of time. It is often more relevant to use the concept of *pure* cyclostationarity (i.e. cumulants instead of moments) because it implicitly removes all cyclostationarities coming from lower order moments [7]. For example, a signal $s(n)$ is said to be purely CS2 if

$$\mathbb{E}\left\{(s(n)-\mathbb{E}s(n))\,(s(n-\tau)-\mathbb{E}s(n-\tau))\right\} \triangleq C_{2s}(n,\tau) \quad (1)$$

is a (quasi) periodic function of n, i.e. if the autocorrelation function of the centered signal $c(n) = s(n) - \mathbb{E}s(n)$ has a Fourier expansion

$$C_{2s}(n,\tau) = R_{2c}(n,\tau) = \sum_{\alpha_k} R_{2c}^{\alpha_k}(\tau) e^{j2\pi\alpha_k n} \quad (2)$$

over a non-empty set of cyclic frequencies $\alpha_k \neq 0$. In the above equation, the $R_{2c}^{\alpha_k}(\tau)$ Fourier-Bohr coefficients are known as the cyclic autocorrelation functions of signal $c(n)$.

2.2. Signal enhancement issue

For the sake of simplicity, the issue is presented here for one sensor output $s(n)$. The admitted model is [2]:

$$s(n) = p(n) + c(n) + n(n) \quad (3)$$

where

- $p(n) = \mathbb{E}p(n)$ is the periodic part,

- $c(n)$ is the pure CS2 part,
- $n(n)$ is the noise which contains all components not pertaining to $p(n)$ or $c(n)$. It is uncorrelated with $c(n)$ - correlated "noise" will be included in $c(n)$. Note that $n(n)$ is not necessarily stationary and can exhibit higher order types of pure cyclostationarity.

As mentioned in the Introduction, it is an easy matter to extract the periodic part p from the measured signals s. Hence, the remainder of the paper will only focus on extracting the pure CS2 part c from the centered signal

$$x(n) = c(n) + n(n) \tag{4}$$

Indeed, just as first-order cyclostationarity is used to estimate p, advantage will now be taken of second-order cyclostationarity to estimate c.

3. PRESENTATION OF THE METHOD

3.1. A solution based on spectral redundancy

Let $X(f)$, $C(f)$ and $N(f)$ be the Fourier transforms of $x(n)$, $c(n)$ and $n(n)$, respectively. By convention $Y(f)$ denotes the spectral increment $dY(f)$ in the Cramér's decomposition of signal $y(n)$ [8]. It can be shown that the second-order cyclostationarity of signal $c(n)$ implies that

$$E\left\{X(f)X^H(f-\alpha_k)\right\} \triangleq \mathbf{S}_{2x}^{\alpha_k}(f) = \mathbf{S}_{2c}^{\alpha_k}(f) \neq 0 \tag{5}$$

for any cyclic frequency $\alpha_k \neq 0$, where the cyclic spectral matrix $\mathbf{S}_{2c}^{\alpha_k}(f)$ has for elements the Fourier transforms of the cyclic autocorrelation functions $R_{2c}^{\alpha_k}(\tau)$. As is well-known, this means that there exist non-zero correlations between spectral components of a CS2 signal spaced apart by α_k [9]. The idea of the proposed method is to exploit these correlations. To do so, the signal is duplicated and shifted in the frequency domain by amounts corresponding to the α_k's. These components are then multiplied by appropriate filter transfer functions and combine together in order to reconstruct the original signal. Figure 1 illustrates this idea on a simplified mono-sensor case dedicated to extracting the pattern at frequency f_1 from the background noise. The reconstruction is limited there to using the two cyclic frequencies $\alpha_1 = f_1$ and $\alpha_2 = f_2$.

Obviously, increasing the number of cyclic frequencies will more efficiently reduce the background noise. At this stage, it is important to point out that the proposed method will do its best to *reduce* the effect of the noise. Perfect *cancellation* is in general not possible and can only be achieved if some regions of the signal and the noise spectra do not overlap.

In general, the reconstruction equation will be:

$$\hat{C}(f) = \underline{G}(f) \cdot \underline{X}_e(f) \tag{6}$$

where

$$\underline{X}_e(f) = \begin{bmatrix} X(f-\alpha_1) & \cdots & X(f-\alpha_K) \end{bmatrix}^T \tag{7}$$

is a $(K \times 1)$ extended vector which contains K cyclic frequencies, $\hat{C}(f)$ is a the estimated CS2 part and $\underline{G}(f)$ is an $(1 \times K)$ transfer vector to be properly designed. The next section will now detail how to identify the transfer vector $\underline{G}(f)$ from the centered signal.

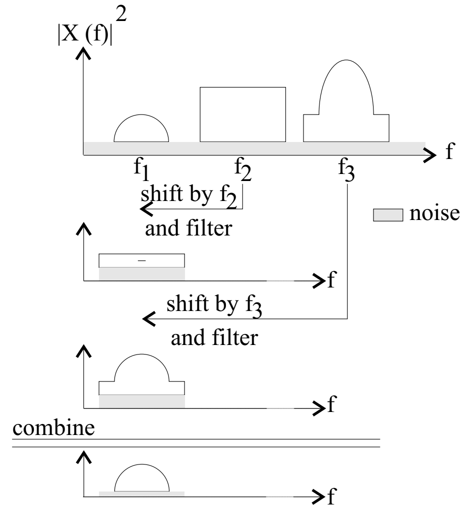

Fig. 1. Noise reduction principle

3.2. Optimal filtering

With scalar formulation, equation (7) becomes:

$$\hat{C}(f) = \sum_{k=1}^{K} G_k(f) \cdot X(f-\alpha_k) \tag{8}$$

When all signals of interest are assumed real, the time domain counterpart of (8) is:

$$\hat{c}(n) = \sum_{k=1}^{K} \{h_k(n) * [x(n)\cdot\cos(2\pi\alpha_k n)] \cdots \\ \cdots + l_k(n) * [x(n)\cdot\sin(2\pi\alpha_k n)]\} \text{ for } \alpha_k \neq 0\ \forall k \tag{9}$$

with h_k and l_k the real and the imaginary parts of the inverse Fourier transform of G_k, respectively. This can be recognized as a MISO (Multiple Input Single Output) structure of LPTV filters, the properties of which have been extensively analysed in [10]. Interestingly enough, LPTV filters can be demonstrated to be optimal among all linear filters for denoising CS2 signals [2].

Let L be the length of the filters. Then, in vector form:

$$\underline{h}_k = \begin{bmatrix} h_k(\tau_1) & \cdots & h_k(\tau_L) \end{bmatrix}^T \tag{10}$$
$$\underline{l}_k = \begin{bmatrix} l_k(\tau_1) & \cdots & l_k(\tau_L) \end{bmatrix}^T \tag{11}$$
$$\tag{12}$$

where the limits τ_1 and τ_L are to be set by the user (in general, the filters will not have to be constrained to be causal). Let us also write

$$a_k(n) = x(n)\cdot\cos(2\pi j\alpha_k n) \tag{13}$$
$$b_k(n) = x(n)\cdot\sin(2\pi j\alpha_k n) \tag{14}$$

for the modulated inputs. More concisely, in vector form:

$$\underline{a}_k(n) = \begin{bmatrix} a_k(n-\tau_L) & \cdots & a_k(n-\tau_1) \end{bmatrix} \tag{15}$$
$$\underline{b}_k(n) = \begin{bmatrix} b_k(n-\tau_L) & \cdots & b_k(n-\tau_1) \end{bmatrix} \tag{16}$$

The estimate $\hat{c}(n)$ is then given by:

$$\hat{c}(n) = \sum_{k=1}^{K} \left[\underline{a}_k(n) \cdot \underline{h}_k + \underline{b}_k(n) \cdot \underline{l}_k \right] \tag{17}$$

The unknown filters in the above equation can be identified by minimizing the squared estimation error $\varepsilon(n)^2$ between the reconstructed signal $\hat{c}(n)$ and the original signal $x(n)$ at different times n. There are $u = 2K \cdot L$ unknown terms, so at least $N_{min} = u + \tau_L - \tau_1$ samples must be used. Considering a signal of N samples ($N \geq N_{min}$), minimizing $\varepsilon(n)^2$ leads to the system of equations:

$$\mathbf{C_{2m}} \cdot \underline{s} = \underline{C}_{m,c} \tag{18}$$

where $\mathbf{C_{2m}} = \mathbf{m}^H \cdot \mathbf{m}$, $\underline{C}_{m,c} = \mathbf{m}^H \cdot \underline{c}$ with

$$\underline{\hat{c}} = \left[\begin{array}{ccc} \hat{c}\left(N - \tau_1\right) & \cdots & \hat{c}\left(\tau - \tau_L\right) \end{array} \right]^T \tag{19}$$

$$\underline{s} = \left[\begin{array}{cccccc} \underline{h}_0 & \cdots & \underline{h}_K & \underline{l}_0 & \cdots & \underline{l}_K \end{array} \right]^T \tag{20}$$

and

$$\mathbf{m}_{i,j} = \begin{cases} \underline{a}_j\left(N - \tau_1 - i + 1\right) & \text{for } j \leq K, \\ \underline{b}_{j-K}\left(N - \tau_1 - i + 1\right) & \text{for } j > K. \end{cases}$$

$$1 \leq i \leq N - \tau_L + \tau_1, \quad 1 \leq j \leq 2K \tag{21}$$

Since the noise is uncorrelated with the signal, the unknown correlation vector $\underline{C}_{m,c}$ can be substituted by $\underline{C}_{m,x} = \mathbf{m}^H \cdot \underline{x}$, provided that the nil cyclic frequency is not taken into account in the filters. Then, the solution to the system of equations

$$\mathbf{C_{2m}} \cdot \underline{s} = \underline{C}_{m,x} \tag{22}$$

finally yields the desired optimal filters $\underline{h}_k$ and $\underline{l}_k$. This can be carried out by direct pseudo-inversion, or efficiently implemented by means of an adaptive LMS-type algorithm. Figure 2 summarizes the algorithm in the simple mono-sensor case.

It is worth pointing out that the proposed method is in essence very similar to a blind Wiener filter. Indeed, equation (6) or (9) could be viewed as special forms of a linear predictive equation right from the beginning. Similarly, the system of equations (22) could be viewed as a special (extended) form of the normal equations.

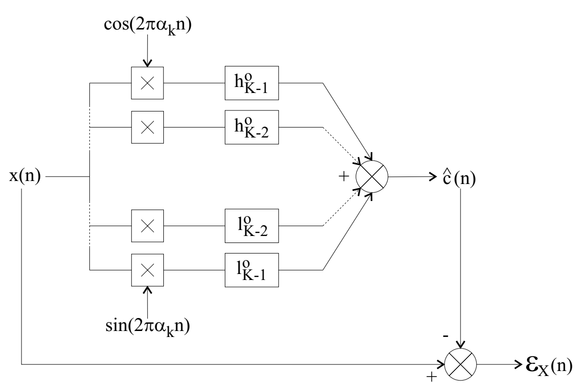

Fig. 2. Scheme for estimating $c(n)$ in the mono-sensor case

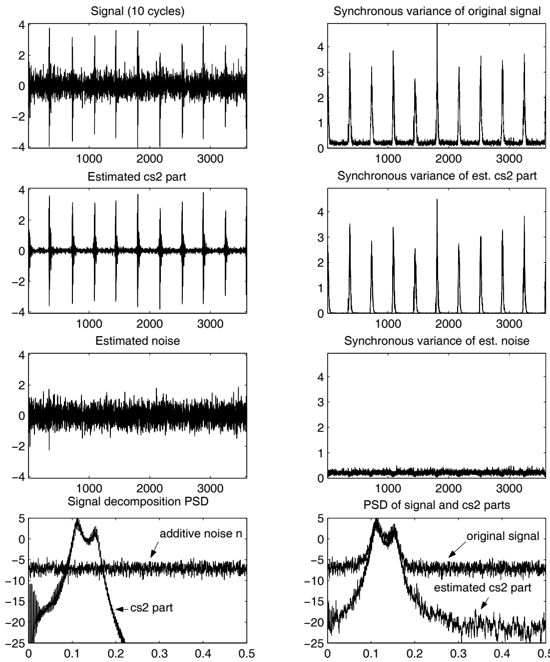

Fig. 3. Simulated signal

4. APPLICATIONS TO SIMULATED AND REAL SIGNALS

The first example demonstrates the proposed method on a synthesized quasi-cyclostationary signal:

$$s(n) = \left[\sum_i A_i \delta\left(n - iT - \tau_i\right) \right] * e(n) + n(n) \tag{23}$$

where:

- $\delta(n)$ is the discrete Dirac impulse,
- $n(n)$ is a white stationary noise,
- T is the mean inter-arrival time,
- A_i is a random amplitude following a Gaussian law $N(\mu_A, \sigma_A^2)$,
- τ_i is a sequence of independent and identically distributed random variables following a Gaussian law $N(0, \sigma_\tau^2)$,
- $e(n)$ is an impulse response resulting from cascading second-order (mass/stiffness/damping) systems.

Signal (23) is a reasonable model for many vibration signals captured on rotating machinery experiencing an internal fault [1, 2, 3]. Due to the random jitter τ_i, it can be shown to have decomposition (3), where the periodic part and the pure CS2 part have the same cyclic fundamental frequency $\alpha_1 = 1/T$. Figure 3 displays the signal over ten cycles before and after the proposed algorithm was used with cyclic frequencies $\alpha_k = k/T$, $k = 1, ..., 6$.

The second example is an application of the proposed technique to an industrial case. The signal of interest is a vibration measurement captured on a rotating machine, which was suspected to have a rolling-element-bearing fault. The characteristic feature of such a fault is to exhibit a series of impacts dominated by the

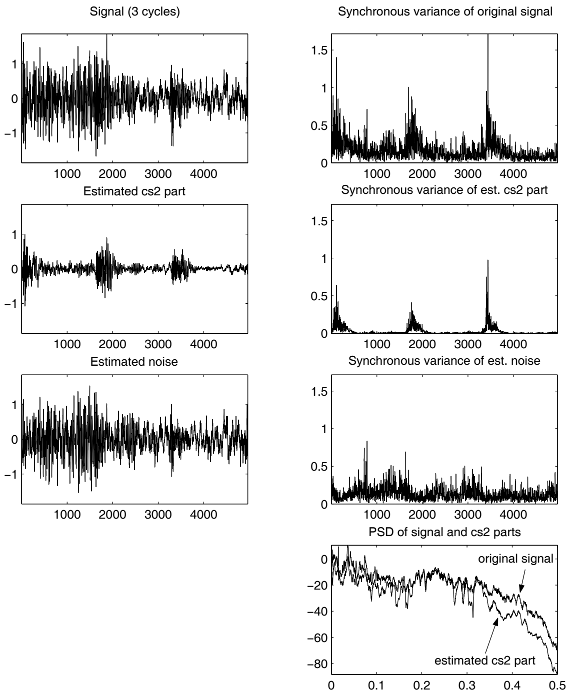

Fig. 4. Real industrial signal

major resonance frequencies of the structure [11]. Such a signal can be well modelled by Eq. (23) and thus experiences second-order cyclostationarity. Angular resampling combined with synchronous averaging was used to estimate and to remove the periodic part, which essentially stemmed from the gears. The resulting centered signal contained the impacts due to the fault, but was highly masked by additive background noise. Figure 4 displays it over three cycles before and after the proposed algorithm was used with cyclic frequencies $\alpha_k = k/T$, $k = 1, ..., 6$. The good enhancement of the first three impacts is obvious.

5. CONCLUSION

Decomposing a cyclostationary signal into its (*i*) periodic part, (*ii*) pure second-order cyclostationary part and (*iii*) noise part may prove very useful in a number of applications since these three components usually carry different types of information. Although many efficient techniques exist for extracting the periodic part, the issue becomes much more complicated when it comes to the pure CS2 part, and actually has rarely been addressed in the literature. The present paper introduced an original method dedicated to this difficult task. It consists in designing an optimal linear-periodically-time-varying Wiener filter, the identification of which was shown to be feasible from the centered signal provided *non-zero* cyclic frequencies are used. The performance of such a filter depends strongly on the spectral supports of the noise and the signal to be filtered out. In general, *perfect* extraction is only possible if those supports do not overlap completely. However, the performance can be shown to increase monotonically with the number of cyclic frequencies taken into account. In practice, this number will be limited by computing costs and memory access. The relevance of the technique was illustrated both on simulated and on actual vibration signals measured on a rotating machine. The filter was able to extract the repetitive patterns resulting from a fault in a rolling element bearing.

6. REFERENCES

[1] J. Antoni, F. Bonnardot, A. Raad, and M. El Badaoui, "Cyclostationary modeling of rotating machine vibration signals," *Mechanical Systems and Signal Processing*, accepted for publication.

[2] J. Antoni, *Apports de l'échantillonnage angulaire et de la cyclostationnarité au diagnostic par analyse vibratoire des moteurs thermiques (in French)*, Ph.D. thesis, INP Grenoble, Grenoble, December 2000.

[3] J. Antoni and R.B. Randall, "Differential diagnosis of gear and bearing faults," *ASME Journal of Vibration and Acoustics*, vol. 124, no. 2, pp. 165–171, 2002.

[4] P.D. McFadden, "A revised model for the extraction of periodic waveforms by the time domain averaging," *Mechanical Systems and Signal Processing*, vol. 1, no. 1, 1987.

[5] J. Antoni and R.B. Randall, "Unsupervised noise cancellation for vibration signals: part i-evaluation of adaptive algorithms," *Mechanical Systems and Signal Processing*, in press 2003.

[6] B.G. Agee, S.V. Schell, and W.A. Gardner, "Spectral self-coherence restoral: a new approach to blind adaptive signal extraction using antenna arrays," *Proceedings of the IEEE*, vol. 78, no. 4, pp. 753–767, April 1990.

[7] W.A. Gardner and C.M. Spooner, "The cumulant theory of cyclostationary time-series, part i : fondations," *IEEE Transactions on Signal Processing*, vol. 42, no. 12, pp. 3387–3409, 1994.

[8] H. Cramér, *Mathematical methods of statistics*, Princeton University, 1946.

[9] W.A. Gardner, *Cyclostationarity in communications and signal processing*, IEEE Press, New York, 1^{st} edition, 1994.

[10] W.A. Gardner, "Cyclic wiener filtering: theory and method," *IEEE Transactions on communication*, vol. 41, no. 1, pp. 151–163, January 1993.

[11] R. B. Randall, J. Antoni, and S. Chobsaard, "The relationship between spectral correlation and envelope analysis in the diagnostics of bearing faults and other cyclostationary machine signals," *Mechanical Systems and Signal Processing*, vol. 15, no. 5, pp. 945–962, September 2001.