SPATIAL AND POLARIZATION CORRELATIONS IN NONSTATIONARY ARRAY PROCESSING

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ABSTRACT

Bilinear synthesis of nonstationary signals impinging on a multi-antenna receiver has been recently introduced. The distinction in the spatial signatures of the sources provides a vehicle to reduce noise and source signal interactions in the time-frequency domain, and hence improves signal synthesis. In this paper, in addition to the spatial domain information, we utilize polarization diversity for enhanced source time-frequency signal representations. It is shown that dual-polarization antennas can be used to mitigate cross-terms via a combined spatial and polarization averaging. Significant reduction in cross-terms can be realized by providing large spatial diversity, large polarization diversity, or a combined moderate values of the respective spatial and polarization correlations.

1. INTRODUCTION

Nonstationary signal analysis can be performed using linear or quadratic distributions. In the case of quadratic time-frequency distributions, the interactions between the signal multi-components due to the bilinear products introduce cross-terms. These terms are undesirable, as they overshadow and obscure the true time-frequency signatures of the signals [1, 2]. There are several effective techniques which have been introduced for cross-term mitigation [1, 3, 4]. However, these techniques are devised for a single antenna receiver and may not be suited for real time signal processing.

Recent work in sensor signal processing has shown how to substantially reduce the cross-terms using antenna arrays [5]. In this case, the multi-component signals arise from the mixture of the mono-component source signals at each antenna. It is shown that by averaging the time-frequency distributions of the sensor data across the array, the spatial separation of the sources can be utilized to suppress the source cross-terms without introducing bias or smoothing of the source auto-terms. This separation is measured by the spatial correlation of the source spatial signatures. The latter depend on the source angular position, the multipath environment, and the array manifold, incorporating mutual coupling and antenna patterns. Weaker correlations, in general, yield stronger suppression of cross-terms. In the case that the sources are closely separated or the array is of a small aperture, the benefits of the antenna array in enhancing the source time-frequency representation become limited. When dual polarized antennas are used, however, the polarization diversity can relax the conditions on the source spatial resolution, and, in essence, work with the available spatial diversity of the array for effective suppressions of cross-terms.

In this paper, we present an analytical framework in which both polarization and spatial diversities are integrated and used for nonstationaty array processing. It is shown that by averaging the time-frequency distributions (TFDs) over the two polarizations across all the elements of the array, both spatial and polarization correlations appear as fractional products, multiplying the distribution cross-terms. As such, significant reduction in cross-terms can be realized by providing large spatial diversity, large polarization diversity, or a combined moderate values of the respective spatial and polarization correlations.

2. SIGNAL MODEL

Assume L source signals $s_l(t), l = 1, ..., L$, are incident on an array with N dual-polarization antenna sensors. We use $^{[p]}$ and $^{[q]}$ to represent the two orthogonal polarizations (e.g., vertical and horizontal polarizations) at each sensor. The received data for each polarization is the linear combination of the same polarization components of the source signals and noise. That is, the signal received at the nth sensor of polarization i,

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where $i = p$ or q, is

$$x_n^{[i]}(t) = \sum_{l=1}^{L} a_{n,l}^{[i]} s_l^{[i]}(t) + n_n^{[i]}(t), \quad (1)$$

where $a_{n,l}^{[i]}$ represents the propagation coefficient of the lth source to the nth sensor with polarization i, and $n_n^{[i]}(t)$ is the noise component at the same polarization.

The data vector at each polarization, $\mathbf{x}^{[i]}(t)$, $i = p$ or q, is expressed as

$$\begin{aligned} \mathbf{x}^{[i]}(t) &= \mathbf{y}^{[i]}(t) + \mathbf{n}^{[i]}(t) \\ &= \mathbf{A}^{[i]}\mathbf{s}^{[i]}(t) + \mathbf{n}^{[i]}(t) \\ &= \mathbf{A}^{[i]}\mathbf{V}^{[i]}\mathbf{s}(t) + \mathbf{n}^{[i]}(t), \end{aligned} \quad (2)$$

where $\mathbf{y}^{[i]}(t)$ and $\mathbf{n}^{[i]}(t)$ are, respectively, the noise-free data vector and the noise vector, $\mathbf{A}^{[i]}$ is the data mixing matrix, and $\mathbf{V}^{[i]} = \text{diag}[v_1^{[i]}, \cdots, v_L^{[i]}]$, with

$$v_l^{[p]} = \cos\gamma_l \quad \text{and} \quad v_l^{[q]} = \sin\gamma_l e^{j\eta_l} \quad (3)$$

representing the coefficients of the lth source for the p and q polarizations. In (3), γ_l determines the magnitude ratio and η_l determines the phase difference between the two polarizations. Without loss of generality, it is assumed that the norm of the spatial signature vector $\mathbf{a}_l^{[i]} = [a_{1,l}^{[i]}, \ \cdots, \ a_{N,l}^{[i]}]^T$ for each source's polarization is N, where $l = 1, \cdots, L$, $i = p$, q, and ^{T} denotes transpose. From (3), it is clear that $\mathbf{v}_l = [v_l^{[p]} \ \ v_l^{[q]}]^T$ has a unit norm. Thus, the strength of the source signals is absorbed in the magnitude of $s_l(t)$.

The combined data vector received at a dual-polarized antenna array is expressed in the following vector format

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}^{[p]}(t) \\ \mathbf{x}^{[q]}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{[p]}\mathbf{V}^{[p]} \\ \mathbf{A}^{[q]}\mathbf{V}^{[q]} \end{bmatrix} \mathbf{s}(t) + \begin{bmatrix} \mathbf{n}^{[p]}(t) \\ \mathbf{n}^{[q]}(t) \end{bmatrix}. \quad (4)$$

For each polarization, the auto- and cross-polarization spatial time–frequency distribution (STFD) in discrete-time is given by

$$\begin{aligned} &D_{\mathbf{x}^{[i]}\mathbf{x}^{[k]}}(t,f) \\ &= \sum_{u=-\infty}^{\infty} \sum_{\tau=-\infty}^{\infty} \phi(t-u,\tau)\mathbf{x}^{[i]}(t+\frac{\tau}{2})(\mathbf{x}^{[k]}(t-\frac{\tau}{2}))^H e^{-j2\pi f\tau}, \end{aligned} \quad (5)$$

where t and f are the time and the frequency variables, respectively, $\phi(t,\tau)$ is the time-frequency kernel, and superscript ^{H} denotes transpose conjugation. Each of i and k takes either value of the polarization index p or q.

The auto- and cross-polarized STFDs can be combined to form the following $2N \times 2N$ spatial polarization time-frequency distribution (SPTFD) matrix,

$$\begin{aligned} &\mathbf{D}_{\mathbf{xx}}(t,f) \\ &= \sum_{u=-\infty}^{\infty} \sum_{\tau=-\infty}^{\infty} \phi(t-u,\tau)\mathbf{x}(t+\frac{\tau}{2})\mathbf{x}^H(t-\frac{\tau}{2})e^{-j2\pi f\tau}. \end{aligned} \quad (6)$$

The noise elements are modeled as stationary and white complex Gaussian processes with zero mean and variance σ^2 in each polarzation, i.e.,

$$E\left[\mathbf{n}(t+\tau)\mathbf{n}^H(t)\right] = \sigma^2\delta(\tau)\mathbf{I}_{2N}, \quad (7)$$

where $\delta(\tau)$ is the Kronecker delta and $\mathbf{I}_{2N}$ denotes the $2N \times 2N$ identity matrix.

3. ARRAY AND POLARIZATION AVERAGING

Similar to the array averaging of the TFDs across the array sensors in [5], the joint array and polarization averaging of the TFDs is defined as the averaging of TFDs across all the array sensors and both polarizations. That is,

$$D(t,f) = \frac{1}{N}\sum_{n=1}^{N}\sum_{i=p}^{q} D_{x_n^{[i]}x_n^{[i]}}(t,f). \quad (8)$$

Without the noise, $D(t,f)$ becomes

$$\begin{aligned} &D(t,f) = \\ &\frac{1}{N}\sum_{n=1}^{N}\sum_{i=p}^{q}\sum_{l=1}^{L}\sum_{m=1}^{L} \left(v_l^{[i]}a_{n,l}^{[i]}\right)\left(v_m^{[i]}a_{n,m}^{[i]}\right)^* D_{s_l s_m}(t,f), \end{aligned} \quad (9)$$

where $D_{s_l s_m}(t,f)$ is the cross-TFD between $s_l(m)$ and $s_m(t)$. Denote

$$\begin{aligned} \mathbf{a}_l &= \left[\left(v_l^{[p]}\mathbf{a}_l^{[p]}\right)^T \quad \left(v_l^{[q]}\mathbf{a}_l^{[q]}\right)^T\right]^T \\ &= [v_l^{[p]}a_{1,l}^{[p]}, \ \cdots, v_l^{[p]}a_{N,l}^{[p]}, \ v_l^{[q]}a_{1,l}^{[q]}, \ \cdots, \ v_l^{[q]}a_{N,l}^{[q]}]^T \end{aligned} \quad (10)$$

as the joint spatial-polarization signature of the lth signal, $l = 1, \cdots, L$, and

$$\begin{aligned} \beta_{l,m} &= \frac{1}{N}\mathbf{a}_m^H\mathbf{a}_l = (v_m^{[p]})^* v_l^{[p]}\beta_{l,m}^{[p]} + (v_m^{[q]})^* v_l^{[q]}\beta_{l,m}^{[q]} \\ &= \frac{1}{N}\left((v_m^{[p]}\mathbf{a}_m^{[p]})^H(v_l^{[p]}\mathbf{a}_l^{[p]}) + (v_m^{[q]}\mathbf{a}_m^{[q]})^H(v_l^{[q]}\mathbf{a}_l^{[q]})\right) \\ &= \frac{1}{N}\sum_{n=1}^{N}\sum_{i=p}^{q}\left(v_l^{[i]}a_{n,l}^{[i]}\right)\left(v_m^{[i]}a_{n,m}^{[i]}\right)^* \end{aligned} \quad (11)$$

as the spatial-polarization correlation coefficient between sources l and m, where $\beta_{l,m}^{[i]} = (1/N)(\mathbf{a}_m^{[i]})^H\mathbf{a}_l^{[i]}$

is the spatial correlation between sources l and m at polarization i. Then, $D(t, f)$ can be expressed as

$$D(t,f) = \sum_{l=1}^{L} \sum_{m=1}^{L} \beta_{l,m} D_{s_l s_m}(t,f). \tag{12}$$

The above equation shows that $D(t, f)$ is a linear combination of the auto- and cross-polarization TFDs of all signal arrivals. It is straightforward to show that for the lth and the mth sources,

$$|\beta_{l,m}| \leq 1, \quad \text{if } l \neq m \quad \text{and} \quad \beta_{l,m} = 1, \quad \text{if } l = m, \tag{13}$$

indicating that the constant coefficients in (12) for TFDs with the same sensor and polarization are always greater than, or at least equal to, those for different sensors or polarizations. For sources with distinct polarizations or distinct spatial signatures, $|\beta_{l,m}| \ll 1$, leading to significant suppression of cross-terms, and thereby enhancing the signal signature estimation.

When the array response is the same for both orthogonal polarizations, then $\mathbf{a}_l^{[p]} = \mathbf{a}_l^{[q]}$. In this case, eqn. (11) can be simplified to

$$\beta_{l,m} = \left(\frac{1}{N} (\mathbf{a}_m^{[p]})^H \mathbf{a}_l^{[p]} \right) \left(\mathbf{v}_m^H \mathbf{v}_l \right) = \delta_{l,m} \alpha_{l,m}, \tag{14}$$

where $\delta_{l,m}$ and $\alpha_{l,m}$, respectively, represent the spatial and polarization correlation of sources l and m. A small value of $\beta_{l,m}$ can be realized with either large spatial diversity or large polarization diversity, or with moderate values of their respective correlations.

The t-f kernel in eqns. (4) and (5), which introduces temporal averaging of the local autocorrelation functions at consecutive time samples, can be selected to reduce the TFD noise effect for the single antenna case. The combined array and polarization averaging reduces the cross-terms and noise variance beyond that achieved in a single antenna (polarization) case without compromising the auto-term resolutions. Even without kernel smoothing, the averaging in eqn. (12) decreases the noise variance and its interaction with the signal components.

4. SIMULATION RESULTS

In this section, we provide computer simulations to demonstrate the improvement gained by the proposed technique in the reduction or elimination of cross-terms and signal synthesis. Two high-order frequency modulated signals are considered on a uniform linear array consisting of four dual-polarization dipoles. The length of the signal sequence is set to $N = 256$. The additive noise at different sensors and polarizations is uncorrelated, zero mean, Gaussian distributed, and white. The input signal-to-noise ratio (SNR) is 3 dB, and the interelement spacing is half wavelength.

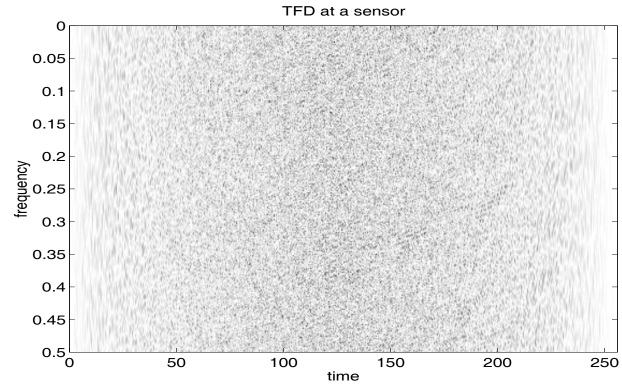

Fig. 1. EDTWVD computed from the signal received at the first vertical polarization antenna.

Fig. 1 shows the extended discrete-time Wigner-Ville distribution (EDTWVD) [6] of the data received at the vertically polarized antenna. With the presence of high-level noise and close t-f signatures, it is very difficult to identify these t-f signatures when only a single-polarization sensor is used.

Next, we consider the result when array and polarization averaging is applied. Three scenarios are considered. The first one represents a situation where the spatial correlation between the two signals is low (i.e., large spatial diversity), whereas in the second scenario the polarization correlation is low (i.e., large polarization diversity). In the third scenario, both spatial and polarization correlations asume moderate values. The parameters are illustrated in Table I. All of the above three scenarios show good cross-term reduction performance. As a result, the signatures of the two signals can be clearly identified and separated. Notice that the array averaging and polarization not only suppresses the cross-terms, but also reduce the noise variance and, as such the signal-to-noise ratio (SNR) is enhanced [5].

With the clearly separated time-frequency signatures of the two signals, we can proceed to mask and synthesize the signal waveforms [5]. With a mask applied to one signal and using standard signal synthesis techniques, a recovery of a high signal quality is expected. Fig. 5 shows the TFD of the synthesized signal waveform of the first signal, synthesized from the averaged TFD of Scenario C.

5. CONCLUSION

Polarization averaging of time-frequency distributions allows effective cross-term reduction and auto-term enhancement, aiding to source time-frequency signature estimations and waveform recovery. Averaging TFDs across polarizations can be performed concurrently with TFD averaging across the array, thereby utilizing both spatial and polarization diversities in syntheses of nonstationary signals. Both types of diversity appear as fractional product factors multiplying the TFD cross-terms. Significant reduction of cross-terms can then be realized by a large diversity of either type or moderate diversities of both. In particular, polarization averaging can be applied alone if the difference in the source spatial signatures is insufficient for cross-term reduction, or the receiver is not equipped with antenna arrays.

Table I. Signal Paramaters

Scenario	DOA (deg)	$\|\delta_{1,2}\|$	Γ (deg)	$\|\alpha_{1,2}\|$
Scenario A	-30, 30	0.000	30, 40	0.985
Scenario B	30, 40	0.879	-45, 45	0.000
Scenario C	30, 60	0.343	-5, 45	0.642

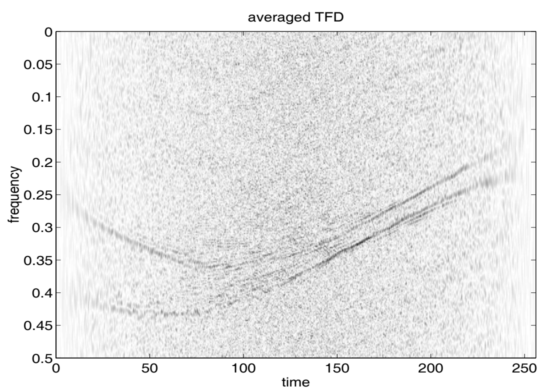

Fig. 2. EDTWVD averaged over sensors and polarizations (Scenario A).

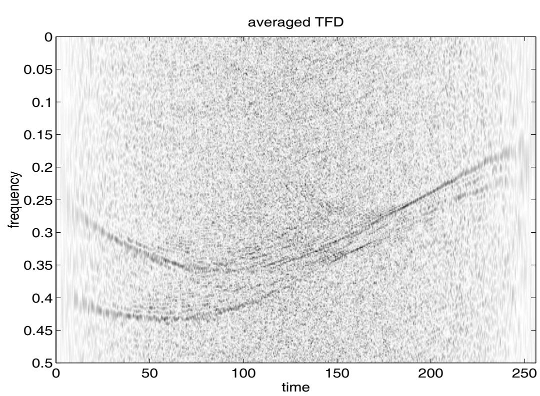

Fig. 3. EDTWVD averaged over sensors and polarizations (Scenario B).

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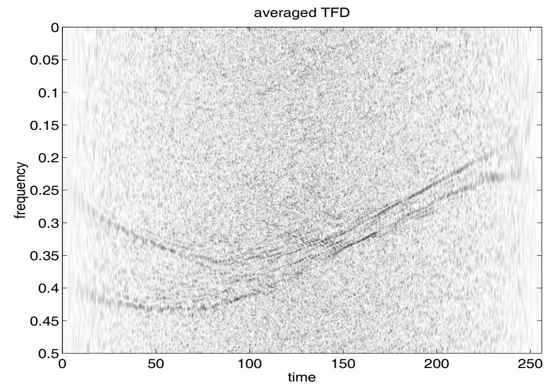

Fig. 4. EDTWVD averaged over sensors and polarizations (Scenario C).

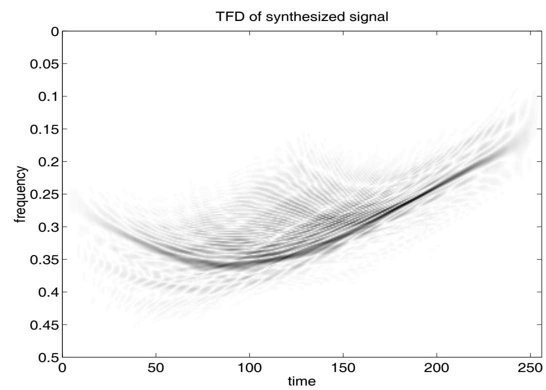

Fig. 5. EDTWVD of the synthesized waveform of the first signal.

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