A QUADRATIC MISO CONTRAST FUNCTION FOR BLIND EQUALIZATION

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ABSTRACT

This paper is concerned with blind separation of convolutive mixtures of mutually independent signals. We consider the MISO extraction of one source signal based on the maximization of a contrast function (CF): a new, so-called "reference" CF is proposed, which is based on cross-statistics between the estimated output and a reference signal. The proposed CF is valid both for i.i.d. and non i.i.d. sources. It presents the advantage over other CFs to be a quadratic function, which makes its optimization much easier to realize. Finally, simulations demonstrate the validity of this CF and show that it leads to improved separation performances.

1. INTRODUCTION

We consider the problem of blind equalization of Linear Time Invariant (LTI) systems (see e.g. [1, 2, 3]). Such a problem is of interest and it appears in its generic form in a wide variety of applications, e.g. array processing, passive sonar, seismic exploration, speech processing, multiuser wireless communications, ... In this latter area in particular, received signals have to be equalized, both in space and time in order to eliminate inter-symbol and co-channel interferences.

Basically, different problems can be considered depending on the characteristics of the input(s) signal(s) and on the number of inputs and outputs of the considered linear system. The classical problem considers a single i.i.d. input and a single output. Recently, more challenging problems have been considered where the number of inputs is greater than one and where the signals are not necessarily assumed i.i.d. [4]. In this paper, we consider a Multi-Input / Single-Output (MISO) system with possibly non i.i.d. input sources, even though we mainly focus our attention on the i.i.d. case.

Our approach relies on inverse filter criteria based on higher-order statistics, see e.g. [2, 5, 6]. We propose a new objective function which makes use of a so-called reference signal and we show that it is a contrast function (CF). Hence the problem can be solved by a (global) maximization of this function. The proposed objective function has the great advantage of depending quadratically on the searched parameters. This leads to a simplified optimization scheme, which thus significantly speeds up the source estimation. Moreover, computer simulations tend to show that the quality of the source estimation is improved in comparison with results provided by former CFs without reference.

2. BACKGROUND

2.1. Model

We consider the *invertible* and *stable* Linear and Time Invariant (LTI) multichannel system described by the input-output relation

$$\mathbf{x}(n) = \sum_{k\in\mathbb{Z}} \mathbf{M}(n-k)\mathbf{s}(k) \tag{1}$$

where $\mathbf{s}(n)$ is the $(N,1)$ vector of source signals, $\mathbf{x}(n)$ is the $(Q,1)$ vector of observations with $Q \geq N$ and $\{\mathbf{M}\} \stackrel{\text{def}}{=} \{\mathbf{M}(n), n\in\mathbb{Z}\}$ is a sequence of (Q,N) complex matrices which corresponds to the impulse response of the so-called LTI mixing system.

Furthermore, the following assumptions are made:
`A1`. The source signals $s_i(n)$, $i \in \{1,\dots,N\}$, are mutually statistically independent.
`A2`. Each source is a zero-mean, unit power and stationary complex random signal with a non-zero fourth order cumulant, *i.e.* for all $i \in \{1,\dots,N\}$

$$\mathrm{C}_4\{s_i\} \stackrel{\text{def}}{=} \mathrm{Cum}\{s_i(n), s_i^*(n), s_i(n), s_i^*(n)\} \neq 0 \,.$$

In addition, the covariance function of each source is a positive definite sequence which is denoted by $\gamma_i(k), k\in\mathbb{Z}$.

2.2. Problem

The considered problem consists in estimating a $(1,Q)$ vector of LTI filters (equalizer) $\{\mathbf{w}\} \stackrel{\text{def}}{=} \{\mathbf{w}(n), n\in\mathbb{Z}\}$ using only the outputs $\mathbf{x}(n)$ of the unknown LTI system $\{\mathbf{M}\}$ in such a way that the signal

$$y(n) = \sum_{k \in \mathbb{Z}} \mathbf{w}(n-k)\mathbf{x}(k) \qquad (2)$$

restores one of the source signals $s_i(n)$, $i \in \{1, \ldots, N\}$.

By defining the $(1, N)$ vector of the global LTI filter $\{\mathbf{g}\} \stackrel{\text{def}}{=} \{\mathbf{g}(n), n \in \mathbb{Z}\}$ as follows

$$\mathbf{g}(n) = \sum_{k \in \mathbb{Z}} \mathbf{w}(k)\mathbf{M}(n-k) \ , \qquad (3)$$

we have

$$y(n) = \sum_{k \in \mathbb{Z}} \mathbf{g}(n-k)\mathbf{s}(k) = \{\mathbf{g}\}\mathbf{s}(n) \ . \qquad (4)$$

Hence we say in general, that the equalization is realized when there exist an index $i_o \in \{1, \ldots, N\}$ and a non-zero scalar unit-norm filter with impulse response $g(n)$ such that the filter components in $\mathbf{g}(n)$ read

$$g_i(n) \stackrel{\text{def}}{=} (\mathbf{g}(n))_i = \alpha g(n)\delta_{i-i_o} \ , \qquad (5)$$

where $\alpha \in \mathbb{C}$ and δ_{i-i_o} stands for the Kronecker symbol, *i.e.* $\delta_{i-i_o} = 1$ if $i = i_o$ and 0 otherwise. The above relation is called the "equalization condition".

Notice that the above equalization criterion can be made more restrictive when the source signals are also assumed to be sequences of independent and identically distributed (i.i.d.) complex random variables. Indeed in such a case, it is classically said that the equalization is realized when the unit-norm scalar filter $g(n)$ reduces to a delay and hence reads $g(n) = \delta_{n-l}$, where $l \in \mathbb{Z}$.

2.3. Normalization

As the source signals are assumed non observable and the mixing system is unknown, we can always assume (without loss of generality) that the source signals are unit power, *i.e.* $\mathrm{E}\{|s_i[n]|^2\} = 1$, for all i. Since one can always work with a normalized output of the equalizer, this implies that the global filter is unit-norm in the sense that:

$$\|\mathbf{g}(n)\|^2 \stackrel{\text{def}}{=} \sum_{i=1}^{N} \sum_{(k_1,k_2) \in \mathbb{Z}^2} g_i(k_1) g_i^*(k_2) \gamma_i(k_2 - k_1) = 1 \ .$$

In the following we denote $\mathcal{G}_1$ the set of unit norm vector filters and $\mathcal{G}_{1e}^{i_o}$ the subset of filters in $\mathcal{G}_1$ satisfying the equalization condition (5). In such a case the complex number α in (5) has unit modulus. Finally we denote $\mathcal{G}_{1ed}^{i_o}$ the subset of $\mathcal{G}_{1e}^{i_o}$ when $g(n) = \delta_{n-l}$. The set $\mathcal{G}_{1e}^{i_o}$ (resp. $\mathcal{G}_{1ed}^{i_o}$) corresponds to the set of admissible equalization solution for any source signals (resp. for i.i.d. source signals).

3. EQUALIZATION CRITERION

3.1. Contrast functions

One of the most appealing approach to the blind equalization problem consists in the use of an appropriate CF. Basically, a CF plays the role of an objective function in the sense that its (global) maximization allows us to solve the problem. Hence the equalization issue becomes an optimization one. Besides, identifiability conditions are provided by the definition domain of the considered CF.

To address our MISO equalization problem, we introduce the following definition of a CF for i.i.d. source signals:

Definition 1 *Let $\mathcal{C}(.)$ be a real function of the signal $y(n)$ (and thus of the $(1, N)$ vector filter $\mathbf{g}$) as defined in (4). $\mathcal{C}(.)$ is called a CF when there exists $i_o \in \{1, \ldots, N\}$ such that:*

p1. *$\exists l \in \mathbb{Z}$ such that for all possible output $y(n)$ of the equalizer:*

$$\mathcal{C}(y(n)) \leq \mathcal{C}(s_{i_o}(n-l)) \qquad (6)$$

p2. *If equality holds in (6), then $\mathbf{g} \in \mathcal{G}_{1ed}^{i_o}$.*

The above definition cannot be used for non i.i.d. source signals since the independence property for such signals leads to the extraction of one source only up to a scalar filter. This is the reason why a generalization of the above definition for non i.i.d. source signals is needed and will be given in Section 4.

3.2. Reference signal

The main contribution of this paper is to consider another equalization vector filter, say $\{\mathbf{w}_r\} \stackrel{\text{def}}{=} \{\mathbf{w}_r(n), n \in \mathbb{Z}\}$, in such a way that its output

$$z(n) = \sum_{k \in \mathbb{Z}} \mathbf{w}_r(n-k)\mathbf{x}(k) \qquad (7)$$

serves to construct cross-statistics with the true equalizer output $y(n)$ in order to facilitate the equalization. It is the reason why it is called a reference signal subsequently.

We also need to define the new global reference vector filter $\{\mathbf{t}(n), n \in \mathbb{Z}\}$ as:

$$\mathbf{t}(n) = \sum_{k \in \mathbb{Z}} \mathbf{w}_r(k)\mathbf{M}(n-k) \ . \qquad (8)$$

3.3. A generalized criterion

The main purpose of this section is to propose a new CF. To this end, we need to introduce a technical assumption about the reference system:

T. $\exists l \in \mathbb{Z}$ such that $\forall (j,k) \in \{1, \ldots, N\} \times \mathbb{Z}$, we have $|t_j(k)| < |t_1(l)|$ when $k \neq l$ or $j \neq 1$.

Define the following function:

$$\mathcal{C}_z(y(n)) \stackrel{\text{def}}{=} |\kappa_4(y(n), z(n))| \tag{9}$$

where $\kappa_4(y(n), z(n)) \stackrel{\text{def}}{=} \text{Cum}\{y(n), y^*(n), z(n), z^*(n)\}$ and $z(n) = \{\mathbf{t}\}\mathbf{s}(n)$ is the reference signal. Notice that the function $\kappa_4(y(n), y(n))$, where the reference signal $z(n)$ is replaced by $y(n)$, was already shown to be a CF in [3] for the case of i.i.d. sources and in [7] for non i.i.d. sources.

Proposition 1 *If the reference system satisfies assumption* `T`*, the function $\mathcal{C}_z(y(n))$ defined in (9) is a CF for i.i.d. sources.*

Proof: Using multi-linearity of the cumulants and independence of the sources, we have:

$$\kappa_4(y(n), z(n)) = \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |g_j(k)|^2 |t_j(k)|^2 \text{C}_4\{s_j\} . \tag{10}$$

It follows:

$$\mathcal{C}_z(y(n)) \leq \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |g_j(k)|^2 |t_j(k)|^2 |\text{C}_4\{s_j\}| . \tag{11}$$

Assuming that $\max_{j=1}^{N} |\text{C}_4\{s_j\}| = |\text{C}_4\{s_1\}|$ (which can be done without loss of generality since the order of the sources is unknown and purely conventional), we obtain:

$$\mathcal{C}_z(y(n)) \leq |\text{C}_4\{s_1\}| \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |g_j(k)|^2 |t_j(k)|^2 . \tag{12}$$

According to assumption `T`, this yields:

$$\mathcal{C}_z(y(n)) \leq |\text{C}_4\{s_1\}| |t_1(l)|^2 \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |g_j(k)|^2 . \tag{13}$$

Now, as the global vector filter has a unit norm and the sources are i.i.d., we have $\sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |g_j(k)|^2 = 1$. In addition, since

$$t_1(l)^2 \text{C}_4\{s_1\} = \kappa_4(s_1(n-l), z(n)) , \tag{14}$$

we finally find

$$\mathcal{C}_z(y(n)) \leq \mathcal{C}_z(s_1(n-l)) . \tag{15}$$

This corresponds to property p1 of the definition of a CF.

Considering now the second property, if we have equality hereabove, then we have equalities in (11), (12) and (13). Focusing on the right hand side expressions of (12) and (13), we obtain:

$$\sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |g_j(k)|^2 |t_j(k)|^2 = |t_1(l)|^2 \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |g_j(k)|^2 \tag{16}$$

and thus:

$$\sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |g_j(k)|^2 \Big(|t_1(l)|^2 - |t_j(k)|^2 \Big) = 0 . \tag{17}$$

According to assumption `T`, we have $|t_1(l)|^2 - |t_j(k)|^2 \geq 0$, and we deduce that, for all $k \in \mathbb{Z}$ and $j \in \{1, \ldots, N\}$,

$$|g_j(k)|^2 \Big(|t_1(l)|^2 - |t_j(k)|^2 \Big) = 0 .$$

Then, for all $j \in \{1, \ldots, N\}$ and for all $k \in \mathbb{Z}$, we have: $|g_j(k)|^2 = 0$ or $|t_1(l)|^2 = |t_j(k)|^2$. Consequently, $|g_1(l)| = 1$ and for all other coefficient $(j,k) \neq (1,l)$, $g_j(k) = 0$. Finally, the equality holds in (15) only if $\mathbf{g} \in \mathcal{G}_{1ed}^{i_o}$. ■

4. GENERALIZATION TO THE CASE OF NON I.I.D. SOURCES

Let us now briefly consider the non i.i.d. case. Since blind MISO separation of non i.i.d. sources leaves a scalar filtering ambiguity, the basic idea consists in allowing the estimation of a non i.i.d. source signal up to any non-zero unit-norm scalar filter.

Definition 2 *The real function $\mathcal{C}(.)$ is called a CF when there exists $i_o \in \{1, \ldots, N\}$ such that:*

p1'. *For all possible output of the equalizer:*

$$\mathcal{C}(y(n)) \leq \sup_{\mathbf{g} \in \mathcal{G}_{1e}^{i_o}} \mathcal{C}(\{\mathbf{g}\}\mathbf{s}(n)) \tag{18}$$

p2'. *If equality holds in (18), then $\mathbf{g} \in \mathcal{G}_{1e}^{i_o}$.*

In addition, the technical assumption `T` has to be strengthened. Let us define the following upper-bound, which is assumed to be reached:

$$\forall i \in \{1, \ldots, N\} \quad \mathcal{M}_i \stackrel{\text{def}}{=} \sup_{\mathbf{g} \in \mathcal{G}_{1e}^{i}} \mathcal{C}_z(\{\mathbf{g}\}\mathbf{s}(n)) \tag{19}$$

Assumption `T` should then be replaced with:
`T'`. $\forall j \neq 1 \quad \mathcal{M}_j < \mathcal{M}_1$

The above assumption is not restrictive. In particular, we have been able to derive a sufficient condition for `T'`. Roughly speaking, this condition can be interpreted as follows: there exists in the reference system $\{\mathbf{t}\}$ a term $t_1(p^\sharp)$ which dominates all the other terms $(t_i(k))_{k \in \mathbb{Z}, 1 \leq i \leq N}$.

5. SIMULATIONS

5.1. Implementation of the algorithm

One of the main advantage of the proposed CF over former CFs such as the fourth-order auto-cumulant lies in the fact that its optimization can be carried out easily. Indeed, in [3] it is proposed to use an iterative, batch, steepest ascent method in order to maximize the CF. In general, this method converges slowly.

On the contrary, since the reference signal $z(n)$ is fixed, one can see that the CF (9) constitutes a quadratic function of the separating filter coefficients. Therefore, the nonlinear optimization problem reduces to the optimization of a quadratic function under a quadratic unit-norm constraint. This can be done making use of the singular value decomposition of a matrix containing cross-cumulants between the observations and the reference signals. An *exact* solution to the optimization problem can hence be obtained within a finite number of operations and an iterative method is not needed any more. This significantly simplifies the optimization task.

5.2. Simulation results

The CF (9) has been tested in various situations. The mixing system has been chosen to be a MIMO FIR filter with more sensors than sources, so as to be sure that it admits a FIR inverse filter. There were either 2 sources and 3 sensors (Fig. 1) or 3 sources and 4 sensors (Fig. 1, 2). The length of the mixing filter was set to 3. The sources were i.i.d. sources, either centered and uniformly distributed or PAM4.

We have first tested the CF with an ideal reference system (*i.e.* the reference signal was one of the sources) and we have observed for sample sizes ranging from 1000 to 10000 points that the optimization of our CF leads to a good estimation of the sources with a quite small mean square error (Fig. 1).

In a practical situation however, one would obviously not know the sources and a first reference signal has to be estimated. We propose to this end to use the MISO source extraction method proposed in [3, 7], which consists in the gradient maximization of $|\kappa_4(y(n), y(n))|$. We then used the resulting signal as a reference and optimized our criterion. As demonstrated by Fig. 2, the use of our CF improves the quality of the separation.

Another possibility consists in stopping the gradient algorithm before it has converged and use the approximate reference to build our CF. We have observed that this solution improves significantly the speed of the source separation and that the quality of the results is not altered in comparison with those in Fig. 2. Finally, note that reference CFs can be useful in semi-blind approaches.

6. REFERENCES

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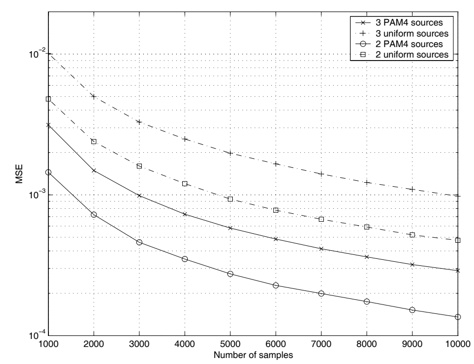

Fig. 1. Average MSE on reconstructed sources versus number of samples for PAM 4 and uniformly distributed i.i.d. sources.

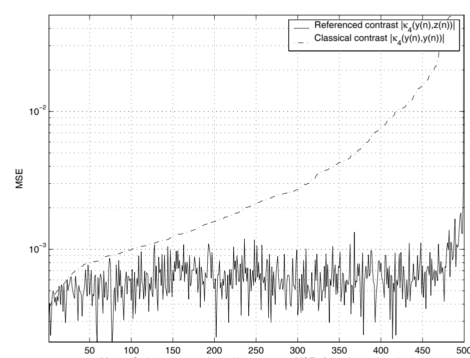

Fig. 2. Average MSE on reconstructed sources (PAM 4 sources, 5000 samples). Comparison on 500 Monte-Carlo realizations of the referenced CF and the CF $|\kappa_4(y(n), y(n))|$. The data have been ordered according to the increasing MSE values for this latter CF.